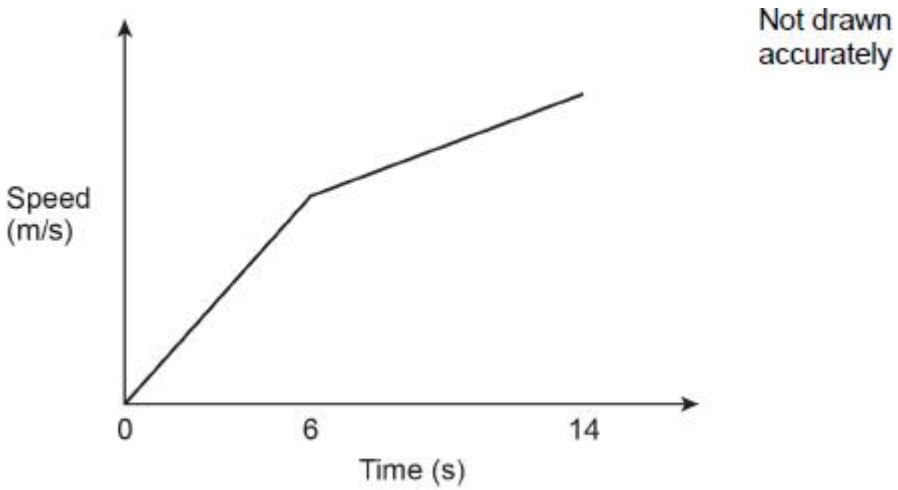
Problem of the Week: Week 7 (Sum2): Year 10: Algebra: Interpreting graphs: Solutions

- plot and interpret graphs (including reciprocal graphs **{and exponential graphs}**) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- **{calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts}**

Lucy's Race

Lucy runs an 80-metre race in 14 seconds.
During the first 6 seconds her speed increases at a constant rate.
During the last 8 seconds her speed increases at a different constant rate.
Her speed at 14 seconds is 2 m/s more than her speed at 6 seconds.

Here is a sketch of her speed-time graph.



Work out her acceleration during the last 8 seconds.
State the units of your answer.

Acceleration = the gradient of the line between 6 seconds and 14 seconds

Acceleration = change in speed ÷ change in time

= 2 ÷ 8 = **0.25 m/s^2**

Exponentials: Solution

Complete the table below for $y = 2^x$ for $-5 \leq x \leq 5$

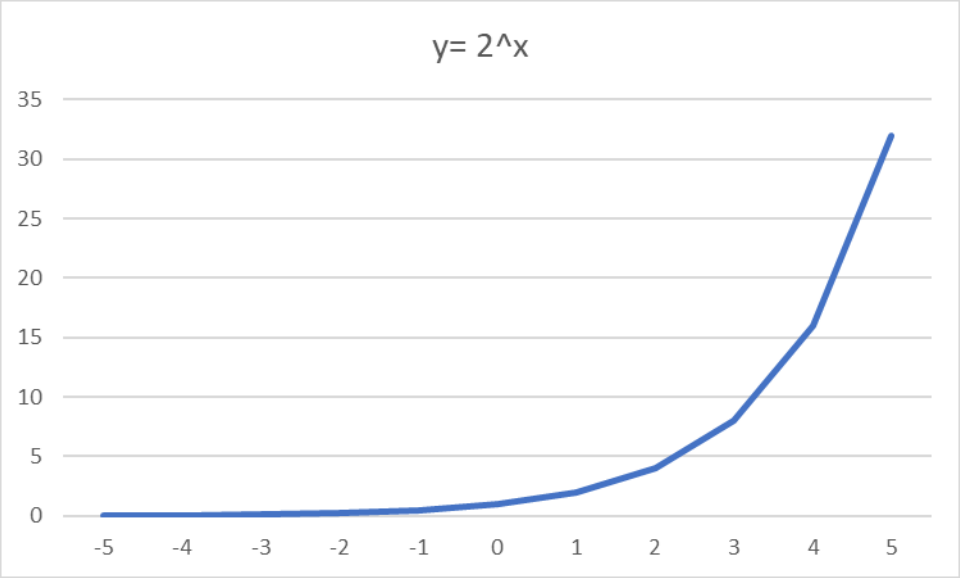
x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$y= 2^x$		0.0625	0.125		0.5	1	2	4			32

Plot the graph of $y = 2^x$ for $-5 \leq x \leq 5$

Explain what happens as x gets far away from zero in the positive and the negative directions

Solution

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$y= 2^x$	0.03125	0.0625	0.125	0.25	0.5	1	2	4	8	16	32



(Note 2^x is the same as 2^x)

As x gets further away from zero in the positive direction, the value of y increases exponentially

(gets very large)

As x gets further away from zero in the negative directions, the value of y approaches, but never gets to, zero

(gets very small but never negative)

Note: Exponential functions can be used to model social, scientific, or personal finance situations. Population growth, radioactive decay, and loan interest rates are a few examples of naturally occurring exponential relationships. Learn how to model these situations using an exponential function to predict behavior, calculate half-life, or plan your budget!