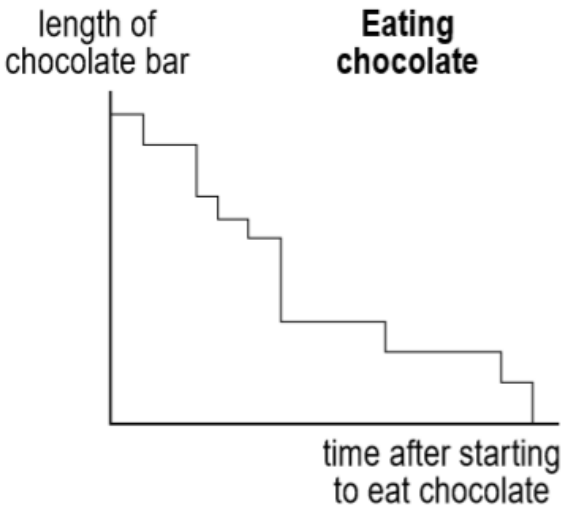
Problem of the Week: Week 2 (Summer 2): Year 8: Algebra: different graphs

- Explore cubic, exponential, reciprocal and piece-wise linear graphs.
- Find approximate solutions to contextual problems using these graphs

Problem 1

Give reasons why the graph is this shape



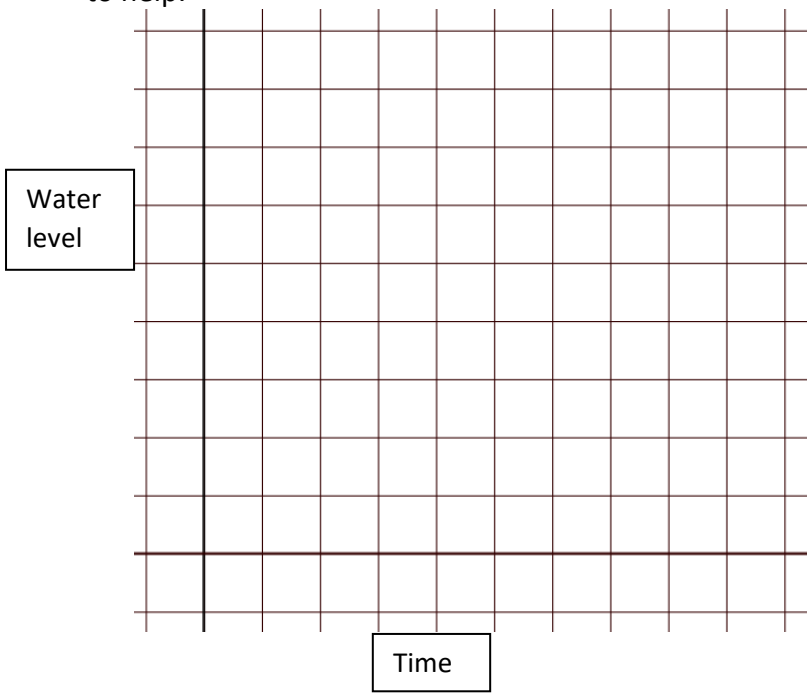
Framework for teaching mathematics: Years 7, 8 and 9 .

Solution

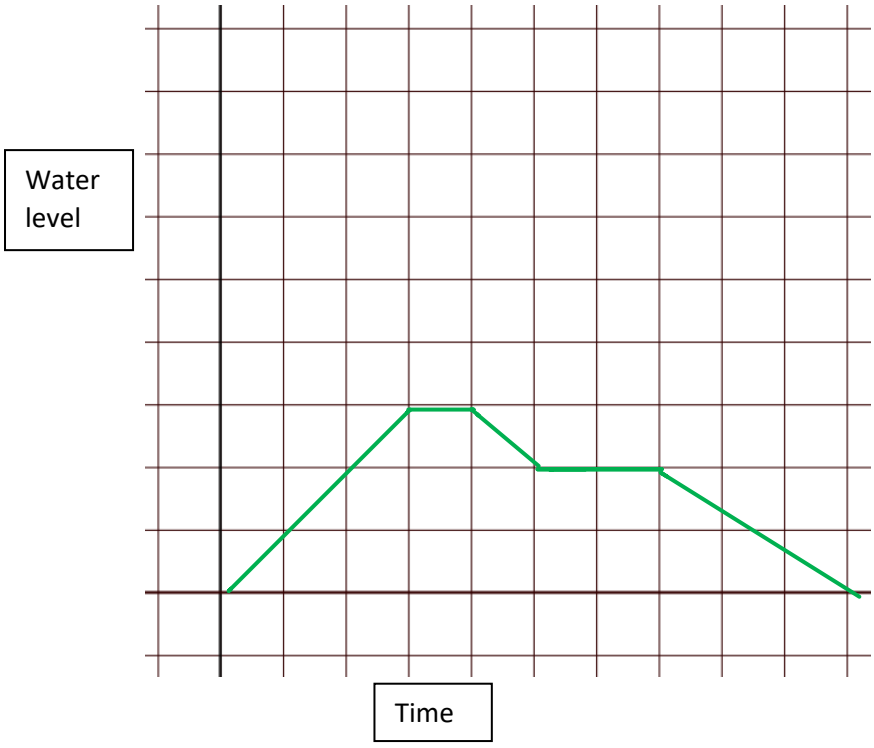
The horizontal lines on the graph show when the person has stopped biting chunks off the chocolate bar, this is probably when they are chewing it. The vertical lines show the chocolate bar getting shorter, about half way through eating the person bites a large part of the chocolate bar off.

