

Problem of the Week: Week 4 (Sum1): Year 10: Algebra: Linear and quadratic graphs : Solutions

- use the form $y=mx + c$ to identify parallel **{and perpendicular}** lines; find the equation of the line through two given points, or through one point with a given gradient
- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function

$$y = \frac{1}{x} \text{ with } x \neq 0, \pm$$

{the exponential function $y = k^x$ for positive values of k , and the trigonometric functions (with arguments in degrees) , $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size}

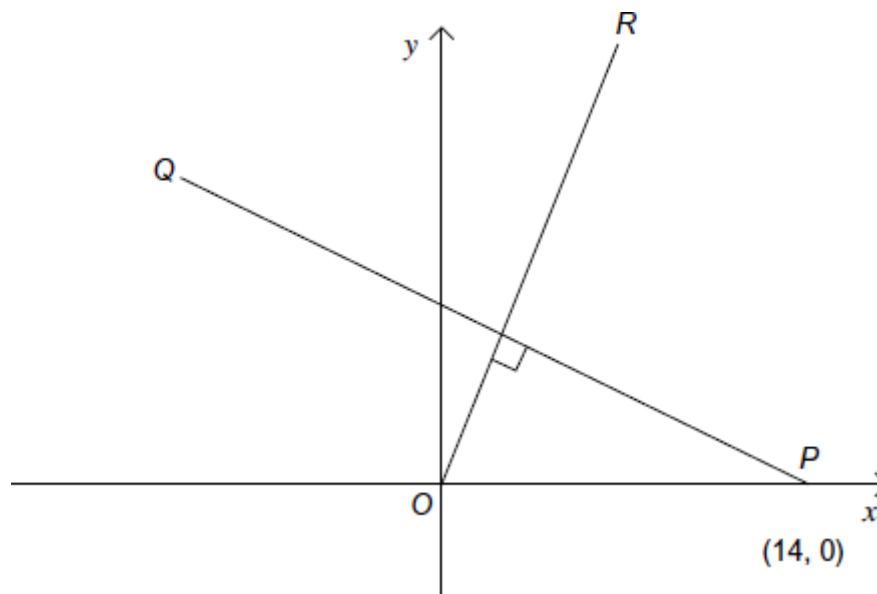
{sketch translations and reflections of the graph of a given function}

Perpendicular Lines

The gradient of line OR is $\frac{7}{4}$

PQ is perpendicular to OR .
 P is the point $(14, 0)$.

Not drawn accurately



Work out the equation of line PQ .

Give your answer in the form $ax + by = c$, where a , b and c are integers.

Solution

Gradient of OR is $\frac{7}{4}$ so the gradient of PQ is $-\frac{4}{7}$

So PQ equation is $y=mx+c$ and this becomes $y = -\frac{4}{7}x + c$

Substitute $(14,0)$ in as $y=0$ and $x = 14$ to give $0 = -\frac{4}{7} \times 14 + c$

