

Reaching the Standard in Mathematics at KS4:
Improving outcomes in mathematics at county and district level

The Starting Point:

In 2016, the LA subject residual for mathematics was +1.30. In 2017, this fell to -0.05. At face value, this means that students performed, on average, one grade above their other subjects in mathematics in 2016 and achieved the same grade in 2017. However, in 2016 75.3% of students in Hampshire achieved a C grade or better in mathematics compared to 75.5% achieving a grade 4 or better with the new grading system for 2017, i.e. students performed to a similar standard in 2016 and 2017. Of course, the new grading system has 9 points, compared to the A*-G system of 8 points. This means that residual figures have been adjusted to account for this and are less comparable and reliable this year. The 9 point system does, however, enable greater differentiation of the most able mathematicians and some fine tuning at the pass/fail boundary. A grade 4 is a GCSE 'standard' pass in mathematics, comparable to the lower end of the old-style grade C. A grade 5 is a GCSE 'strong' pass in mathematics, comparable to the standard achieved in HPJs (high performing jurisdictions such as Singapore and Shanghai). Both 4+ and 5+ results will be published at school level by the DfE. In 2017, 53.1% of students in Hampshire achieved a grade 5 or better.

From a cohort of 12476 entries, 9419 students achieved a grade 4 or better and 6630 achieved a grade 5 or better, indicating 2789 students achieved a grade 4 (22%). In addition to this 1396 students achieved a grade 3 (11%).

There are two initial challenges:

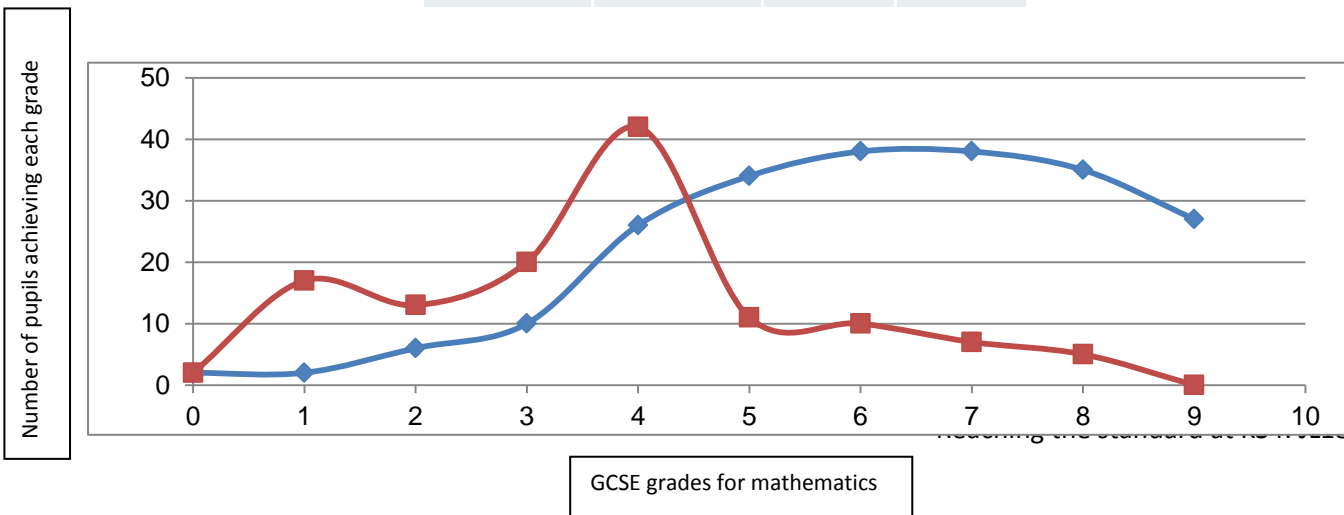
1. How do we enable the 11% at the grade 3 level to access grade 4 success, since this is the door opener to financial viability in adult life and
2. How do we convert the 22% who are grade 4 pupils into grade 5 'strong' passes, since this enables students to access a greater range of post-16 courses from a more secure base. This would also mean that figures in Hampshire for grade 5+ would be comparable with previous C+ figures.

