Problem of the Week: Week 1 (Sum2): Year 10: Number: Standard form and Accuracy

- calculate with numbers in standard form A 10n, where $1 \leq A < 10$ and n is an integer
- **{change recurring decimals into their corresponding fractions and vice versa}**
- apply and interpret limits of accuracy when rounding or truncating, **{including upper and lower bounds}.**

Big and small numbers in the physical world

https://nrich.maths.org/7278

Think about how best to approximate these things from the physical world around us. You will need to make some estimations and find information from friends or other sources, as would any scientist! Take care to represent all of your answers using a sensible number of decimal places and be sure to note all of your assumptions clearly.

1.Light travels at $c=3\times10^8$ metres per second. How fast in this in miles per hour? How many times faster is this than a sports car?

2.The Milky Way is a spiral galaxy with diameter about 100,000 light years and thickness about 1000 light years. There are estimated to be between 100 billion and 400 billion stars in the galaxy. Estimate the average distance between these stars.

3.Density of lead 11.34g/cm^3. How big would a tonne of lead be?

4.How many AA batteries contain enough charge between them to run a laptop for an hour?

NOTES AND BACKGROUND

An obvious part of the skill with applying mathematics to physics is to know the fundamental formulae and constants relevant to a problem. By not providing these pieces of information directly, you need to engage at a deeper level with the problems. You might not necessarily know all of the required formulae but working out which parts you can and cannot do is all part of the problem solving process!

Solutions

1. Let's change miles per hour to meters per second: 1 mile = 1.61 km and 1 hour = 3600 seconds. Thus $c=3\times10^8$m/s = $3\times10^8\times3600/1610$ mph =6.7×10^8mph. The fastest car in 2010 was the Bugatti Veyron with max speed 250 mph. The speed of light is 6.7×10^8 / 250 = 2.7×10^6 times bigger than the speed of a sport car.

2. We are given that a diameter of the galaxy is D=100,000 light years (ly) and a thickness about d=1000ly. We can approximate our galaxy as a cylinder then calculate a volume of the galaxy. $V=d\times\pi(D/2)^2$. An average number of starts in the galaxy is N=250 billion stars. Now, we can find out a density of the stars: $N/V = (250\times10^9):(1000\times \pi(100{,}000/2)^2) = 0.032$ stars/ly^3. Thus, one star occupies the volume which is equal 1/0.032 ly^3=31ly^3. In order to find the average distance between stars we take a cube root of 31 to get that the distance is about 3.2 light years.

3. Mass of lead m=1 t=1000 kg=1×10^6 g. Density of lead 11.34g/cm^3. Thus, the volume of 1 tonne of lead is V=$(1\times10^6)/(11.34)$ cm^3 = 88180 cm^3.

We can imagine this volume as a cube with side length 44.5 cm or as a ball with diameter 55.2cm, for a comparison the diameter of football ball is 14-16 cm.

4. One AA zinc-carbon battery has a capacity of 1100 mAh. For example, one AA Alkaline battery has much bigger capacity of 2700 mAh. A power being used by a laptop is about 60 watts and AC adapter changes voltage from 240 volts to average 20 volts. The formula which relates power with the voltage and the current is P=I×U where P is power, I the current and U is the voltage. The electric current is a flow of electric charge. Thus, the charge required is 60/20×1=3Ah. We can conclude that we need about three AA zinc-carbon batteries or one good Alkaline battery to run a laptop for one hour.

Recurring Mean

What is the mean of $1.\dot{2}$ and $2.\dot{1}$?

This problem is taken from the UKMT Mathematical Challenges.

Solution

Answer: $1.\dot{6}$ or $\frac{5}{3}$ or $1\frac{2}{3}$

Decimals

The mean is $(1.\dot{2}+2.\dot{1})\div 2 = 3.\dot{3}\div 2 = 1.\dot{6}$

Fractions

$1.\dot{2} = 1\frac{2}{9}$

$2.\dot{1} = 2\frac{1}{9}$

$1\frac{2}{9}+2\frac{1}{9} = 3\frac{3}{9} = 3\frac{1}{3}$

$3\frac{1}{3}\div 2 = 1\frac{1}{2}+\frac{1}{6} = 1\frac{2}{3}$