Problem of the Week: Week 5 (Sum2): Year 10: Algebra: Functions

- where appropriate, interpret simple expressions as functions with inputs and outputs; **{interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'}**
- simplify and manipulate algebraic expressions (including those involving surds **{and algebraic fractions}**) by:
  - factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares**; {factorising quadratic expressions of the form $ax^2 + bx + c$ }**
  - simplifying expressions involving sums, products and powers, including the laws of indices

Warm up

Functions

$f(x) = 2x^2 + 7$

What is the value of:

a) f(3)
b) f(-10)
c) f (1/2)
d) f (√3)

Solutions

a) f(3) = 2 x 3 x 3 + 7 = **25**
b) f(-10) = 2 x -10 x -10 +7 = **207**
c) f(1/2) = 2 x ½ x ½ +7 = **7 ½**
d) f(√3) = 2 x √3 x √3 +7 = 2 x 3 +7 = **13**

Challenge

Composite functions

$f(x) = 2x + c$

$g(x) = cx + 5$

$fg(x) = 6x + d$

c and d are constants.

Work out the value of d.

Solution

fg (x) = f (cx + 5)

fg(x) = 2 (cx + 5) + c

Also fg (x) = 6x + d

So 6x + d = 2 (cx + 5) + c (expand the brackets)

6x + d = 2cx + 10 + c (compare coefficients)

6 x = 2c x and d = 10 + c

6 = 2 c

3 = c (substitute c=3 into d = 10 +c)

d = 13

Warm up

Difference of two squares

The area of a rectangle is given as $x^2 - 169$

(a) Find the side lengths of the rectangle in terms of x
(b) If x is given as 25, find the numerical value of the area of the rectangle
(c) If the area is given as 120 square units, find x

Solution

(a) Using the 'difference of two squares', $x^2 - 169 = (x + 13)(x - 13)$

The two side lengths are **x + 13 and x – 13**

(b) If x = 25, then the numerical area = (25 + 13) (25 – 13) = 38 x 12 = **456**

Or 625 – 169 = **456**

(c) $x^2 - 169 = 120$ (rearrange)

$x^2 = 169 + 120$

$x^2 = 289$ (square root)

x = 17

Challenges

Factors and Primes

a) Factorise the expression $9x^2 - 1$
b) Use your answer the find the prime factors of 899
c) Factorise the expression $4x^2 - 49$
d) Use your answer to find the three unique prime factors of 39 951

Solutions

a) **$(3x - 1)(3x + 1)$**
b) Let $x = 10$
$9x^2 - 1 = 9 \times 10 \times 10 - 1 = 900 - 1 = 899$
$9x^2 - 1 = (3x - 1)(3x + 1) = (30 - 1)(30 + 1) =$ **29×31**
c) $(2x - 7)(2x + 7)$
d) Let $x = 100$
$4x^2 - 49 = 4 \times 100 \times 100 - 49 = 40\,000 - 49 = 39\,951$
e) $4x^2 - 49 = (2x - 7)(2x + 7) = (2 \times 100 - 7)(2 \times 100 + 7) = (200-7)(200+7) = 193 \times 207$
193 is prime
$207 = 23 \times 3 \times 3$
$39\,951 = 193 \times 23 \times 3^2$