Hampshire Services
HIAS SCHOOL IMPROVEMENT

SERVICES FOR SCHOOLS

Hampshire mathematics planning tool for pupils with SEND

Year 3

Hampshire County Council

www.hants.gov.uk

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Continuing Professional Development Standard

National Centre for Excellence in the Teaching of Mathematics

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Introduction

The Planning Tool for pupils with SEND has been developed by Hampshire Inspector Advisory Service (HIAS) to aid teachers in meeting the learning needs of pupils with Special Educational Needs and Disabilities (SEND) whose current attainment is well below that of their peers, in some or all aspects of the curriculum.

The Rochford Review (2016) recognised the need for pupils with SEND, working below age related expectations to have 'the opportunity to demonstrate both attainment and progress' taking account of the 'potential differences in the way these pupils learn'.

> 'There has always been a proportion of pupils for whom we cannot use these statutory assessments as they have not completed the relevant programmes of study when they reach the appropriate chronological age.
>
> The Rochford Review recognises that age-related expectations are not appropriate for a significant proportion of pupils working below the standard of the national curriculum tests, many of whom have SEND that affect their rate of cognitive development or speed of learning. It is important that they have the opportunity to demonstrate both attainment and progress, and that the way we measure their progress accounts for potential differences in the way these pupils learn.'
>
> **The Rochford Review 2016**

The planning tool supports teachers to identify an appropriate starting point and plan aspirational expectations of progress over time. It is intended that the tool is used to create suitably personalised learning for individual pupils with the expectation this is integrated into whole class teaching by removing barriers and enabling participation in whole class learning.

In creating the plan, teachers need to use diagnostic information gathered through a range of activities including day to day interactions with the pupil, information from parents and other professionals working alongside the pupil.

For example, assessment may show a child in Year 5 is working within the national curriculum expectations for year 2 in maths, year 2 in some domains for writing and year 3 in some domains for reading. When using the Year 2 or Year 3 planning tool it is expected that the pupil has mastered the curriculum content in a particular strand from the previous year group.

The following planning tools are included in the pack:

	Pre-Year 1 to Year 1	Year 1	Year 2	Year 3
Mathematics	✓	✓	✓	✓

Principles

Members of the Rochford Review agreed a set of principles to inform the development of their recommendations. HIAS have also adopted those principles that are relevant to underpin the pedagogical thinking behind the planning tools.

Principles from The Rochford Review (2016)

- Every pupil should be able to demonstrate his or her attainment and progress.
- Parents and carers should receive meaningful information about the achievement and progress their child makes and should be involved appropriately in assessment processes.
- Equality is not always about inclusion. Sometimes equality is about altering the approach according to the needs of the pupils.
- Curriculum should drive assessment and not the other way around.
- Key milestones should be clear and unambiguous.
- It should also be possible to assess the application of knowledge, understanding and skills in a range of different contexts.
- The language used to describe the achievements and progress of these pupils should always be positive, inclusive and should be jargon free.

In addition, the following principles informed by SEND Code of Practice underpin the guidance for the use of the English and the mathematics SEND planning tools.

Pupils make most progress when **class teachers**:

- remain responsible for all the pupil's learning – including directing the work of teaching assistants, utilising the advice of specialist staff, and ensuring pedagogy is consistent across all learning provision
- set high expectations for every pupil, whatever their prior attainment
- use high quality teaching as the first step in responding to pupils who have or may have SEND
- plan to address potential areas of difficulty and to remove barriers to pupil achievement where possible
- SENCOs and TAs have regular opportunities to update their pedagogical knowledge of effective teaching of pupils with SEND
- use high quality formative assessment information to inform teaching and learning, based on activities that involve interaction and dialogue between teachers and pupils and between pupils, including assessing and evaluating the impact of targeted support and interventions
- are supported by Senior Leaders and the SENCO in making decisions

(SEND Code of Practice 2015)

In addition class teachers should:

- teach pupils to become metacognitive learners
- develop and enhance pupil's self-esteem through the quality of interaction and appropriate level of challenge
- use pupil's own real-life experiences, familiar contexts and interests to engage and motivate
- develop efficient communication strategies between all adults, to ensure new skills are applied and embedded in classroom work
- meet with parents, at least 3 times a year to review progress towards learning outcomes and set new ones

Pupils make most progress in Mathematics and English when:

- pupils have the opportunity to become fluent in the fundamentals of the subject through varied and frequent practice with increasingly complex tasks over time - ensuring pupils have opportunities to generalise and apply their learning in a range of contexts
- tasks are used to develop conceptual understanding
- pupils are supported to recall and apply knowledge increasingly rapidly, accurately and reliably over time
- pupils are expected to communicate their thinking using appropriate subject specific language and communication (signs, symbols or technology)
- pupils are supported to use discussion in order to learn; they should be able to elaborate and explain clearly their understanding and ideas

Target group

Teachers will need to make bespoke use of the planning tools to target individual pupil's specific areas of need. The document is not designed to be used to inform teaching and learning for the majority of pupils in mainstream school.

As for the guiding principles, HIAS have used the SEND Code of Practice and The Rochford Review to identify the target group of pupils:

6.17 Children who are making less than expected progress given their age and individual circumstances.
This can be characterised by **progress** which:

- is significantly slower than that of their peers starting from the same baseline
- fails to match or better the child's previous rate of progress
- fails to close the attainment gap between the child and their peers
- widens the attainment gap'

(SEND Code of Practice 2015)

Children who will not have completed the relevant programme of study when they have reached the appropriate chronological age.
(Rochford Review)

Pedagogical thinking

The SEND planning tool is a curriculum document that breaks down barriers to understanding and therefore progress. It can be used in its entirety or just the relevant sections for each individual child. For example, a child on the autistic spectrum may be operating at age related expectations in many areas of the English curriculum, but assessment has identified elements of reading comprehension where the child is operating well below their peers.

An element of pedagogical thinking behind the planning tools is that pupils with SEND may never achieve the full curriculum, however what they do learn is learnt with depth and rigour. Therefore slowing down on content, teaching less but teaching it well is essential. Many approaches can be used to achieve this such as precision teaching, pre-teaching and overlearning to ensure success.

Overlearning

Use an overlearning approach to secure and embed the concept. Over learning is 'the systematic use of repetition, both within and between lessons, in the attempt to ensure that newly acquired skills and material are automised, consolidated in memory, so that they can be readily applied or recalled when needed, and will not be disrupted by subsequent lesson material.

Rose, Jim Identifying and Teaching Children and Young People with Dyslexia and Literacy Difficulties, 2009

This means that what pupils with SEND do achieve, they achieve more deeply by working through the learning hierarchy, over an appropriate time scale. They may not learn everything - but what they do learn, they learn well and will be able to apply with some independence.

Pupils may demonstrate this deeper learning through some of the following responses:

- describing it in his or her own words
- representing it in a variety of ways (e.g. using concrete materials, pictures and symbols)
- explaining it to someone else
- making up his or her own examples (and non-examples) of it
- seeing connections and make links
- recognising it in new situations and contexts
- applying it independently in a range of situations

(Teaching for Mastery, OUP, 2015)

General guidance

Ensure you know the whole child

It is essential that the adults working with the child have a clear understanding of the pupil's special educational need or disability:

- who is their favourite cartoon character / football player / pop star? Keep up to date as preferences change
- how do they spend their evenings and weekends?
- what is the length of their attention span?
- does technology motivate them?
- are they a visual, kinaesthetic or auditory learner?
- what barriers to learning does their special need create?
- are they motivated by competition eg against a peer / against the clock or not?

Developing and maintaining self-esteem and self-worth:

- use pre teaching to enable pupils to grow in confidence with new concepts, texts, vocabulary in preparation for a sequence of learning
- encourage pupils to look back on their achievements to see how far they have progressed
- how does the pupil see themselves as a learner?
- does the pupil regularly experience success?
- are there opportunities to build on prior success?
- are their successes valued and celebrated alongside their peers, wider school community and family?
- are their concept successes shared regardless of skill gaps e.g. celebrate the content even if spelling is poor
- are teachers designing tasks that pupils can achieve with independence?
- are they intrisically or extrinsically motivated? While pupils are extrinsically motivated use a clear rewards system whilst encouraging intrinsic motivation
- are teachers considering *where* they are carrying out their learning?
- is the pupil's voice actively heard in decision making about their learning?
- give meaningful feedback at an appropriate level to develop independence.

Create learning that is perceived more achievable:

- is the cognitive load of the task itself too much to enable learning to take place?
- consider how abstract the task is and therefore level of difficulty for some pupils
- consider alternative ways of recording outcomes in ways that work for them e.g. diagrams, bullet points, mind map
- facilitate opportunities for pupils to link new concepts to prior learning
- be explicit about next steps in learning and ensure they are clear about their success
- use language pupils relate to e.g. refer to ‘levelling up’ which is terminology used in computer gaming
- chunk up tasks e.g. chop up an A4 sheet and only give one section at a time,
- use speech-to-text software or a recording device/ app to capture learning
- schedule breaks if appropriate for the child.