The Issues:

What happens in schools that had strong residuals and attainment outcomes in 2017 in mathematics?

Consider the following two schools:

| School | Residual | % 4+ | % 5+ |
|--------|----------|-------|-------|
| Red | -0.55 | 57.7% | 25.4% |
| Blue | +0.37 | 90.8% | 78.9% |



We can see that the red school in negative residual with a grade 4+ achievement of 57.7 % (17.8% behind the Hampshire average) has a high proportion of grade 3 and grade 4 students, compared to the blue school with a positive residual and a grade 4+ figure that exceeds the LA average by 15.3%. The crossover comes at grade 5 (the strong pass), when the blue school almost levels out across the higher grades. Meanwhile, the red school drops drastically and continues to fall away with no student achieving the top grade 9.

Of course, there are many reasons for these results, including:

- Geographical location
- Proportion of disadvantaged groups in respective cohorts
- Level of staff turnover
- Expertise of maths teachers
- Proportion of non-specialists teaching in the department.
- Quality of leadership at department and school level
- Stability of the department (linked to turn-over, leadership, budget etc.)

There are other considerations, given the new grading system and the changes in the national curriculum that affect the teaching and learning of mathematics:

- Has the scheme of work been updated to take account of a different approach to teaching, learning and assessing mathematics from Y7 to Y11?
- Does the department have regular (weekly) time to engage in their own CPD in terms of actually doing some mathematics?
- Is there any cross-institution involvement to support changes in thinking, such as attendance at network meetings or local cluster groups?
- Who supports non-specialist teachers in the department? Have they been left behind because their priorities are elsewhere in the school?
- Does the KS3 delivered curriculum take account of formative assessment and an appropriate balance of problem solving and reasoning to better prepare students for the new style GCSE papers?
- What is the most appropriate tier of entry for students? Have some schools been more savvy than others?

The secondary maths team has found the following issues are often present in supported schools:

- Teachers feel pressured to deliver the GCSE specification in a modular way, with unit assessments as a written test. This can mean that teachers default to teaching in a procedural way in the hope that students perform well in each test. When students do not do so well, this is often qualified with students having had a 'poor year' somewhere within their secondary journey due to weak teaching. Teachers believed that the gaps in knowledge exhibited by students could only be addressed with short term fixes (learn the procedure) by the time they arrived in KS4.
- Often due to the pressure to produce results, lower attaining students can be expected to access end of Y10/11 concepts during the autumn term. This can lead to confusion for the students when asked to apply their knowledge to non-routine problems. This has been observed in both mixed attainment KS3 classes and in ability sets.
- Formative teacher assessment does not match the test outcomes. Where this was the case, books were reviewed. It was found that tests were often a feature of ongoing teacher assessment and evidence of formative assessment in the form of short tasks or annotated pupil work was minimal. Most departments follow a GCSE specification scheme of work,

which includes some sort of testing regime. Teachers feeling pressured to make positive predictions regarding student's end of KS4 outcomes find themselves teaching to a unit test to ensure success at that point. This procedural approach does not embed concepts in long term memory and students do not make links across mathematics to enable them to use skills in a range of problems at a later date.

