Problem of the Week: Week 6 (Sum2): Year 10: Number: Integers, powers and roots: Solutions

- **{estimate powers and roots of any given positive number}**
- calculate with roots, and with integer **{and fractional}** indices
- calculate exactly with fractions, **{surds}** and multiples of π; **{simplify surd expressions involving squares [for example** $\sqrt{12}=\sqrt{4\times3}=\sqrt{4}\times\sqrt{3}=2\sqrt{3}$ **] and rationalise denominators}**

Estimating roots and powers

$\sqrt{225} = 15$ {since $15^2 = 15 \times 15 = 225$}. We can also write this as $225^{1/2} = 15$

$\sqrt[3]{27} = 3$ {since $3^3 = 3 \times 3 \times 3 = 27$}. We can also write this as $27^{1/3} = 3$

a) Estimate the value of $\sqrt{82}$ using a known square number.
$\sqrt{81} = 9$ so **$\sqrt{82} \approx 9.1$**
To check : 9.1 x 9.1 = 82.81 , which is a bit over
The actual solution is 9.055……. (to four s.f)

b) Estimate the value of 8.2^4
$8.2 \approx 8$
$8^4 = 8 \times 8 \times 8 \times 8$
$= 64 \times 64$
$\approx 60 \times 70$
= 4200
(actual value is 4521 to 4 s.f.)

c) Estimate the cube root of 3250
$3250 = 3.25 \times 1000$
$\sqrt[3]{1000} = 10$
$\sqrt[3]{1} = 1$
$\sqrt[3]{8} = 2$
$\sqrt[3]{3.25}$ is between 1 and 2 , but slightly nearer to 1 than 2
$\sqrt[3]{3.25} \approx 1.4$
$\sqrt[3]{3250} = \sqrt[3]{3.25} \times \sqrt[3]{1000}$
$\approx 1.4 \times 10$
=14
(actual value is 14.8 to 3 s.f)

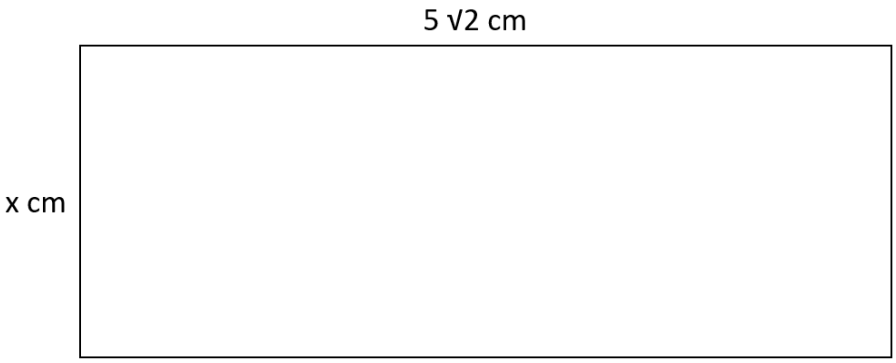
d) Estimate the value of $\sqrt{820,000}$
$\sqrt{820,000} = \sqrt{82} \times \sqrt{10,000}$
$\sqrt{82} \approx 9.1$ (from part (a))
$\sqrt{10,000} = 100$
$\sqrt{820,000} \approx 9.1 \times 100$
= 910
(actual value is 905.5 to 4 s.f)

Surd Area

The area of this rectangle is 60 cm^2

Find the value of x

Give your answer in the form a√b where a and b are integers



Solution

Area = length x width

Area = 60

Area = 5√2 x

5√2 x = 60

x = 60 / (5√2)

$= \frac{60}{5} \times \frac{1}{\sqrt{2}}$ rationalise the denominator and divide 60 by 5

$= 12 \times \frac{\sqrt{2}}{2}$ divide 12 by 2

= 6√2 so a is 6 and b is 2