Created learning that is perceived as more desirable:

- ensure the learning is appropriately challenging and engaging
- offer different medium eg different type of pen / coloured paper
- ensure tasks are age appropriate e.g. use celebrities / football teams / computer games / current films to hunt for phonemes with older pupils, use current chart songs to find rhyming words
- give a clear purpose to learning eg a real reason to publish
- be clear in expectations for success in every lesson eg ‘there is no writing in this lesson today’ or ‘I would like 3 sentences in this session’
- link learning to real life where possible eg identify phonemes in family members names for writing birthday cards
- use concrete resources effectively eg talking clipboards to reduce load on working memory
- create age appropriate tasks eg use a rap not a song, avoid APPs that are too young
- avoid as far as possible tasks being too ‘different’ from their peers
- use multisensory learning approaches e.g. letter formation on a touch screen, in the sand, chalk on a wall
- give pupils some choice e.g. ‘you can do it like this or with this’.

Create task design and resources that are more accessible for SEND pupils:

- use adult modelling widely
- enlarge tasks onto A3
- secure background subject knowledge by using multimedia approaches e.g. online clips, graphic novels etc
- avoid too much text on one page
- support words with visuals alongside whenever possible
- highlight key words in the instructions
- provide a word bank to support
- ensure order of questions / tasks is very clear i.e. left to right and top to bottom
- break tasks into manageable goals

Adapt adult language to encourage participation and success:

- refer to ‘tasks / activities’ rather than ‘work’
- ensure work is never used as a punishment
- allow plenty of time for pupil to process instructions.

The Mathematics Planning Tool for pupils with SEND

The Planning Tool supports pupils with SEND in developing mathematical understanding in line with the aims of the mathematics national curriculum to:

> - 'become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
> - reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
> - solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.'
>
> **Mathematic National Curriculum 2013**

The Mathematics Planning Tools focus on the following domains: number and place value; addition and subtraction; multiplication and division; fractions. Each domain also has some 'Key Concepts' pupils should know by the end of the year in order to meet end of year expectations as suggested in the national curriculum documents.

For each strand some key ideas have been identified which underpin the national curriculum statements. The Planning Tool offers ideas for teachers for potential gaps in curriculum knowledge, misconceptions and or skills that pupils need to gain to meet a national curriculum statement.

For example, the key ideas for number and place value include the following:

- counting
- comparing numbers
- identifying, representing and estimating numbers
- reading and writing numbers
- understanding place value

These key ideas are further broken down into some component parts to enable teachers to fine tune planning needed in areas for which a particular pupil needs further support. Pupils need to make progress in all these components parts to make progress overall in number and place value.

In each domain consideration is given to the importance of problem solving. This might be annotated as to the types of problems pupils have experience of and can solve confidently and independently. It might be annotated to identify the phrases pupils can interpret successfully for the mathematics needed eg 'altogether', 'total', 'spent' etc. It is an essential consideration for assessment in each domain.

Spoken Language

The Early Years Foundation Stage and national curriculum for English and mathematics reflects the importance of spoken language in pupils' development across the whole curriculum- cognitively, socially and linguistically.

The Hampshire Reading and Writing planning tools should be referred to for additional support when planning to meet the needs of individual pupils.

> The development of children's spoken language underpins all seven areas of learning and development. Children's back-and-forth interactions from an early age form the foundations for language and cognitive development. The number and quality of the conversations they have with adults and peers throughout the day in a language-rich environment is crucial. By commenting on what children are interested in or doing and echoing back what they say with new vocabulary added, practitioners will build children's language effectively.
>
> Communication and Language
>
> ***Statutory framework for the Early Year's Foundation Stage, September 2021***

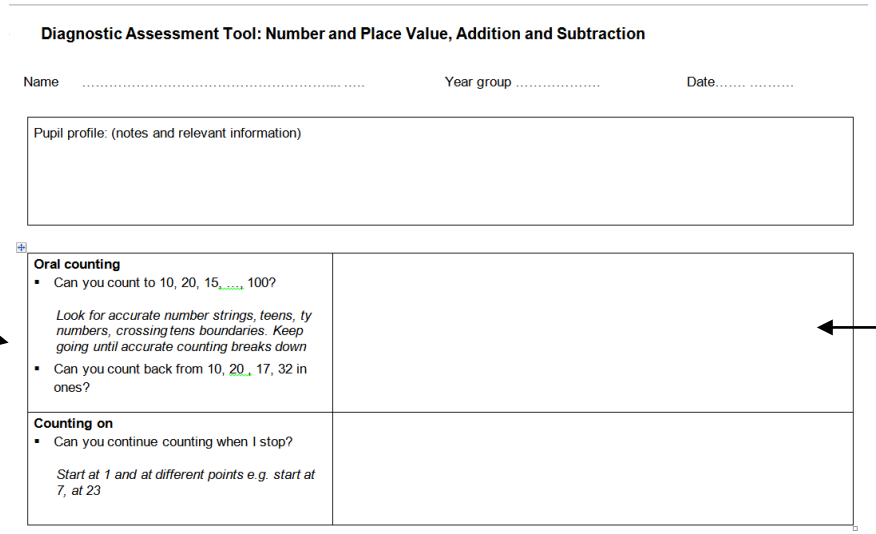
> The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof. They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions.
>
> ***Mathematics National Curriculum* 2013**

> Spoken language underpins the development of reading and writing. The quality and variety of language that pupils hear and speak are vital for developing their vocabulary and grammar and their understanding for reading and writing. Teachers should therefore ensure the continual development of pupils' confidence and competence in spoken language and listening skills.'
>
> ***English National Curriculum* 2013**

Identifying a starting point

Included with the Planning Tool is a sample set of diagnostic questions that can be used to identify starting points for individual pupils. These questions support assessment in each of the mathematics curriculum domains in the Planning Tool. They are indicative of the types of questions which are useful to ask rather than exhaustive. Generally, pupil responses to these questions provides sufficient information to identify a starting point for individual pupils. The intended approach is of a 'conference', rather than a 'test', with the teacher focused on finding out what the pupil *can* do. Qualitative assessment information about the pupils' responses to questions used should be recorded. The questions can be used as part of planning as key assessment questions, in one session or used across several sessions/ lessons. The set of questions used needs to match with the domain(s) of mathematics the pupil will be about to work on in class. The diagnostic assessment questions can be used again after a period of time, appropriate to the pupil, as an indicator of progress through comparison with previous responses and to contribute to identifying starting points for next units of work. During the year, ongoing day to day assessments may mean that a teacher uses these questions only once in the year. Professional judgement should be used to determine how and when these questions are used.

Suggested diagnostic question →



Diagnostic Assessment Tool: Number and Place Value, Addition and Subtraction

Name ..... Year group ..... Date..... .....

Pupil profile: (notes and relevant information)	
Oral counting ▪ Can you count to 10, 20, 15,...., 100? *Look for accurate number strings, teens, ty numbers, crossing tens boundaries. Keep going until accurate counting breaks down* ▪ Can you count back from 10, 20, 17, 32 in ones?	
Counting on ▪ Can you continue counting when I stop? *Start at 1 and at different points e.g. start at 7, at 23*	

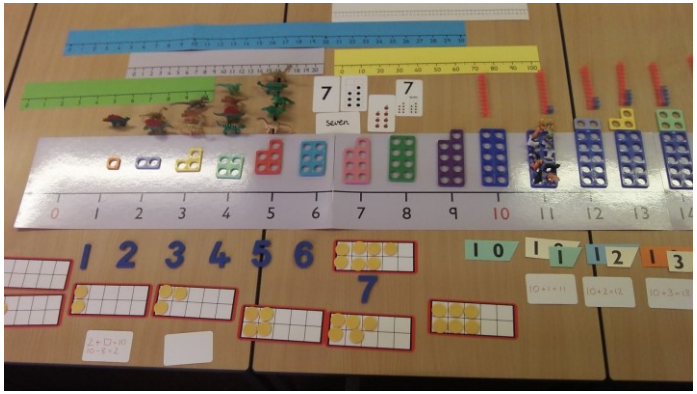
← Qualitative notes about pupil responses

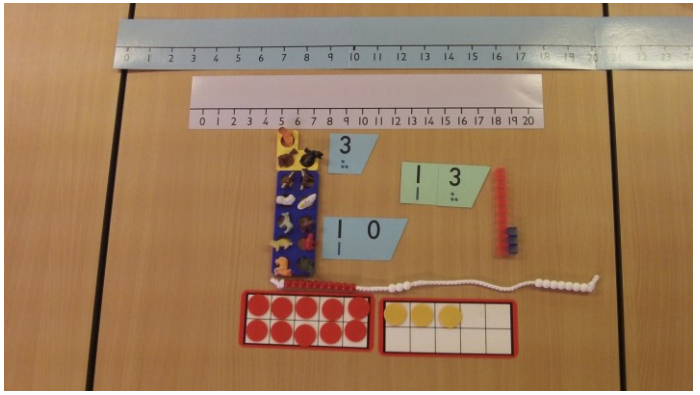
Schools may also use commercially available tests to inform starting points with the SEND Planning Tool.