- A lack of breadth and depth is often evident in student's books. These books are usually characterised by a large number of calculations with little exploration or diagrammatic representations to expose structure. This could be due to some missing subject knowledge on the part of the teachers.
- As in all secondary mathematics departments, a disparity in quality of teaching is often a feature. Department residuals in supported schools sometimes revealed one particular group performing less well than their peers compared to the other grades they achieved. A strong head of department, well supported by senior management, is needed in this case, to address quality of teaching and performance, appropriate CPD and robust monitoring. Department leaders often need to make best-fit decisions as to how best to allocated their teachers, which can lead to the best mathematicians teaching the best behaved 'top sets' and non-specialists with good behaviour management skills teaching the students who would most benefit from a teacher with a secure understanding of the subject.
- Silo teaching: It was noted that the GCSE curriculum was often delivered in silos. Teachers react to the topic heading and the sample GCSE questions, delivering them individually, rather than making rich connections across mathematics. This led to grade 3/4 students failing to see links. Consequently, they were unable to be flexible and show fluency during the exam and were less successful on papers 2 and 3, the problem solving and reasoning papers. This is linked to procedural teaching.
- Initial Paper 1 analysis identified inaccurate calculation due to a bad choice of method and a lack of fluency with fractions as the problem areas for last year's cohort. All error analysis (mainly from discussions at network groups) pointed to a lack of understanding of multiplicative relationships with too much drill and practice and insufficient insightful models and images. Examiner's reports will reveal further areas for development.
- The mastery approach, developing the philosophy of a conceptual approach in tandem with an attention to mathematical procedures and methods, is not yet part of a general secondary approach. This approach is behind the construction of the 2014 national curriculum and associated assessment points. It needs some different pedagogical approaches in KS3 and KS4. Discussions with teachers showed that the CPA approach (concrete, pictorial, abstract) is seen as and is often presented as a hierarchical system rather than an opportunity to provide a conceptual insight. This means that the concrete, and sometimes visual, representations are rarely utilised and so revealing structure to enable reasoning is not possible.
- In many cases marking and feedback policies in schools were ineffective in that students did not value them. When teachers gave written feedback that included challenging questions, students who did not reach the standard often did not appear to have responded or reacted. There was a sense that marking was 'done' rather than seeing feedback as a learning dialogue between teacher and pupil. For grade 3/4 students, the impact of such marking appeared to be minimal on the next steps in learning.

The Fixes:

How do we achieve success and access for all?

Reviewing best practice in schools with a high percentage of students achieving grade 5+, alongside current research, the following suggested actions are offered for consideration:

Assessment, marking and feedback, including questioning:

- Use assessment to build on students' existing knowledge and understanding. Teachers may need to try a different approach if what they did first time did not work. This assessment is not formal but is formative and 'organic', in that it is a direct response to what students are doing in the lesson.
- Use assessment of students' strengths and weaknesses to inform selection and use of tasks. Tasks and resources are tools which need to be deployed effectively to have a positive impact on learning.
- If a student has not understood the concept in the lesson, a different approach, model or image is likely to be required. Interventions need to be anticipated by the teacher so that they enable students to be ready for the next lesson. This is the idea of 'keep-up not catch-up'.
- Offer same day, in class, feedback. Effective feedback and responses will enable teachers to have an intimate and immediate insight into what students know and understand. Written feedback at point of instruction should add value to a learning conversation and has more impact and value than summative, end of day, marking. Always model responses for students to show them how to respond. Schools should be careful to avoid onerous marking policies that detract from adding value to the learning conversation between teacher and pupil.
- Address misconceptions immediately. A misconception is an understanding that leads to a 'systematic pattern of errors'. For example, multiplying by 10 can be done quickly by adding a zero (is this true for 0.5×10 ?). Teacher subject knowledge needs to be sufficiently robust to ensure that common misconceptions are planned for and are not sidestepped or ignored.
- Do the mathematics! Whenever a lesson is planned, or a task designed, teachers should rehearse the mathematics to ensure that it works and that they understand the mathematical structure that underpins their lesson content. This preparation will enable teachers to use time in lessons that was previously spent checking answers or creating a new challenge for the 'speedy' pupil to ask insightful questions. Questioning in mathematics needs to start a learning conversation.

The use of manipulatives and different representations

- Teachers should appreciate that manipulatives and representations are tools that reveal structure. They should have a clear rationale for their choice of representation to teach a specific mathematical concept.
- Manipulatives should be used to provide insights into increasingly sophisticated mathematics. Teachers should enable students to see links between the resource and the ideas that they represent.
- Use a number line. Evidence indicates that number lines are a particularly effective representation for a wide range of ideas in mathematics.
- Use arrays (open and closed). The use of diagrams that reveal structure support students to make links across multiplicative relationships and also to model the laws of arithmetic (commutative, associative and distributive).

