Problem of the Week: Week 3 (Sum1): Year 10: Proportion: Direct and inverse: Solutions

- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity (including trigonometric ratios)
- understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; **{construct and}** interpret equations that describe direct and inverse proportion.

➢ Build a Wall

To build a particular wall, a supervisor knows that these builders work at the following rates:

a) Ali and Bill take 12 days
b) Ali and Charlie take 15 days
c) Bill and Charlie take 20 days

Assuming that the rates at which builds work are not affected by their companion:

1. How long would it take each of them working alone to build the wall?
2. How long would it take to build the wall if they all worked together?

{taken from 'Problem Pages'; edited by Charlie Stripp and Steve Drape; published by The Mathematical Association. ISBN: 0 906588 45 6}

Solution

➢ Let a be the number of days it takes Ali to build the wall, so Ali builds $1/a$ of the wall each day

Similarly, Bill builds $1/b$ of the wall each day and Charlie builds $1/c$ of the wall each day

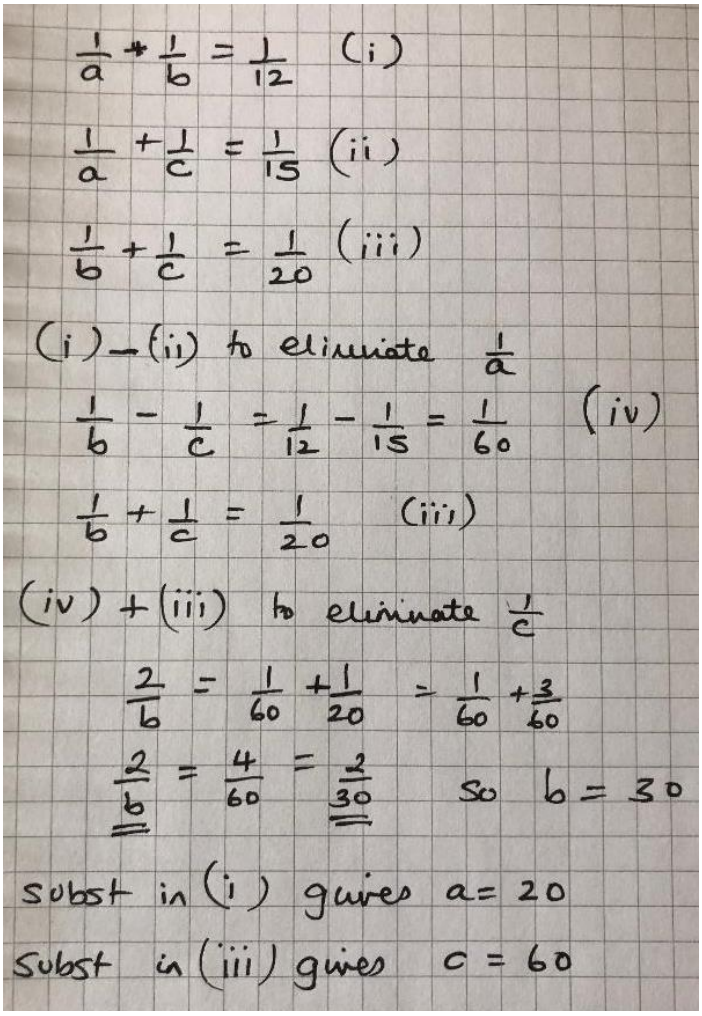
We know that Ali and Bill build 1/12 of the wall each day, so $1/a + 1/b = 1/12$

Similarly, $1/a + 1/c = 1/15$

And $1/b + 1/c = 1/20$

Solving these three simultaneous equations gives $a = 20$, $b = 30$, $c = 60$

(see below for method)

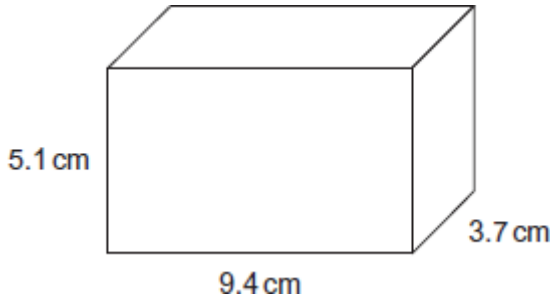


- Suppose that, working together, Ali, Bill and Charlie take *x* days
  Then $1/x = 1/20 + 1/30 + 1/60$
  $1/x = 3/60 + 2/60 + 1/60 = 6/60$
  $X = 10$

  So if they all work together, it will take 10 days to build the wall.

Similar Cuboids

The diagram shows a cuboid.



Work out the surface area and the volume of the cuboid (A)

The cuboid is enlarged by scale factor 4.

Work out the surface area and volume of the enlarged cuboid (B).

Calculate how many times bigger the surface area and the volume of the enlarged cuboid compared to the original cuboid.

Solution

Surface Area $_A$ = (2 x 5.1 x 9.4) + (2 x 5.1 x 3.7) + (2 x 3.7 x 9.4)

= 95.88+37.74+69.56 = 203.18 cm^2

Volume $_A$ = 5.1 x 3.7 x 9.4 = 177.378 cm^3

Dimensions $_B$ = (4 x 5.1) and (4 x 9.4) and (4 x 3.7) = 20.4 and 37.6 and 14.8 (cm)

Surface Area $_B$ = 4^2 x Surface Area $_A$ = 16 x 203.18 = 3250.88 cm^2

Volume $_B$ = 4^3 x Volume $_A$ = 64 x 177.378 = 11352.192 cm^3

Length $_A$: Length $_B$

1 : 4

Area $_A$: Area $_B$

1^2 : 4^2

Volume $_A$: Volume $_B$

1 : 4^3

So the new area is 16 times larger and the new volume is 64 times larger