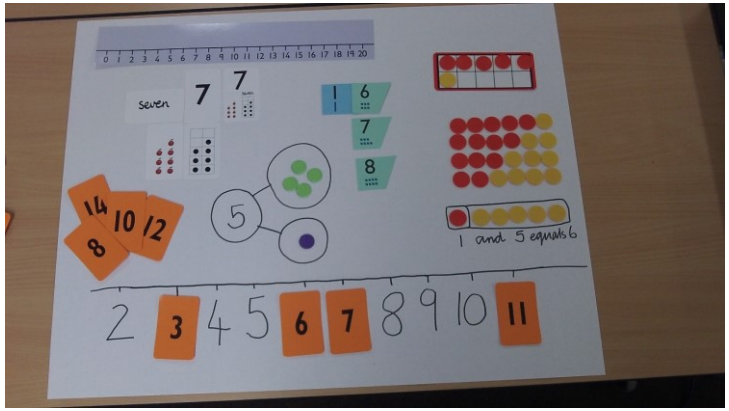
Effective Mathematics Pedagogy

It is assumed that teachers and adults will use a concrete, pictorial abstract (CPA) approach to teaching and learning. This involves the use of a range of unstructured and structured concrete resources (eg counters, place value cards, Dienes etc) and the use of a range of mathematical models and diagrams (eg number lines, bar models, arrays etc). The context used for mathematical reasoning and problem solving should make sense to the pupil.

Pupils need focused continuous opportunities to develop their mastery of mathematical language from 'simple' everyday words and phrases to more formal mathematical terms and symbols. Individual pupil's developing understanding of the multiple connections between words, symbols, meanings, contexts and procedures related to operations and domains provides important assessment information about progress and attainment. Teacher assessment should consider to what extent the pupil is able to apply the mathematics to solve problems. The context used for mathematical reasoning and problem solving should be accessible and make sense to the pupil.







How to use the planning tool documents

Understanding the layout of the planning tools

National curriculum non-statutory guidance

Key concepts

National curriculum statements

Number and Place Value

Year 1: National Curriculum notes and guidance (non-statutory)

Pupils practise counting (1, 2, 3…), ordering, (eg first, second, third…), and to indicate a quantity, (eg 3 apples, 2 centimetres), including solving simple concrete problems, until they are fluent.

Pupils begin to recognise place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100, supported by objects and pictorial representations.

They practise counting as reciting numbers and counting as enumerating objects, and counting in twos, fives and tens from different multiples to develop their recognition of patterns in the number system, (eg odd and even numbers), including varied and frequent practice through increasingly complex questions.

They recognise and create repeating patterns with objects and with shapes.

Key concepts

- The order of numbers enables comparison between numbers.
- As you count on the quantity represented by the number becomes larger and becomes smaller as you count back.
- Numbers greater than 9 are formed by combining more than one digit and numbers between 10 and 20 start with a '1'.
- The position of a digit in a number indicates its value.
- The place value system is based on units of 10.
- Knowing number names /reading teens numbers can be confusing in terms of place value, eg 11, 12, 13, 14.

Curriculum strands

Within the document, the national curriculum programme of study domain number and place value is broken down into smaller curriculum strands to support precise identification of need. The curriculum strands identified are:

- counting
- comparing numbers
- identifying, representing and estimating numbers
- reading and writing numbers
- understanding place value

Problem-solving

Teacher assessment should consider to what exte
solve problems.

Skills, knowledge and concepts		NC expectations – Year 1
Use concrete resources to model and record addition and subtraction calculations (U +/- U) using +/- and = signs.	Uses structured number lines to show addition calculations (U + U).	• Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs
Explain and use concrete resources to model commutativity with addition.	Use structured number lines to show subtraction calculations (U - U).	
Explain using concrete resources that subtraction is not commutative, eg 9 - 6, 6 - 9.	Use structured number lines to show addition calculations (TU + U) bridging through 10.	
Use diagrams, eg bar models and concrete resources to explain inverse.	Use structured number lines to show subtraction calculations (TU - U) bridging through 10.	
Identify addition number sentence to solve a simple word problem 3-7 (8-10, 11-20).	Identify subtraction number sentence to solve a simple word problem 3-7 (8-10, 11-20).	

Strategies:

- *support pupils to 'talk out loud' when recording number sentences*
- *make explicit links between number sentences and number line recording and or bar model diagrams*
- *ensure pupils can relate 'numbers' used to the problem context.*

Each strand or domain includes '**Skills, knowledge and concepts**. These are not intended to be linear or that every child will need to be taught each element to achieve the full statement. Some are ideas for teaching, other ideas for assessment to identify barriers to learning for individuals

Each strand or domain includes '**Strategies'** ideas of approaches for teachers to try that may suit a particular pupil more aptly

Within the planning tool, each skill, knowledge and concept point is written as a pupil 'can' statement. This is to acknowledge and celebrate the pupil's strengths and understanding – giving a positive starting point and mind-set for pupils and adults involved in supporting learning. They are suggested not exhaustive ideas.

Process for using the Planning Tools

1. Complete diagnostic assessment activities including day-to-day AfL to identify starting points.
2. Identify focus domains where the planning tools would support pupil progress.
3. Narrow down the focus to key strands that will inform planning and teaching. Develop a plan that where ever possible follows the whole class planning.
4. Identify skills, knowledge and concepts to inform bespoke planning, alongside strategies to teach new learning.
5. The planning tool can be used as an annotated working document to show areas in which the pupil is making progress.

Year 3 Planning Tool

Number and Place Value

Year 3: National Curriculum notes and guidance (non-statutory)

Pupils now use multiples of 2, 3, 4, 5, 8, 10, 50 and 100.

They use larger numbers to at least 1000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2, (eg 146 = 100 + 40 and 6, 146 = 130 + 16).

Using a variety of representations, including those related to measure, pupils continue to count in ones, tens and hundreds, so that they become fluent in the order and place value of numbers to 1000.

Key concepts

- The value of a digit is determined by its position in a number.
- Understanding place value supports understanding of where a number is in relation to multiples of 10 and 100.
- Numbers ending in 99 are 'nearly' multiples of 100.
- Place value supports addition and subtraction calculations involving +/- 1, 10 and 100 and multiples of 1, 10 and 100.

Curriculum strands

Within the document, the national curriculum programme of study domain number and place value is broken down into smaller curriculum strands to support precise identification of need. The curriculum strands identified are:

- counting
- comparing numbers
- identifying, representing and estimating numbers
- reading and writing numbers
- understanding place value.

Problem-solving

Teacher assessment should consider to what extent the pupil is able to apply conceptual understanding of number and place value to solve problems.

Curriculum strand – Counting

Skills, Knowledge and Concepts		NC expectations – Year 3
Can say any part of the number sequence from 1-200 (1-500; 1-1000).	Can say any part of the number sequence backwards from 200-1 (500-1; 1000-1).	Count from 0 in multiples of 4, 8, 50 and 100.
Within the range 1-200 (1-500; 1-1000) can count forwards from a given number to another given number.	Within the range 200-1 (500-1; 1000-1) can count backwards from a given number to another given number.	
Can say the number after a given number in the range 1-200 (1-500, 1-1000) without dropping back to 1.	Can say the number before a given number in the range 1-200 (500, 1000) without counting up through all numbers first.	Find 10 or 100 more or less than a given number.
Recognises patterns in the number sequences from 1-20 (1-100) and uses this to say them/ self-correct.	Can find 10 more than any given number (0-100) fluently then 0-200, 0-500, 0-1000.	
Can count accurately in multiples of 100 using concrete objects 500 (1000).	Can find 10 less than any two-digit number (100-0) fluently then 200-0; 500-0; 1000-0.	
Can count in multiples of 2, 5 and 3 fluently.	Uses step counting to count larger groups of objects, inc coins in context of money all coins 1p, 2p, 5p,10p, 20p, 50p.	
Can count in multiples of 50 to 200 (500, 1000).	Can count in multiples of 4 to tenth multiple	
	Can count in multiples of 4 and 8 to tenth multiple (notice links)	

Strategies:

- *use counting object and structured resources*
- *use structured/ unstructured number lines with some numbers blanked out/covered over*
- *use packs of number cards to create a number line model*
- *use packs of number cards, (shuffled or in order) to turn top card over; ask which number comes next? 10 more/less?*
- *use blank number lines to place number cards on, fill in missing gaps by writing the number.*

