**Problem of the Week: Week 6 (Sum2): Year 9:** Number: Inters, powers and roots

- Use integer powers and roots to solve problems

**Power Mad**

Powers of numbers behave in surprising ways. Take a look at the following and try to explain what's going on:

Work out $2^1, 2^2, 2^3, 2^4, 2^5, 2^6 \ldots$
**For which values of $n$ will $2^n$ be a multiple of $10$?**

**For which values of $n$ is $1^n + 2^n + 3^n$ even?**

Work out $1^n + 2^n + 3^n + 4^n$ for some different values of $n$.
**What do you notice?**

**What about $1^n + 2^n + 3^n + 4^n + 5^n$?**

**What other surprising results can you find?**

Here are some suggestions to start you off:

$4^n + 5^n + 6^n$
$2^n + 3^n$ for odd values of $n$
$3^n + 8^n$
$2^n + 4^n + 6^n$
$3^n + 5^n + 7^n$
$3^n - 2^n$
$7^n + 5^n - 3^n$

**Can you justify your findings?**

**https://nrich.maths.org/6401**

**Possible solutions:**

Multiple of 10:

| Power of 2 | Answer | Units Digit |
|---|---|---|
| 1 | 2 | 2 |
| 2 | 4 | 4 |
| 3 | 8 | 8 |
| 4 | 16 | 6 |
| 5 | 32 | 2 |
| 6 | 64 | 4 |
| 7 | 128 | 8 |
| 8 | 256 | 6 |

For which values of n will $2^n$ be a multiple of 10?

As can be seen from the following table, the unit digits of the powers of two are in a repetitious pattern of 2, 4, 8, 6, 2, 4, 8, 6…

All multiples of 10 have a unit digit of 0. However, as seen from the pattern, none of the powers of 2 have their unit digit ending in a 0. Therefore, no power of 2 is a multiple of 10.

Even:

The unit digits of $2^n$ for n=1,2,3… are 2, 4, 8, 6 then repeat. For $3^n$ they go 3, 9, 7, 1 then repeat. If n is odd, the units that are being added are either 2 + 3 or 7 + 8, which both end in 5. So $2^n + 3^n$ where n is odd always ends in 5. This is a stronger conclusion than saying 'it's a multiple of 5' as a multiple of 5 can also end in 0.

If n is a multiple of 4 the units being added are 6 + 1 so in this case it will always end in 7.

$1^n + 2^n + 3^n$ is even for all values of n. This is obvious because $1^n$ and $3^n$ are always odd and $2^n$ is always even, and odd + odd + even =even.

Surprising results:

If the power n in $4^n + 5^n + 6^n$ is an odd number, then the last digit of this sum is 5; if the power n is even then the last digit of the sum is 7.

$3^n + 8^n$: For consecutive values of n (n = 1, 2, 3, …) the last digit of the sum goes in a pattern of 1, 3, 9, 7.

$2^n + 4^n + 6^n$: For consecutive values of n (n = 1, 2, 3, …) the last digit of the sum goes in a pattern of 2, 6, 8, 8.

$3^n + 5^n + 7^n$: For consecutive values of n (n = 1, 2, 3, …) the last digit of the sum goes in a pattern of 5, 3, 5, 7.

$3^n - 2^n$: For consecutive values of n (n = 1, 2, 3, …) the last digit of the sum goes in a pattern of 1, 5, 9, 5.

**Powerful order**

List the following three numbers in increasing order:

$2^{25}$ $8^8$ $3^{11}$ **https://nrich.maths.org/7153**

**Possible solution**

We have $8^8 = (2^3)^8 = 2^{24} = (2^2)^{12} = 4^{12} > 3^{11}$
So $3^{11} < 8^8$

Also $2^{24} < 2^{25}$ so $8^8 < 2^{25}$

So the order is $3^{11} < 8^8 < 2^{25}$

**Roots near 9**

Given that $n$ is an integer, and the difference between $\sqrt{n}$ and $9$ is less than $1$, how many different possibilities are there for $n$?

**https://nrich.maths.org/13713**

**Possible solution**

**Answer**: $35$ numbers

$\sqrt{n}$ is less than $1$ from $9$
$\sqrt{n}$ is between $8$ and $10$
$n$ is between $64$ and $100$ (but not $64$ or $100$)

$$65, 66, 67, 68, \ldots, 99$$

$$\underbrace{\underbrace{1, 2, 3, \ldots, 64}_{\text{64 numbers}}, 65, 66, 67, 68, \ldots, 99}_{\text{99 numbers}}$$

$99 - 64 = 35$