Problem 2

Draw a graph to show the water level of a bath, from putting the plug in to fill it, having a bath and letting the plug out at the end. Give reasons for the shape of the graph. Use the axes below to help.



Solution



This graph shows the bath is empty to begin with and then fills at a constant rate, the taps are then turned off and the water level remains the same for a short period of time. A small amount of water is let out of the bath, the water level then stays the same, until finally the plug is taken out and the bath is emptied.

Problem 3

$$y = \frac{2}{x} \qquad y = \frac{x}{2} \qquad y = \frac{5}{x} \qquad y = \frac{x}{3}$$

What is the same and what is different with these equations?
Sort them into two groups.
Make up two more equations to go in each group
Explore the graphs of each group, what do you notice? (geogebra can be used to explore the graphs: https://www.geogebra.org/graphing it is a free download)

Solution
Group 1:

$$y = \frac{x}{2} \qquad y = \frac{x}{3} \qquad y = \frac{x}{5} \qquad y = \frac{x}{27}$$

These both have a number as the denominator and x as the numerator. The graphs will be a straight line as the equations are equivalent to:

$$y = \frac{1}{2}x \qquad y = \frac{1}{3}x$$

which are in the form y=mx, the gradient is less than one, which gives a graph that is less steep than y=x

Group 2:

$$y = \frac{2}{x} \qquad y = \frac{5}{x} \qquad y = \frac{10}{x} \qquad y = \frac{50}{x}$$

These both have a number as the numerator and x as the denominator. The graphs are of the form:

$$y = 2\left(\frac{1}{x}\right) \qquad y = 3\left(\frac{1}{x}\right)$$

This means that the graphs have asymptotes.

Problem 4

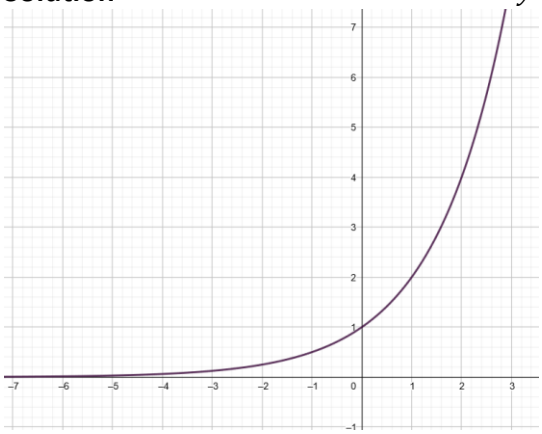
$y = 2^x$ $y = \frac{2}{x}$

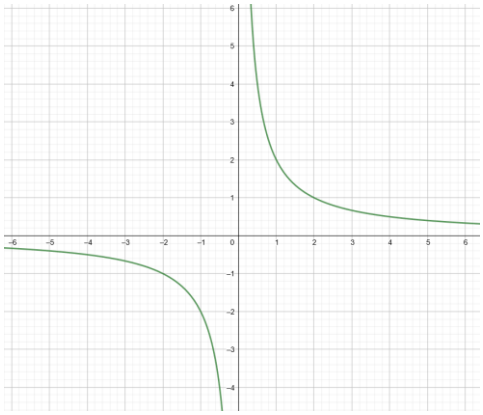
Use a graphing tool to draw these graphs (geogebra can be used)
What is the same and what is different with the two graphs?
Using what you have noticed, sketch graphs of the following:

$y = 3^x$ $y = \frac{3}{x}$

Solution

$y = 2^x$





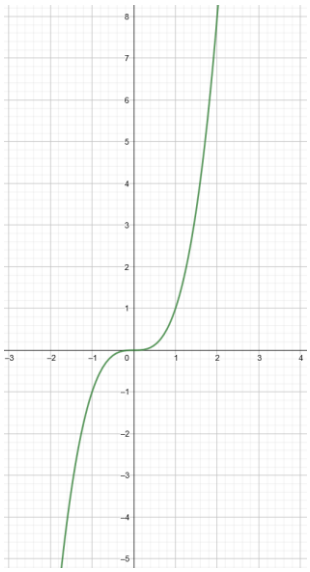
$y = \frac{2}{x}$

The graphs both tend towards the y-axis.

$y = \frac{2}{x}$ also tends towards the x-axis. The graph $y = 2^x$ cuts the y-axis at (0,1) and goes through the point (1,2). The graph $y = \frac{2}{x}$ does not cross either axis, and goes through the points (2,1) and (-2,-1).

Problem 5

Describe this graph, its is the graph of $y = x^3$



Solution
The graph is an 'S' shape.
As the value of x increases the value of y increases very quickly, this is also true as the negative value of x increases.
The y values change from negative to positive values at the point (0,0)