Curriculum strand – Comparing numbers

<table>
<tr><th colspan="2">Skills, Knowledge and Concepts</th><th>NC expectations – Year 3</th></tr>
<tr><td>Can use a structured (then empty) number line to compare position of two numbers 0-200 (0-500, 0-1000.</td><td rowspan="2">Can say whether two numbers are close together or far apart (through oral counting or number line model).</td><td rowspan="4">Compare and order numbers up to 1000.</td></tr>
<tr><td>Can use language of ‘more/ less’ to describe two sets of objects with links to > < signs.</td></tr>
<tr><td>Can use structured resources, eg dienes to compare numbers 1-200 (1-500, 1-1000).</td><td>Can order numbers 1-200 (1-500, 1-1000).</td></tr>
<tr><td>Can use language of ordinality up to twentieth.</td><td>Can use a structured (empty) number line to compare position of two numbers (multiples of 100, non-multiples of 100 from 0-200 (0-500, 0-1000) related to multiples of 100.</td></tr>
</table>

Strategies:

- *use counting objects, counters and structured resources*
- *use structured number lines with some numbers blanked out/covered over*
- *use packs of number cards to create a number line model*
- *use packs of number cards, (shuffled or in order) to turn top card over; ask which number comes next? One more/less?*
- *use blank number lines to place number cards on, fill in missing gaps by writing the number*
- *use > < = signs on cards to place between two numbers, or quantity of objects or mix of two if vocabulary linked to the symbols is secure*
- *use structured resources such as dienes for two and three digit numbers.*

Curriculum strand – Identifying, representing and estimating numbers

Skills, Knowledge and Concepts		NC expectations – Year 3
Makes a reasonable estimate up to 200 (500, 1000) using counting objects, coins and dienes.	Can mark numbers on structured number lines 0-200 (0-500, 0-1000) showing awareness of position of multiples of 100.	Identify, represent and estimate numbers using different representations.
Can fluently systematically subitise small numbers in all different ways 3-7 (8-10) and record using + and = signs. Recognises patterns linking multiples of 10 and 100 3 + 8, 30 + 80, 300 + 800 etc.	Can use tens arrays, dienes and numicon to represent same number in different ways and compare different numbers using two tens arrays and other structured resources, including place value cards.	
Can use structured resources, eg dienes to represent any three-digit number explaining choices.	Can use structured resources to show understanding of flexible place value partitioning, eg 56 = 50 + 6, 40 + 16, 30 + 26 etc.	

Strategies:

- *use counters and a range of objects to arrange in patterns*
- *estimate numbers of objects in different sized jars and boxes, use images of quantities*
- *use structured resources like Dienes to show flexible place value partitioning, eg 56 = 50 + 6, 40 + 16, 30 + 26 etc.*

Curriculum strand – Reading and writing numbers

Skills, Knowledge and Concepts		NC expectations – Year 3
Read all number words to 500 (0-1000).	Show all numbers using place value cards and structured resources from oral instruction 0-200 (0-500, 0-1000).	Read and write numbers up to 1000 in numerals and in words.
Write all numbers using symbols and words without zero as place holder.	Write all numbers using symbols and words including with zero as place holder. eg 904.	

Strategies:

- *play pelmanism games with cards showing number words and number symbols*
- *place collections of objects, structured resources such as dienes on cards showing number words or number symbols*
- *write number/ word labels for collections of objects and structured resources*
- *use structured number lines to 1000 as a reference for number symbols*
- *use structured resources in combination with place value arrow cards to support reading and writing of numbers*
- *add number word cards to structured and semi structured, (eg multiples of 10, multiples of 100) number lines.*

Curriculum strand – Understanding Place Value

Skills, Knowledge and Concepts		NC expectations – Year
Can draw an empty number line and mark where three-digit numbers would be and explain the position compared to multiples of 100. Understands the significance of the order of digits, ie 141 and 411 are in different positions on a number line related to cardinal value (HTU) and ordinal value - position compared to other numbers.	Can confidently find two and three-digit numbers to 500 (1000) on a structured number line relative to multiples of 10 and 100.	Recognise the place value of each digit in a three-digit number (hundreds, tens, and ones)
Can use a combination of a range of structured resources, eg straws, dienes, place value cards, place value counters, Numicon to demonstrate 1-200 (501-1000).	Can continually 'subtract 10/100' from a three-digit number recognising the oral counting pattern, eg 307, 207,107, 7 using structured resources, eg numicon, dienes, place value cards and number lines to model the numbers and explain place value patterns.	
Recognise and use rounding to nearest multiple of 10 fluently, eg + 9, + 19, + 29 as 10 - 1; 20 - 1, 30 - 1 etc.	Recognise and use rounding to nearest multiple of 100, eg +/- 99, +/- 999 as 100 – 1, 1000 – 1, 299 /300 -1 etc, including in the context of money and units of measurement.	

Strategies:

- *use appropriate strategies linked to place value when counting quantities of objects, eg coins (same coin or mixture)*
- *use a range of structured resources to show place value*
- *use blank and structured number lines to look at position of number compared to multiples of 10 and 100*
- *make near multiples of 10 and 100 using concrete structured resources.*
- *make all numbers in a sequence using structured resources starting from any number, eg 106-123 or 223 -247 to model the relationship between numbers (1more/less, 10 more/less, 100 more/less)*
- *use unstructured number lines to show '10more/less and 100 more/ less' patterns and combine with representing those numbers with structured resources to enable pupils to see patterns, eg 107,207,307,307; 324, 314, 294 etc.*

Addition and Subtraction

Year 3: National Curriculum Notes and Guidance (non-statutory)

Pupils practice solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100.

Pupils use their understanding of place value and partitioning, and practice using columnar addition and subtraction with increasingly large numbers up to three-digits to become fluent (see Mathematics Appendix 1).

Key concepts

- Patterns in calculation are used to support reasoning about reasonable answers, eg 2 + 5, 20 + 50, 200 + 500.
- Key number facts for 100 and 1000 are used to support calculation, eg 50 + 50, 500 + 500, 25 + 75, 250 + 750, 10 + 90, 100 + 900 particularly in the context of measures.
- Using number bonds of 10 and 100 used to bridge to multiples of 10 and multiples of 100 to support calculation.

Curriculum strands

Within the document, the national curriculum programme of study domain number and place value is broken down into smaller curriculum strands to support precise identification of need. The curriculum strands identified are:

- number bonds
- mental calculations
- written recording
- inverse operations, estimating and checking answers.

Problem-solving

Teacher assessment should consider to what extent the pupil is able to apply conceptual understanding of addition and subtraction to solve problems

Curriculum strand – Number bonds

Skills, Knowledge and Concepts		NC expectations – Year 3
Can fluently subitise small numbers in different ways (3, 4, 5, 6, 7, 8, 9 and 10) using counting objects and structured resources, eg Numicon. Can record all the different partitions of numbers (3-20) using + and = signs and identify the linked subtraction facts.	Can use bar models and two-part diagrams fluently to show partitions of all units numbers (10, 11-20) identifying the related subtraction fact with each addition fact relating this to multiple of 10 (100). 3 + 7, 30 + 70, 300 + 700 in context of money and measures.	Ensure fluency with use of number bonds to 20 relating numbers to 5 and 10, bridging through 10 (links to unit fractions and decimals). Ensure fluency with use of number bonds to 20 relating numbers to 5 and 10, bridging through 10 (links to unit fractions and decimals).
Understands X + 1 can be interpreted as 'next number' and '1 more' without the need to count all. (0-200, 500, 1000) including context of money and measure.	Can use bar models and part/ whole diagrams fluently to show place value partitions of three-digit numbers. 458 = 400 + 50 + 8, 450 + 8, 400 + 58 etc.	
Understands X - 1 can be interpreted as 'number before' and '1 less' without the need to take away and then count all (0-1000).	Understands three-digit number subtract 100 and multiples of 100 = two-digit/ units digit and can use structured resources to explain, eg Numicon, place value cards, eg 340 - 300 = 40.	
Understands X + U where X = 100 (or multiple of 100) can be calculated using place value without the need to count all (0-200, 0-500, 0-1000), eg 200 + 9 = 209.	Understands the pattern linking number bonds to 10 with number bonds to 20 (addition) 100 and 1000 3 + 7 = 10, 13 + 7 = 20, 53 + 47 = 100, 530 + 470 = 1000.	
Knows doubles of all numbers to 50 and can show using resources and on a bar model. Relates to three-digit numbers, eg 13 + 13 = 26, 130 + 130 = 260.	Understands the pattern linking number bonds to 10 with number bonds to 20 (addition and subtraction) and 100 (1000), eg 1 + 9, 11 + 9, 110 + 90, 910 + 90.	

Skills, Knowledge and Concepts		NC expectations – Year 3
Knows halves of all numbers to 50 and can show using resources and on a bar model.	Can use structured resources to show flexible partitioning of two (three)-digit numbers, eg 65 = 60 + 5, 50 + 15, 40 + 25 etc. Relates this to finding multiples of 2, 5, 10, 3, 4, 8 to support division.	
Ensure fluency with use of number bonds to 20 relating numbers to 5 and 10, bridging through 10 and 100.		

