Problem of the Week: Week 2 (Sum2): Year 10: Probability

- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}.**

Takeaway Time

https://nrich.maths.org/7178

35 teenagers were asked what takeaway meals they liked to eat.

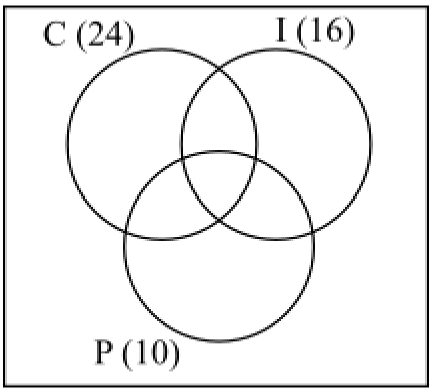
24 answered Chinese food
16 answered Indian food
10 answered pizza

None of the teenagers liked all three.
All who liked pizza also liked Chinese.
9 of the Chinese fans didn't like either Indian or pizza.

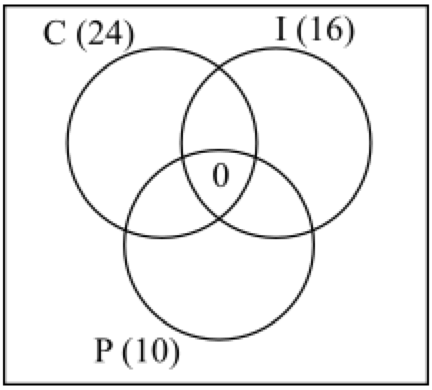
If all the teenagers liked at least one, how many liked only Indian?

Solutions

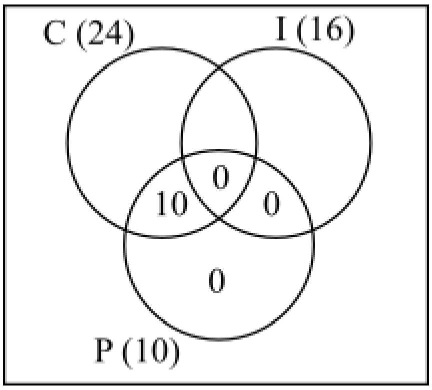
The circles on this Venn diagram represent students who liked Chinese, Indian and pizza.
The totals are shown in brackets.



None of the teenagers liked all three, so there are 0 students in the intersection of all three.

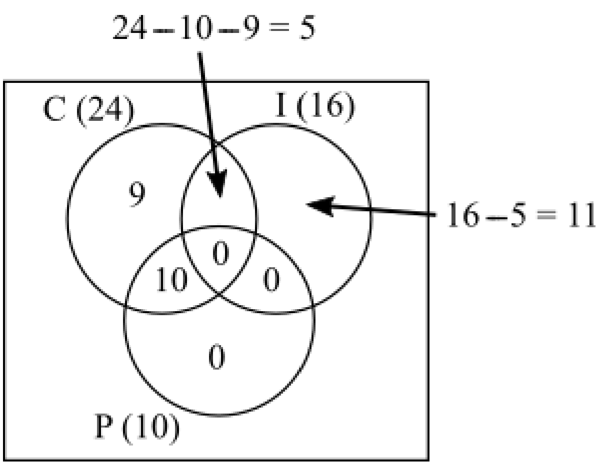


All who liked pizza also liked Chinese, so all 10 are in this overlap, with none in the other parts of the pizza circle.



9 of the Chinese fans didn't like either Indian or pizza.

That gives enough information to fill in more regions, and see that 11 teenagers liked Indian only.



Counting students who liked Chinese and pizza

There were 35 teenagers altogether.

24 liked Chinese

16 liked Indian

10 liked pizza

Since all the students who liked pizza also liked Chinese, and no student liked all three, we can just think about the remaining 25 students.

Of these, 14 liked Chinese and 16 liked Indian.

Since each student liked at least one of these, and there are a total of 30 'likes', 5 students must like both Indian and Chinese.

This leaves 11 students who liked Indian only.

Thinking about the students who liked Chinese

Let's think about the 24 students who liked Chinese:

The number who liked Chinese, Indian and pizza: 0

The number who only liked Chinese: 9

The number who liked Chinese and pizza: 10

So the number who liked Chinese and Indian: $24-9-10=5$

Therefore $16-5=11$ liked Indian only.

Odd Dice

https://nrich.maths.org/13666

Three fair, six-sided dice are numbered as follows:

A: 1, 1, 1, 2, 2, 2
B: 3, 3, 4, 4, 5, 5
C: 6, 7, 7, 8, 8, 8

The three dice are rolled once. What is the probability that the sum obtained is an odd number?