So $c = 8$

PQ is $y = -\frac{4}{7}x + 8$

Now multiply everything by 7 to remove the fraction

$$7y = -4x + 56$$

Now rearrange to give **$4x + 7y = 56$**

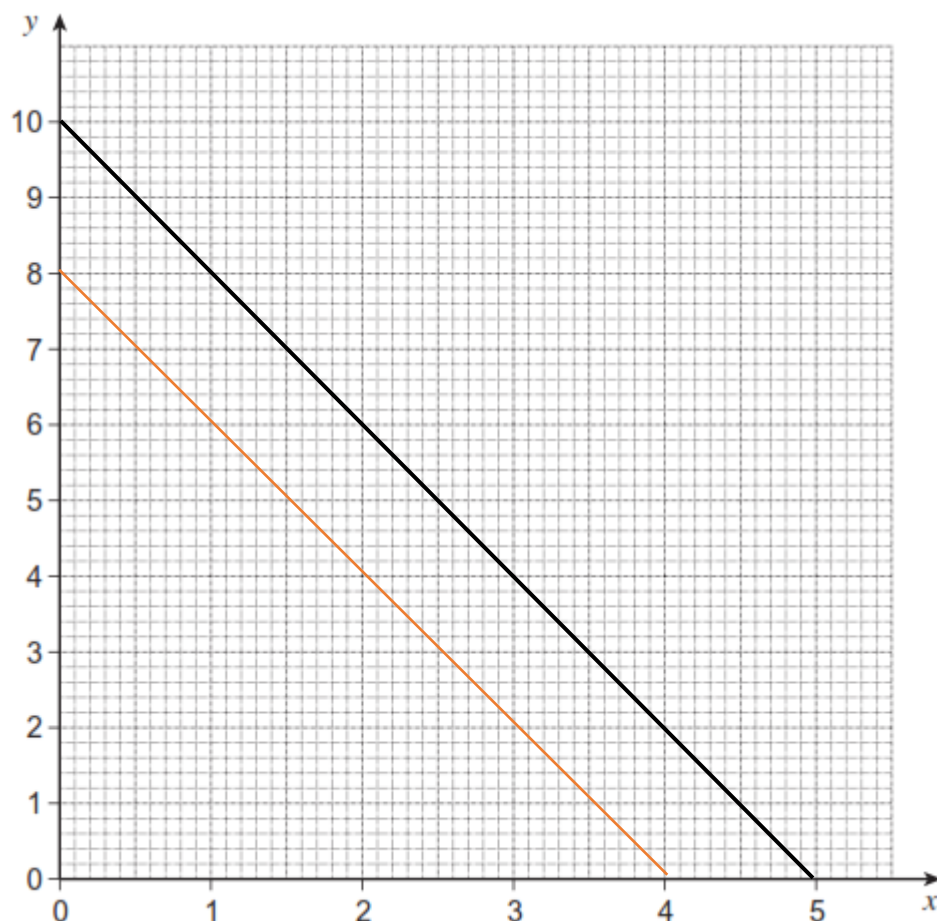
Parallel Lines

Solution

Complete the table of values for $2x + y = 10$

| | | | | | | |
|-----|----|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y | 10 | 8 | 6 | 4 | 2 | 0 |

On the grid draw the graph of $2x + y = 10$ for values of x from 0 to 5.



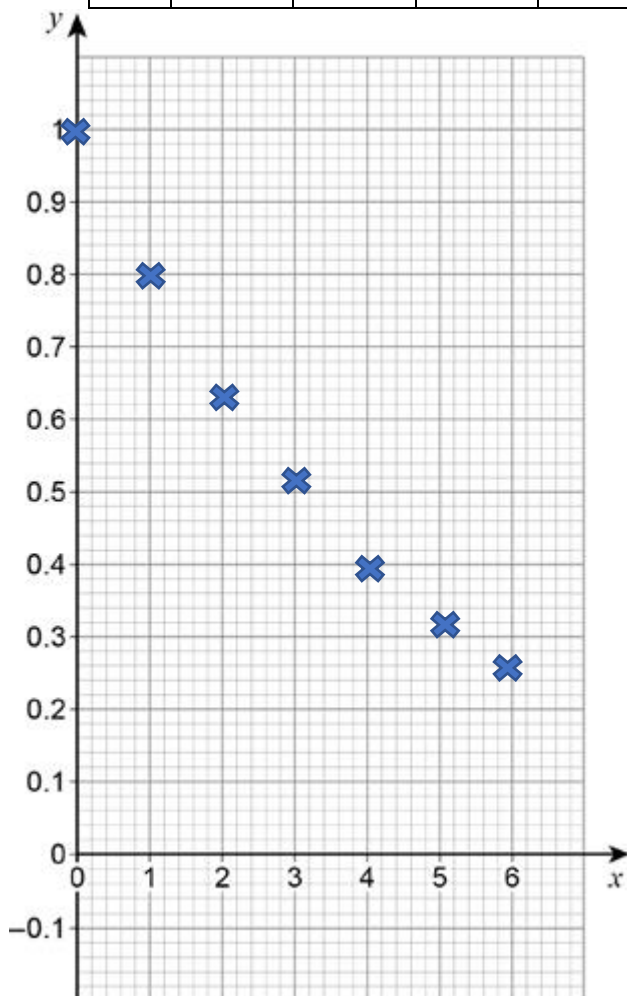
Draw three other lines that are parallel to $2x + y = 10$ and state their equations.

For example $2x + y = 8$; $2x + y = 4$; $2x + y = 6$ (all lines with a gradient of 2 are parallel)

Challenge: The exponential function

Draw the graph of $y = 0.8^x$ for values of x from 0 to 6

| | | | | | | | |
|-----|---|-----|------|-------|--------|---------|----------|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 1 | 0.8 | 0.64 | 0.512 | 0.4096 | 0.32768 | 0.262144 |



Exponential function (2)

The point $\left(3, \frac{1}{64}\right)$ lies on the curve $y = k^x$ where k is a constant.

Show that the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ lies on the curve.

Solution

$$y = k^x$$

$$1/64 = k^3$$

$$1/4^3 = k^3 \quad \text{note that '1' is the same value as '1^3'}$$

$$1^3/4^3 = k^3 \quad \text{cube root both sides}$$

$$1/4 = k$$

So the equation is $y = (1/4)^x$

Now substitute in $(1/2, 1/2)$ to give

$$1/2 = (1/4)^{1/2} \quad \text{Note that raising a number to the power of one half is the same as square rooting the number}$$

$$(1/4)^{1/2} = 1^{1/2} / 4^{1/2} = 1/2$$

Therefore $(1/2, 1/2)$ is on the curve $y = (1/4)^x$