Strategies

- *use place value counters, to show part whole combinations linked to bar models fluently*
- *use counters or structured resources to model all number sentences with explicit focus on language use, eg 'if I know 130 + 40 equals 170 then 170 take away 40 must equal 30'*
- *use place value cards alongside other resources to show place value calculations*
- *use combinations of tens arrays, dienes to link number bonds to 10 with number bonds to 20 and bonds to 100.*

Curriculum strand – Mental calculations

Skills, Knowledge and Concepts		NC expectations – Year 3
Use recall of number bonds to 20 to check solutions (addition).	Group calculations into solutions > < then = to 100, 1000.	Add and subtract numbers mentally, including • a three-digit number and ones • a three-digit number and tens • a three-digit number and hundreds.
Use recall of number bonds to 20 to check solutions (subtraction).	Uses inverse to solve missing box calculations.	
Use known fact to solve related fact, eg if 300 + 400 = 700 then 300 + 500 must be 800, 247 + 200 = 447 then 247 and 300 must be 547.	Identify calculations that can be worked out easily with place value counting forwards/ backwards, ie +/- 10s (+/- 100s).	

Skills, Knowledge and Concepts		NC expectations – Year 3
Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot (HTUs).	Recognise and use rounding to nearest multiple of 100, eg + 99, + 199, + 299 as 100 - 1; 200 - 1; 300 - 1 etc.	

Strategies:

- *model the number sentence with counters, objects structured resources and cover over leading to missing box representations*
- *focus on the language of part-part-whole to support inverse link to recording with a bar model*
- *model how one number fact is related to another*
- *use rounding to nearest multiple of 100 appropriately*
- *use units in Dienes and then 10s and 100s to model 2 + 3, 20 + 30, 200 + 300 etc.*

Curriculum strand – Written recording

Skills, Knowledge and Concepts		NC expectations – Year 3
Use concrete resources to model and record addition and subtraction calculations (HTU +/- U, HTU +/- TU, HTU +/- HTU) using +/- and = signs.	Use unstructured number lines to show addition and subtraction calculations (HTU +/- U, HTU +/- TU, HTU +/- HTU).	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction* **pupils need to know and understand KS2 curriculum by the end of key stage 2.*
Explain and use concrete resources to model commutativity with addition.	Use unstructured number lines to show addition calculations (HTU + U/TU) bridging through 100, eg 398 + 6, 398 + 16.	

Skills, Knowledge and Concepts		NC expectations – Year 3
Explain using concrete resources that subtraction is not commutative, eg 90 -60, 60 – 90.	Use unstructured number lines to show subtraction calculations (HTU - U/TU) bridging through 100, eg 403 - 6, 403 – 16.	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction* **pupils need to know and understand KS2 curriculum by the end of key stage 2.*
Use diagrams, eg bar models and concrete resources to explain inverse (HTU +/- TU, HTU +/- HTU).		
Identify addition number sentence to solve a simple word problem 0-1000 including context of money (£).	Identify subtraction number sentence to solve a simple word problem 0-1000 including context of money (£).	

Strategies:

- *support pupils to 'talk out loud' when recording number sentences*
- *make explicit links between number sentences and number line recording and or bar model diagrams*
- *ensure pupils can relate 'numbers' used to the problem context*
- *ensure pupils using facts rather than counting in ones when adding and subtracting and record jumps on number lines with multiples of 10s and 100s.*

Curriculum strand – Inverse operations, estimating and checking answer

Skills, Knowledge and Concepts		NC expectations – Year 3
Use bar models to explain using correct vocabulary how an addition fact linked to a subtraction fact (bonds to 20, 100), eg 23 + 7 = 30 so 30 – 23 = 7 and 30 – 7 = 23.	Solve missing box calculations using bar model diagrams to support reasoning about calculation as 'part/whole' model.	Estimate the answer to a calculation and use inverse operations to check answers.

Strategies:

- *focus on the language of part-part whole to support inverse link to recording with a bar model*
- *model using counters and structured resources how one number fact is related to another*
- *use units in Dienes and then 10s and 100s to model 2 + 3, 20 + 30, 200 + 300 etc*
- *place counters and or structured resources on blank bar model templates before recording in books.*

Curriculum strand – Problem-solving

Key ideas to develop skills, knowledge and concepts when problem-solving involving addition and subtraction.

<table>
<tr><th colspan="2">Skills, Knowledge and Concepts</th><th>NC expectations – Year 3</th></tr>
<tr><td>Identify number sentence needed and show solution on a number line.</td><td>Identify number sentence needed and show solution on an unstructured number line and a bar model.</td><td rowspan="2">Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.</td></tr>
<tr><td>Use bar models to solve missing box calculations, eg 125 + ? = 200, 390 = 410 - ?.</td><td>Solve two step problems involving addition and subtraction.</td></tr>
<tr><td colspan="3">Strategies:
Key ideas to develop skills, knowledge and concepts when problem solving involving addition and subtraction:
• addition, using aggregation, subtraction using partitioning
• addition using augmentation, subtraction using reduction
• ensure pupils can identify contexts in which comparison of two numbers is required and use number line model and bar models to represent the difference between two numbers
• ensure pupils can relate the numbers in a number sentence to the problem context
• ensure pupils understand the problem before working on the arithmetic required to solve the calculation
• ensure pupils can identify the steps needed in a solution if working on a multi-step problem.</td></tr>
</table>

Multiplication and Division

Year 3: National Curriculum Notes and Guidance (non-statutory)

Pupils continue to practice their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity, (eg 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts, (eg using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts, (eg 30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (eg four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects, (eg 3 hats and 4 coats, how many different outfits?, 12 sweets shared equally between 4 children, 4 cakes shared equally between 8 children).

Key concepts

- There are links and relationships between counting in steps of 2, 4 and 8, eg doubling and halving, commutativity.
- Each tables fact can be represented with an array, number line and bar model.
- Knowing a tables fact mean you can derive a division fact using inverse.
- Division can be sharing or grouping.
- Use the language of 'equal groups of', when multiplying or dividing.
- Knowing how each number in a multiplication and division number sentence relates to a problem, eg There were 24 oranges put into bags with 4 in each bag. How many bags are needed? 24 ÷ 4 = 6, where 24 = the oranges, 4 is the number in each bag and 6 is the number of bags needed.

Curriculum strands

Within the document, the national curriculum programme of study domain number and place value is broken down into smaller curriculum strands to support precise identification of need. The curriculum strands identified are:

- multiplication and division facts
- mental calculations
- written calculations
- inverse operations, estimating and checking answers.

Problem-solving

Teacher assessment should consider to what extent the pupil is able to apply conceptual understanding of multiplication and division to solve problems.

Curriculum strand – Multiplication and division facts

Skills, Knowledge and Concepts		NC expectations – Year 3
Can count in 4s (8s) to tenth multiple.	Can identify multiples of 4 (8) on a number line.	Count from 0 in multiples of 4, 8, 50 and 100 Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
Can recognise patterns between 4 and 8 tables.	Can organise multiples of 4 (8) counters into an array (grouping) recording as x (÷) number sentence.	
Can count in multiples of 50 (100) to tenth multiple and record on a number line and with structured resources, eg dienes, 50p, £1 coins.	Can record x and ÷ number sentences to describe an array (2, 5, 3, 4, 8).	
Can use an array (number line) to find out how many groups of 3 (4, 8) are in a given multiple.	Can use counting objects to put into groups of 3 (4, 8) (sharing) and record as both x and ÷ sentences.	
Can use an array (number line) to find out how many groups of 3 (4, 8) are in a given number (non-multiples) to identify remainders.	Can recognise non-multiples of 5 x and 10 x fluently.	

