

Problem of the Week: Week 5 (Sum1): Year 10: Proportion: Compound units

- convert between related compound units (speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

Medals

The volume of a medal is 45 cm^3

The medal is made from copper and tin.

$\text{volume of copper} : \text{volume of tin} = 22 : 3$

The density of copper is 8.96 g / cm^3

The density of tin is 7.31 g / cm^3

Work out the mass of the medal.

Solution

Density = mass $\div$ volume so mass = density $\times$ volume

volume of copper : volume of tin = $22 : 3$ (so 25 'shares')

$$45 \div 25 = 1.8 \text{ per share}$$

So the volume of the copper is $22 \times 1.8 = 39.6 \text{ cm}^3$

and the volume of the tin is $3 \times 1.8 = 5.4 \text{ cm}^3$

So mass of copper is $8.96 \times 39.6 = 354.816 \text{ g}$

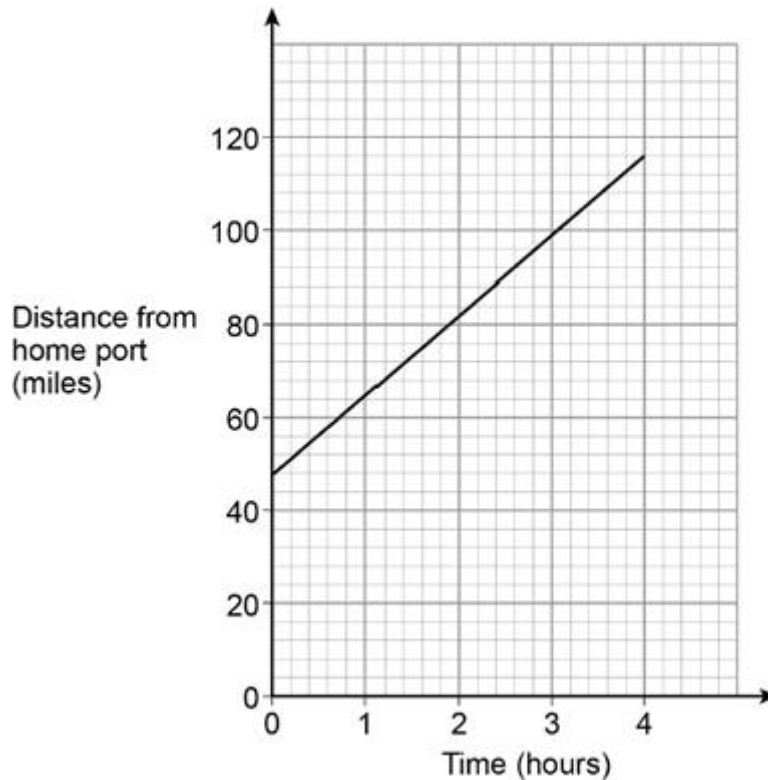
And mass of tin is $7.31 \times 5.4 = 39.474 \text{ g}$

Add together for total mass: $34.816 + 39.474 = \underline{\underline{394.29 \text{ g}}}$

Sailing Ship

A ship is sailing in a straight line from its home port.

The distance-time graph shows 4 hours of the journey.



Work out the speed of the ship during these 4 hours.

Solution

From the graph

at 0 hours [46, 50]

at 1 hour [63, 67]

at 2 hours [80, 84]

at 3 hours [96, 100]

at 4 hours [114, 118]

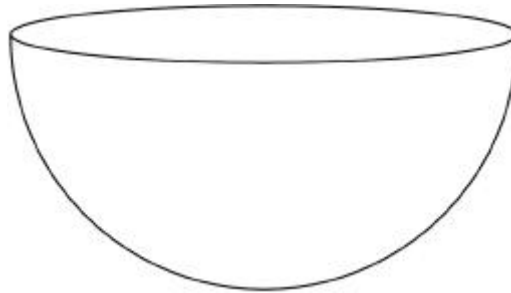
To find the speed we need to use the formula $\text{Speed} = \text{distance} \div \text{time}$

For example speed = $(63 - 46) \div 1 \text{ hour} = \underline{\underline{17 \text{ mph (miles per hour)}}$

Challenge: Filling a hemisphere

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3 \text{ where } r \text{ is the radius}$$

A container is a hemisphere of radius 30 cm



Sand fills the container at a rate of 4000 cm³ per minute.

Does it take **less than** a quarter of an hour to fill the container?

You **must** show your working.

Solution

Volume of the hemisphere is half the volume of a sphere

$$\text{Volume of the hemisphere} = \frac{1}{2} \times \frac{4}{3} \times \pi \times 30^3 = 56\,546.67 \text{ cm}^3$$

Every minute 4000 cm³ is poured into the hemisphere

$$4000 \times 15 \text{ minutes} = 60\,000 \text{ cm}^3 \text{ of sand every 15 minutes}$$

Therefore **yes, it takes less than 15 minutes to fill the container**

Challenge: Earth's orbit

The distance from the Earth to the Sun is 93 million miles.

Assume

it takes 365 days for the Earth to travel once around the Sun

the Earth travels in a circle with the Sun at the centre.

Work out the average speed of the Earth in miles per hour.

Solution

Speed = distance $\div$ time

Circumference of a circle with a radius of 93 000 000 miles = $2\pi r = 2\pi \times 93\,000\,000$

This is the distance the Earth travels every 365 days = 584 336 233.5677015 miles

The time in hours in one year = $365 \times 24 = 8\,760$ hours

Speed = $584\,336\,233.57 \div 8760 = \underline{\underline{66\,705\text{ mph} \approx 67\,000\text{ miles per hour}}}$