This problem is taken from the World Mathematics Championships

Solution

To find out if a sum of three numbers is odd or even, you don't actually need to know the numbers. You only need to know if they are odd or even.

This means that we can replace the numbers on the dice with 'odd' or 'even' (or O and E) without losing any information.

A: 1, 1, 1, 2, 2, 2 becomes O, O, O, E, E, E
B: 3, 3, 4, 4, 5, 5 becomes O, O, E, E, O, O
C: 6, 7, 7, 8, 8, 8 becomes E, O, O, E, E, E

Using a sample space diagram

All possible outcomes can be shown in two tables like the ones below. Two tables are necessary because the outcomes of only two dice can be shown on each table. They can be split only by the outcome on A because it is equally likely to be odd or even.

	A	B					
	even	O	O	E	E	O	O
	E	O	O	E	E	O	O
	O	E	E	O	O	E	E
C	O	E	E	O	O	E	E
	E	O	O	E	E	O	O
	E	O	O	E	E	O	O
	E	O	O	E	E	O	O

	A	B					
	odd	O	O	E	E	O	O
	E	E	E	O	O	E	E
	O	O	O	E	E	O	O
C	O	O	O	E	E	O	O
	E	E	E	O	O	E	E
	E	E	E	O	O	E	E
	E	E	E	O	O	E	E

Some of the rows and columns are repeated. A simpler version of the tables is shown below, with fewer rows and columns - but the ratio between the odd and even outcomes on each dice is preserved.

	A	B		
	even	O	O	E
	O	E	E	O
C	E	O	O	E
	E	O	O	E

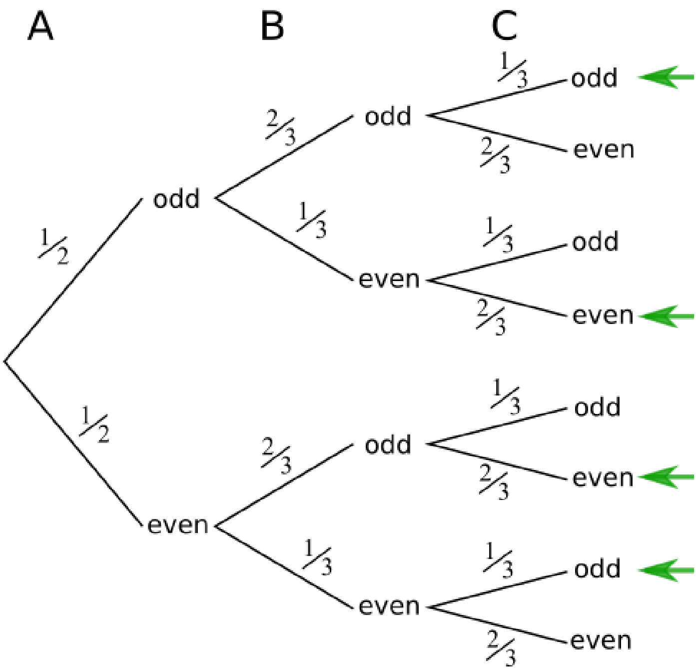
	A	B		
	odd	O	O	E
	O	O	O	E
C	E	E	E	O
	E	E	E	O

Using the second pair of tables, there are 18 possible outcomes. 9 of them are 'O'. So the probability of getting an odd sum is 1/2 .

Using a tree diagram

The tree diagram below shows the possible outcomes.

The outcomes where the sum is an odd number are indicated with a green arrow.



So the total probability is:

$(1/2\times2/3\times1/3)+(1/2\times1/3\times2/3)+(1/2\times2/3\times2/3)+(1/2\times1/3\times1/3)=2\times(1/2\times2/3\times1/3)+(1/2\times2/3\times2/3)+(1/2\times1/3\times1/3)=2\times(1\times2\times1)/(2\times3\times3)+(1\times2\times2)/(2\times3\times3))+(1\times1\times1)/(2\times3\times3)=2\times2/18+4/18+1/18=(4+4+1)/18$

$=9/18=\underline{\mathbf{1/2}}$

Using combinations of products

To add three numbers and get an odd number, the numbers must have all been odd, or two evens and an odd.

The probability that all of the numbers are odd is $1/2\times2/3\times1/3=1/9$.

There are three ways of getting two even numbers and an odd number. The odd number could be on any of the three dice.

Die A odd: $1/2\times1/3\times2/3=1/9$

Die B odd: $1/2\times2/3\times2/3=2/9$

Die C odd: $1/2\times1/3\times1/3=1/18$

So the probability of getting an odd sum is:

$\mathbf{1/9+1/9+2/9+1/18=2/18+2/18+4/18+1/18=9/18=\underline{1/2}}$