Strategies:

- *use counters, objects to show groups, place on structured number lines*
- *transform groups of counters on structured number lines into arrays and vice versa*
- *use 2p, 5p, 10p coins to support counting in 2s, 5s and 10s*
- *use counters to represent multiplication/ division facts on paper - annotate with multiplication and division number sentences focusing on language of 'equal parts'*
- *use a problem context and label the numbers in the number sentence to the context.*
- *develop reasoning focused on the relationships between counting in 2s and 4s, 4s and 8s, 5s and 10s using concrete resources, arrays, number line recording and bar models to make these connections explicit.*

Curriculum strand – Mental calculations

<table>
<tr><th colspan="2">Skills, Knowledge and Concepts</th><th>NC expectations – Year 3</th></tr>
<tr><td>Can recall fluently 2, 5, 10, 3, 4, 8 multiples as tables facts.</td><td rowspan="2">Can use known facts to work out next product in 3 (4, 8) times tables, eg 4 x 2 = 8 so 4 x 3 = 12 as 4 more than 8 (4 x 2).</td><td rowspan="2">Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers x one-digit numbers, using mental and progressing to formal written methods*

*Ensure fluency with multiple representations for x/÷ facts before beginning work on more formal recording as an additional representation (not replacement).</td></tr>
<tr><td>Using arrays can use 10 multiplied by 3 (4, 8) to work out 9 multiplied by each number.</td></tr>
</table>

Strategies:

- *represent multiplication and division facts using arrays and number lines*
- *ensure pupils connect counting with calculations*
- *'read' arrays, bar models and number lines as both multiplication and division*
- *model how groups of 5 relate to groups of 10, groups of 2 relate to groups of 4, groups of 4 relate to groups of 8 using structured resources and counters.*

Curriculum strand – Written calculations

<table>
<tr><th colspan="2">Skills, Knowledge and Concepts</th><th>NC expectations – Year 3</th></tr>
<tr><td>Can draw an array to show any multiples of 3 (4, 8).</td><td rowspan="2">Can show repeated addition on a structured (unstructured) number line to find out how many groups of 3 (4, 8) there are in a given multiple up to twelfth (12th) multiple.</td><td rowspan="4">Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.</td></tr>
<tr><td>Can use bar models to show sharing (grouping) for division facts 3 (4, 8).</td></tr>
<tr><td>Can use counting objects, dienes, cuisenaire to show a two-digit multiplied by a one-digit number.</td><td rowspan="2">Can show repeated addition on a structured (unstructured) number line to find out how many groups of 3 (4, 8) there are in a given two-digit number.</td></tr>
<tr><td>Can use partitioning into 10s and 1s alongside arrays to multiply a two-digit number by a one-digit number (2, 3, 4, 5, 8).</td></tr>
</table>

Strategies:

- *support pupils to 'talk out loud' when recording number sentences referring to group size and how many groups needed*
- *make explicit links between number sentences and number line recording, arrays and or bar model diagrams*
- *ensure pupils can relate 'equal groups' used to the problem context*
- *ensure pupils understand and can model with resources, pictures and symbols the link between repeated addition and multiplication*
- *ensure pupils understand and can model with resources, pictures and symbols the link between repeated subtraction* and division.

Curriculum strand – Inverse operations, estimating and checking answers

<table>
<tr><th colspan="2">Skills, Knowledge and Concepts</th><th>NC expectations – Year 3</th></tr>
<tr><td rowspan="2">Recognise and use patterns from knowing sequences of multiples to check answers for 3 (4, 8) timetable facts.</td><td>Recognise multiples of 2 (4, 8) will be even.</td><td rowspan="2">Estimate the answer to a calculation and use inverse operations to check answers (from addition and subtraction)</td></tr>
<tr><td>Anticipate remainders when working with non-multiples of divisor (2, 5, 10, 3, 4, 8).</td></tr>
</table>

Strategies:

- *focus on the language of part-part-whole to support inverse link to recording with a bar model*
- *model using counters and structured resources how one number fact is related to another*
- *place counters and or structured resources on blank bar model templates before recording in books.*

Curriculum strand – Problem-solving

Skills, Knowledge and Concepts	NC expectations – Year 3
Use problem solving contexts to develop fluency with tables facts and linked division facts and understanding of the role of each numbers in a number fact, ie group or multiplier?	Solve problems, including missing number problems involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Strategies:

- *ensure pupils can model number sentence solutions to problems involving multiplication and division using pictures, concrete resources linked to mathematical models such as arrays and bar models*
- *ensure pupils develop conceptual understanding of the link between counting along a number line in multiples of 2,5 and 10 and multiplication facts*
- *ensure pupils are able to draw pictures and use mathematical representations such as bar models and arrays to solve problems involving sharing structure for division*
- *ensure pupils can relate numbers used in number sentences to the problem context.*
- *develop pupil's use of vocabulary and phrases related to the multiplication and division symbol.*

Fractions

Year 3: National Curriculum Notes and Guidance (non-statutory)

Pupils connect tenths to place value, decimal measures and to division by 10.

They begin to understand unit and non-unit fractions as numbers on the number line, and deduce relations between them, such as size and equivalence. They should go beyond the (0, 1) interval, including relating this to measure.

Pupils understand the relation between unit fractions as operators (fractions of), and division by integers.

They continue to recognise fractions in the context of parts of a whole, numbers, measurements, a shape, and unit fractions as a division of a quantity.

Pupils practise adding and subtracting fractions with the same denominator through a variety of increasingly complex problems to improve fluency.

Key Concepts

- Fractions are equal parts of a whole.
- The denominator represents the number of equal parts.
- There are links and relationships between unit fractions, eg $^1/_2$, $^1/_4$ and $^1/_8$.
- Equal parts of shapes do not need to be congruent but need to be equal in area.
- $^1/_{10}$ is the same as 0.1.
- Bar models are useful representations to support comparison and equivalence of fractions.
- The number line is a useful representation to show fractions are numbers and can be placed on the number line.

Curriculum strands

Within the document, the national curriculum programme of study domain number and place value is broken down into smaller curriculum strands to support precise identification of need. The curriculum strands identified are:

- counting in fractional steps
- recognising fractions – unit fractions
- comparing fractions
- equivalence
- addition and subtraction of fractions.

Problem-solving

Teacher assessment should consider to what extent the pupil is able to apply conceptual understanding of fractions to solve problems.

Curriculum strand – Counting in fractional steps using an number line

Skills, Knowledge and Concepts		NC expectations – Year 3
Can record counting in any unit fractions on a number line up to one whole (beyond one whole).	Can record counting in tenths on a number line up to one whole (beyond one whole).	Count up and down in tenths

Strategies:

- *use concrete objects and shapes to model counting in fractional steps with reference to a number line model*
- *use unstructured number lines to record position of unit fractions compared to one whole*
- *record using pictures of wholes and part-wholes to discuss how many equal parts.*

Curriculum strand Recognising fractions (halves, quarters – see Year 1 Planning Tool; thirds, two quarters, three quarters – see Year 2 Planning Tool)

Skills, Knowledge and Concepts		NC expectations – Year 3
Can relate $^1/_2$, $^1/_4$, $^1/_5$, $^1/_{10}$, $^1/_3$, $^1/_8$ with division facts.	Knows how to find non-unit fractions by counting/ repeated addition, eg $^4/_5 = {}^1/_5 + {}^1/_5 + {}^1/_5 + {}^1/_5$.	Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators. Recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10.
Knows how to find non-unit fractions by subtraction, eg $^4/_5 = {}^5/_5 - {}^1/_5$.	Can use arrays and objects for multiples of 3, 4, 8 to show fractions of an amount including non-unit fractions, (eg $^5/_8$).	
Can use resources to show tenths by dividing into equal parts.		

Skills, Knowledge and Concepts		NC expectations – Year 3
Can relate finding a tenth/ dividing by 10 to place value charts, eg 5 to 0.5, 12 to 1.2.	Can relate finding a tenth with measurement, (eg mm/cm) and scales.	Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators.
Know 'one tenth' can be recorded as $^{1}/_{10}$ and 0.1.	Can describe diagrams (bar models and fraction walls) and shapes using any unit fraction (non-unit fraction).	
Can recognise/ reason when a shape or quantity is 'not' the given fraction.		

Strategies:

- *ensure pupils are working practically before and alongside any recording using appropriate vocabulary to talk about what they are doing*
- *ensure pupils use whole sentence answers, eg 'if the circle is the whole shape then this is one tenth of the shape/circle'*
- *make links with everyday tasks and routines*
- *use blank template bar models with resources before recording in books*
- *fold shapes, including strips in half repeatedly to notice families of fractions, eg 1/3 and 1/6, 1/5 and 1/10, label the equal parts.*

Key contexts for developing skills, knowledge and concepts of fractions:

- *shape*
- *number*
- *time*
- *length*
- *capacity and volume.*

Curriculum strand – Comparing fractions

Skills, Knowledge and Concepts		NC expectations – Year 3
Can use fraction wall (bar models) families to compare fractions (more/ less than a half? – then compared to other unit fractions).	Can use a number line to mark unit fractions and fractions with same denominator relative to one whole.	Compare and order unit fractions and fractions with the same denominators.
Understand that when comparing unit fractions a larger denominator means that unit fractions is a smaller part of the same whole.	Know that two halves = 1 whole, four quarters = 1 whole, three thirds = 1 whole	

Strategies:

- *make fraction wall models starting with the same length strip*
- *use language of 'equal parts', more than/ less than and equal to.*

Develop conceptual understanding of comparison of fractional parts within one whole:

- *more/less than half*
- *how many more equal parts to one whole?*

Curriculum strand – Equivalence

<table>
<tr><th colspan="2">Skills, Knowledge and Concepts</th><th>NC expectations – Year 3</th></tr>
<tr><td rowspan="2">Can use bar models to show families of fractions:
Whole, $^1/_2$, $^1/_4$, $^1/_8$
Whole, $^1/_2$, $^1/_5$, $^1/_{10}$
Whole, $^1/_2$, $^1/_3$, $^1/_6$.</td><td>Can identify fractions = to one half and talk about relationship of numerator/ denominator in these fractions.</td><td rowspan="2">Recognise and show, using diagrams, equivalent fractions with small denominators.</td></tr>
<tr><td>Can use different rectilinear shapes and circles to show/ explain equivalent fractions, eg $^2/_6 = {}^1/_3$.</td></tr>
</table>

