Problem of the Week: Week 4 (Sum2): Year 10: Geometry: Area and perimeter

- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- calculate arc lengths, angles and areas of sectors of circles
- calculate surface areas and volumes of spheres, pyramids, cones and composite solids

Useful formulae:

Volume of a sphere $= \frac{4}{3} \pi r^3$

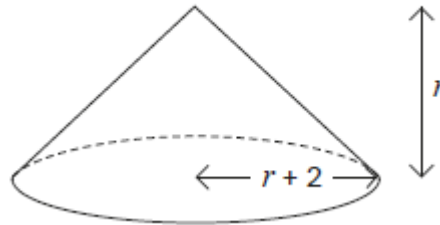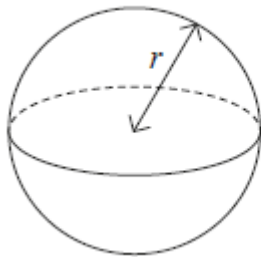Volume of a cone $= \frac{1}{3} \pi r^2 h$

(where h is the perpendicular height of the cone and r is the radius of the cone)

Volume of a pyramid $== \frac{1}{3} \times$ base area $\times$ height

Equal Volumes

The volume of the sphere is equal to the volume of the cone.

Work out the value of *r*.

Do **not** use trial and improvement.

You **must** show your working.

Hint

Make sure you use the correct radius for the cone according to the diagram

Solution

$\frac{4}{3} \pi\, r^3 = \frac{1}{3} \pi\, (r + 2)^2\, r$ (x3 and expand brackets)

$4\, \pi\, r^3 = \pi r\, (r^2 + 4r + 4)$ (÷πr)

$4\, r^2 = r^2 + 4r + 4$ (subtract r^2)

$3\, r^2 = 4r + 4$ (rearrange)

$3\, r^2 - 4r - 4 = 0$ (factorise)

$(3r + 2)\, (\,r - 2\,) = 0$

Solutions are r = - 2/3 (not possible) and **r = 2**
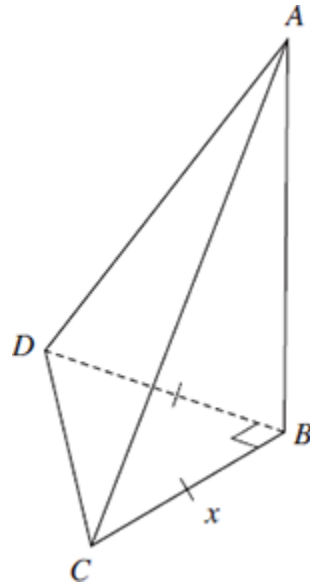
Pyramid Length

ABCD is a triangular based pyramid.
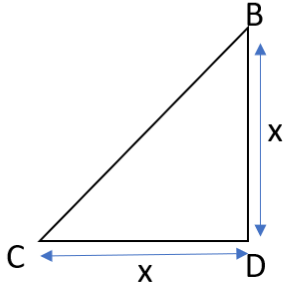The base *BCD* is a right-angled triangle.

A is directly above *B*.
BC = *BD*
AB = 2 × *BC*
The volume of the pyramid is 72 cm³.

Calculate the length of *BC*, labelled *x* in the diagram

Solution

B

x

C — x — D

Area of the base of the pyramid
= ½ x base x height of BCD
= ½ x^2

Perpendicular height of pyramid is AB
AB = 2 BC
AB = 2x

Volume of pyramid = 1/3 x area of base x height
Volume = (1/3) (½ x^2) (2x)
Volume = 72 cm^3

So 72 = (1/3) (½ x^2) (2x)
    72 = 1/6 ($2x^3$)
    72 = 1/3 (x^3)
   216 = x^3
     6 = x   (since 6x6x6 = 216)

Length x = 6cm