Problem Solving

- Teach students strategies for problem solving. These are general strategies that can be applied to solving a variety of different problems such as *identify a simpler problem first*, *make a table/ be systematic* or *trial and improvement*.
- Organise teaching so that problems with *similar* structures and *different* contexts are presented together to allow students to develop strategies and recognise types of problems.
- Encourage students to use visual representations. Help students make use of appropriate diagrams to provide insight into the structure of a problem (bar models are a good example here).
- Develop evaluative thinking. Expect students to ask questions such as, “What am I trying to work out?”, “How am I going to go about it?”, “Have I seen a problem like this one before?” and “What approaches could I try?”

Mathematical Networks:

- Teaching should always emphasise the many connections between different mathematical facts, procedures and concepts.
- Planning a new unit of work should be done together as a department. It should include an element of brainstorming to ensure that teachers are able to see the rich mathematical connections before teaching. For example, doubling and halving is linked to other multiples, factors and fractions. Fluency with doubling and halving enables access to other multiplication and associated division facts.
- Ensure that students develop fluent recall of number facts by making connections with other areas of mathematics. Time spent on multiple representations and images enables students with a range of learning preferences to commit number facts to long term memory.
- Ensure that students know and understand that fractions and decimals extend the number system beyond whole numbers. Exploring the idea of ‘part / whole’ for calculations and for discrete numbers should be a continual part of the mathematical experience for all students.
- Teach students to understand procedures. For example the decomposition method of subtraction is only possible if one can partition numbers in different ways and appreciate that “one of these is worth ten of these”. For some students, it is best understood by modelling the calculation with base ten materials.

Independence and Motivation:

- Support students to develop metacognition (the ability to independently plan, monitor and evaluate their own thinking and learning). This is **not** ticks against predetermined success criteria. It can be done through encouraging students to explain their thinking to other students and through providing appropriate models and images to facilitate that evaluative thinking. Students should be encouraged to ask themselves whether or not their answer makes sense, could they show it in a different way, do they know good strategy to try, have they seen a problem like this one before and so on.
- Collaborative working. Pupil independence is developed when they feel safe to learn. This is often when working in pairs or small groups to agree a solution or strategy. Students will need teachers to model effective discussion and dialogue.
- Give students time. Regardless of the concept being taught, students need significant time to imitate, internalise and independently apply strategies. **Fewer things in greater depth.**
- **Teachers should behave like mathematicians.** Placing value on the subject, its rich connections and processes changes attitudes. A ‘can do’, growth mind-set approach opens up new possibilities for a learner.

Interventions:

- Use **structured** interventions to provide additional support. These should happen early to reduce pupil's anxiety and ensure that misconceptions do not become entrenched.
- Interventions should include explicit and systematic teaching. If this is not delivered by a teacher, then it should be planned and monitored by the teacher.
- Ensure that connections are made between any intervention and whole-class instruction. Interventions should work alongside the main lesson.
- Interventions should motivate students. They must add value to learning so that the pupil feels supported and able to return to the main group.

In summary:

The recommendations made in this document are not intended to be a 'fix-all'. Departments should select areas to work on as required, some of which are listed here. It is a well-known adage that if we want to change something, we should do something differently. The same is true of teaching and learning in mathematics. As teachers of mathematics, we should always ask ourselves the question, "How do we ensure access and success for all?" This in turn should encourage the planning, rehearsing and teaching of a wide range of carefully considered approaches that pays due attention to the many different learning preferences in our classrooms. The idea of fewer things in greater depth to encourage metacognition, leading to independence and motivation is a key factor for many students. It is hoped that this document provides some ideas and starting points for professional dialogue in departments. In addition to this, it is hoped that it supports the development of consistently excellent, well-thought through maths teaching that creates positive experiences and excellent outcomes for all students at the end of KS4.

References:

Improving Reasoning: Analysing Alternative Approaches: Malcolm Swan

'You Cubed': Changing Mathematical Mindsets: Jo Boaler

Improving Mathematics in Key Stages Two and Three: Education Endowment Foundation(2017)

Key understandings in mathematics learning: Nunes, Bryant, Watson (2009)