Strategies:

- *place concrete resources on to large blank templates for bar models and fraction walls*
- *ensure pupils 'talk out loud' using 'equal parts' language when representing number sentences*
- *make links with counting in fractional steps on number lines.*
- *develop conceptual understanding of equivalence in the context of number and shape using bar models and arrays.*

Curriculum strand – Addition and subtraction of fractions

<table>
<tr><th colspan="2">Skills, Knowledge and Concepts</th><th>NC expectations – Year 3</th></tr>
<tr><td>Can use bar models to write addition and subtraction number sentences, eg $^1/_3 + {}^2/_3 = {}^3/_3 = 1$ whole, eg $^5/_7 - {}^2/_7 = {}^3/_7$.</td><td rowspan="2">Can recognise patterns with knowing number bonds to all numbers to 10 with writing fraction addition and subtraction number sentences.</td><td rowspan="3">Add and subtract fractions with the same denominator within one whole, (eg $^5/_7 + {}^1/_7 = {}^6/_7$).</td></tr>
<tr><td>Use fraction walls to give two fractions with the same denominator that total one, eg $^2/_5 + {}^3/_5 = {}^5/_5 = 1$.</td></tr>
<tr><td>Know that when adding and subtracting fractions with the same denominator the denominator doesn’t change.</td><td>Use a fraction wall to find what is left when subtracting a fraction from 1, eg $1 - {}^1/_4 = {}^3/_4$.</td></tr>
</table>

Strategies:

- *construct bar models using families of facts using paper strips, shapes and rods*
- *relate part-whole reasoning for addition and subtraction.*
- *developing conceptual understanding of addition and subtraction of fractions within one whole.*

Supporting Resources

Diagnostic Assessment Tool: Number and Place Value, Addition and Subtraction

Name …….... ….. Year group ……. Date……. …….

Pupil profile: (notes and relevant information)

Oral counting ▪ Can you count to 10, 20, 15, …, 100? *Look for accurate number strings, teens, ty numbers, crossing tens boundaries. Keep going until accurate counting breaks down* ▪ Can you count back from 10, 20, 17, 32 in ones?	
Counting on ▪ Can you continue counting when I stop? *Start at 1 and at different points, eg start at 7, at 23*	

Object counting ▪ How many dinosaurs/objects do you think are here? ▪ Can you count them? Choose a handful ie. 3 – 10, 10 – 20, 20 – 30 of interesting objects to count e.g. dinosaurs not multilink *Look for accurate counting, moving, touching objects whilst saying the number name – one to one correspondence* ▪ Can you give me 6 beads? *Does the child know when to stop the count?*	
Writing numbers ▪ Can you write 2, 5, 8 etc (numbers to 10, numbers to 20, 2-digit numbers to 100)? *Look for reversals, lack of confidence, looking at resources to copy numbers from, reversals of 2-digit numbers, eg 52 ad 25*	

Number before, number after ▪ Can you say the number after/before … *In the range 0-10; 0-20; 0-30, 0-50, 0-100 etc* *Does the child start at one or can they say the next number in the sequence?*	
Number recognition 0 – 10, 20, 100 ▪ Can you read this number? ▪ Can you find this number? Using a set of out of order number cards eg 0 – 20, 100 … which is number 12? Number 21? …. *Can the child find a given number? Distinguish between: 13, 30 and 31 etc* *Know names for multiples of 10 to 100?*	
Number sequencing ▪ Can you put these numbers in order from the smallest to the largest? *Using a set of shuffled consecutive numbers, then a set of random number cards 0 – 100 (as appropriate) can the child order consecutive numbers then random numbers?*	

Place value ▪ Does the child understand the value of each digit in 2-digit (then 3-digit) numbers? *Using structured, eg Numicon, Dienes and unstructured eg bundles of straws, bags of 10 objects, coins and arrow cards can the child make the numbers in the previous three sections? Can they talk about the value of the digits and find their position on a structured number line?*	
Counting in steps ▪ Starting from 0 can you count forward/backward in 2s, 10s? – Starting from zero. – Starting from a single digit number. – Starting from any number.	

The following are examples of calculations in a context. You will need to adjust the numbers in the calculations according to responses to previous questions.

Addition: If I have 4 dinosaurs and 3 more come along, how many will I have altogether? (5 add 3, 9 +4, 9+10, 13 +9, 13 +19 etc) Does the child: • use objects, count both sets • use pictorial recording • write a number sentence to match the calculation needed • count on in ones • use known number facts • count on from the biggest number • count on using a structured number line • count on in tens/ones using an empty number line • use any other recording to help with the calculation?	
Subtraction: If I have 7 dinosaurs and 4 go off for a walk, how many will I have left? (8 subtract 4, 10-7, 23-4, 23-10) Does the child: • use objects, count a set, take some away, count how many are left • use pictorial recording • write a number sentence to match the calculation needed • count back in ones • count back in ones using a structured number line • use known number facts • count back in tens/ones using a empty number line • count on when appropriate • use any other recording to help with the calculation?	

Summary notes: Addition and Subtraction

Diagnostic Assessment: Multiplication and Division

Name…… Year group ……. ……. Date ……

Pupil profile: (notes and relevant information)

Counting in steps ▪ Can you count forward/backward in 2s, 10s? Starting from zero (multiples). Starting from a single digit number. Starting from any number. ▪ Can you count forward/ backward in multiples of 5s?	
Doubling and halving What is double this number? eg 3, 5, 7,10 What is half this number? eg 4, 8, 10, 20	

What strategies does the child use? If this is half of the number can you tell me what the whole number was? *(Could have half of the quantity 'hidden' under a cover)* What is half of this whole shape? (*paper shapes needed to enable pupils to fold or draw lines to show half)*	

The following are examples of calculations in a context. You will need to adjust the numbers in the calculation according to responses to previous questions.

Multiplication I have 4 sweets in one party bag, how many sweets would be in 2 bags? (use other multiples, where known facts might be used and where children have to use other strategies) Does the child: • use objects to solve the calculation • use pictorial recording • write a number sentence to match the calculation needed • use repeated addition • count in multiples • use multiplication facts • use a number line or other informal recording?	

Division: If I have 6 cakes and share them between 2 people, how many will each person have? (sharing) I have 6 cakes for a party and I need to put 2 cakes on each plate, how many plates will I need? (grouping) Does the child: • use objects to solve the calculation • draw a picture or other informal recording • use a number line • use repeated addition/subtraction • count in multiples • write a number sentence to match the calculation needed • use multiplication/division facts?	

Summary notes

Mathematical Diagnostic Assessment: Fractions

Name …….. Year group …… Date ……..

Pupil Profile (notes and relevant information)

Half (quarter) of a shape Can you fold • this square (triangle, rectangle, circle, etc) in half (quarters)? • these strips of paper (different lengths) in half (quarters)? Give different shapes with folds (not all that fold into halves/ quarters), • can you show me which shapes have been folded in half (quarter)? • How do you know these are folded in half (quarter)?	
Symbolisation • How do you record one half (one quarter, one third) in symbols?	

Half/quarter of a quantity • Show me half of these fish (start with 6 fish, build up quantity as appropriate) Pupil to use chosen objects to show how they are working out their answer. • Show me one quarter of…. • Shade half of a shape, when the shape is divided into an even number of equal pieces. • Shade one quarter of a shape when the shape is divided into 8 (12, 16, 20, etc) pieces.	
Counting in halves/quarters • How many halves make one (two, three, etc) whole? • How many halves do you have if you have two and a half apples? • How many quarters make one whole one? • Can you count in halves (quarters)? • If one quarter of a pizza is eaten, what fraction of the pizza is left? • If one quarter of children go home from school for lunch, what fraction of children have lunch at school?	
Fractions of a quantity (Bar model) • Use a bar model to show half of eight (10, 12, 18, etc). • Pupils use objects to show solution Use a bar model to show one quarter of 8 (12, 16, 20, etc).	

Comparing Fractions Using a shape • Is a quarter of this shape more or less than half of this shape? • Is a third of this shape more or less than a half of this shape? Using a quantity, eg 12 counters • Is half of these 12 counters more or less than one quarter of these 12 counters? • Is one third of these 12 counters more or less than one half of these 12 counters?	

Summary notes: Fractions

Counting

Recognising (shape/number)

Comparing (shape/number)

Equivalence

The following are examples of calculations in a context. Adjust the fractions used and numbers in the calculations according to responses to previous questions. Provide opportunities to use objects, draw pictures and or diagrams to show solutions.

Halves and quarters of one whole • If one half of a pizza is eaten, what fraction of the pizza is left? • If one quarter of a pizza is eaten, what fraction of the pizza is left? • If two quarters of a pizza is eaten, what fraction of the pizza is left?	
Halves and quarters of a number • If one quarter of children go home from school for lunch, what fraction of children have lunch at school? • If half of class are boys, and there are ten boys. How many children are there in the class? • One quarter of the cookies are chocolate and there are five chocolate cookies. How many cookies are there altogether?	
Third of one whole • If one third of a pizza is eaten, what fraction of the pizza is left?	
Thirds of a number • There were 12 cakes. The girls ate one third of them. How many did they eat?	

Summary notes: Fractions

National Curricum 2014 – Year 1

Year	Number and place value	Addition and subtraction	Multiplication and division	Fractions	Measurement	Geometry	
						Properties of shapes	Position and direction
YEAR 1	Pupils should be taught to: ▪ count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number ▪ count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens ▪ given a number, identify one more and one less ▪ identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least ▪ read and write numbers from 1 to 20 in numerals and words.	Pupils should be taught to: ▪ read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs ▪ represent and use number bonds and related subtraction facts within 20 ▪ add and subtract one-digit and two-digit numbers to 20, including zero ▪ solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9.	Pupils should be taught to: ▪ solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Pupils should be taught to: ▪ recognise, find and name a half as one of two equal parts of an object, shape or quantity ▪ recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.	Pupils should be taught to: ▪ compare, describe and solve practical problems for: ▪ lengths and heights (e.g. long/short, longer/shorter, tall/short, double/half) ▪ mass or weight (e.g. heavy/light, heavier than, lighter than) ▪ capacity/volume (full/empty, more than, less than, quarter) ▪ time (quicker, slower, earlier, later) ▪ measure and begin to record the following: ▪ lengths and heights ▪ mass/weight ▪ capacity and volume ▪ time (hours, minutes, seconds) ▪ recognise and know the value of different denominations of coins and notes ▪ sequence events in chronological order using language such as: before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening ▪ recognise and use language relating to dates, including days of the week, weeks, months and years ▪ tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.	Pupils should be taught to: ▪ recognise and name common 2-D and 3-D shapes, including: ▪ 2-D shapes (e.g. rectangles (including squares), circles and triangles) ▪ 3-D shapes (e.g. cuboids (including cubes), pyramids and spheres).	Pupils should be taught to: ▪ describe position, directions and movements, including half, quarter and three-quarter turns.

National Curriculum 2014 – Year 2

Year	Number and place value	Addition and subtraction	Multiplication and division	Fractions	Measurement	Geometry		Statistics
						Properties of shapes	Position and direction	
YEAR 2	Pupils should be taught to: ▪ count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward ▪ recognise the place value of each digit in a two-digit number (tens, ones) ▪ identify, represent and estimate numbers using different representations, including the number line ▪ compare and order numbers from 0 up to 100; use <, > and = signs ▪ read and write numbers to at least 100 in numerals and in words ▪ use place value and number facts to solve problems.	Pupils should be taught to: ▪ solve problems with addition and subtraction: ▪ using concrete objects and pictorial representations, including those involving numbers, quantities and measures ▪ applying their increasing knowledge of mental and written methods ▪ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 ▪ add and subtract numbers using concrete objects, pictorial representations, and mentally, including: ▪ a two-digit number and ones ▪ a two-digit number and tens ▪ two two-digit numbers ▪ adding three one-digit numbers ▪ show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot ▪ recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.	Pupils should be taught to: ▪ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers ▪ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs ▪ show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot ▪ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	Pupils should be taught to: ▪ recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity ▪ write simple fractions e.g. $^1/_2$ of 6 = 3 and recognise the equivalence of $^2/_4$ and $^1/_2$.	Pupils should be taught to: ▪ choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels ▪ compare and order lengths, mass, volume/capacity and record the results using >, < and = ▪ recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value ▪ find different combinations of coins that equal the same amounts of money ▪ solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change ▪ compare and sequence intervals of time ▪ tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times. • Know the number of minutes in an hour and the number of hours in a day	Pupils should be taught to: ▪ identify and describe the properties of 2-D shapes, including the number of sides and symmetry in a vertical line ▪ identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces ▪ identify 2-D shapes on the surface of 3-D shapes, for example a circle on a cylinder and a triangle on a pyramid ▪ compare and sort common 2-D and 3-D shapes and everyday objects.	Pupils should be taught to: ▪ order and arrange combinations of mathematical objects in patterns ▪ use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise).	Pupils should be taught to: ▪ interpret and construct simple pictograms, tally charts, block diagrams and simple tables ▪ ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity ▪ ask and answer questions about totalling and comparing categorical data.

National Curriculum 2014 – Year 3

Year	Number and place value	Addition and subtraction	Multiplication and division	Fractions	Measurement	Geometry: Properties of shapes	Statistics
YEAR 3	Pupils should be taught to: ▪ count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number ▪ recognise the place value of each digit in a three-digit number (hundreds, tens, ones) ▪ compare and order numbers up to 1000 ▪ identify, represent and estimate numbers using different representations ▪ read and write numbers up to 1000 in numerals and in words ▪ solve number problems and practical problems involving these ideas.	Pupils should be taught to: ▪ add and subtract numbers mentally, including: ▪ a three-digit number and ones ▪ a three-digit number and tens ▪ a three-digit number and hundreds ▪ add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction ▪ estimate the answer to a calculation and use inverse operations to check answers ▪ solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.	Pupils should be taught to: ▪ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables ▪ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods ▪ solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which n objects are connected to m objects.	Pupils should be taught to: ▪ count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 ▪ recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators ▪ recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators ▪ recognise and show, using diagrams, equivalent fractions with small denominators ▪ add and subtract fractions with the same denominator within one whole (e.g. $^{5}/_{7} + {}^{1}/_{7} = {}^{6}/_{7}$) ▪ compare and order unit fractions, and fractions with the same denominators ▪ solve problems that involve all of the above.	Pupils should be taught to: ▪ measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml) ▪ measure the perimeter of simple 2-D shapes ▪ add and subtract amounts of money to give change, using both £ and p in practical contexts ▪ tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks ▪ estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes, hours and o'clock; use vocabulary such as a.m./p.m., morning, afternoon, noon and midnight ▪ know the number of seconds in a minute and the number of days in each month, year and leap year ▪ compare durations of events, for example to calculate the time taken by particular events or tasks.	Pupils should be taught to: ▪ draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them ▪ recognise angles as a property of shape or a description of a turn ▪ identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle ▪ identify horizontal and vertical lines and pairs of perpendicular and parallel lines.	Pupils should be taught to: ▪ interpret and present data using bar charts, pictograms and tables ▪ solve one-step and two-step questions such as 'How many more?' and 'How many fewer?' using information presented in scaled bar charts and pictograms and tables.

Bibliography

General

https://www.sendgateway.org.uk/

https://thesendhub.co.uk/

https://www.gov.uk/government/publications/rochford-review-final-report

https://www.gov.uk/government/publications/send-code-of-practice-0-to-25

Mathematics

https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/335158/PRIMARY_national_curriculum_-_Mathematics_220714.pdf

https://www.ncetm.org.uk

https://www.ncetm.org.uk/resources/46689 Primary Assessment materials

https://www.ncetm.org.uk/resources/27041/ What makes a good resource? Special Educational Needs sector

https://www.gl-assessment.co.uk/products/sandwell-early-numeracy-test-sent/

https://www.nfer.ac.uk/for-schools/products-services/primary-schools/nfer-tests/

Cockburn A D and Littler G, *Mathematical Misconceptions* (Sage, 2008, ISBN: 9781847874412)

Haylock D and Cockburn A D, *Understanding Early Years Mathematics* (Sage, 1989, ISBN: 1853960748)

Haylock D and Cockburn A D, *Understanding Mathematics for Young Children: A guide for foundation and lower primary teachers* (Sage, 2008, ISBN: 141294726X)

Heibert J (et al), *Making sense: teaching and learning mathematics with understanding* (Heinemann, 1997, ISBN: 2854186052)

Houssart J, *Low Attainers in Primary Mathematics: The Whisperers and the Maths Fairy* (Routledge, 2004, ISBVN: 0-415-31554-9)

Hughes M, Children and Number (Blackwell, 1986, ISBN: 0-631-13581-2)

Nunes T and Bryant P, *Children Doing Mathematics* (Blackwell, 1998, ISBN: 0-631-184722-4)

Thompson I (eds), *Enhancing Primary Mathematics Teaching* (OUP, 2003, ISBN: 0-335-21375-8)

Thompson I (eds), *Teaching and Learning Early Number* (OUP, 1997, ISBN: 0-335-19851-1)

Worthington M and Carruthers E, *Children's Mathematics* (Paul Chapman, 2003, ISBN: 0-7619-4070-7)