

integer

-15, 7, 43
are all **integers**

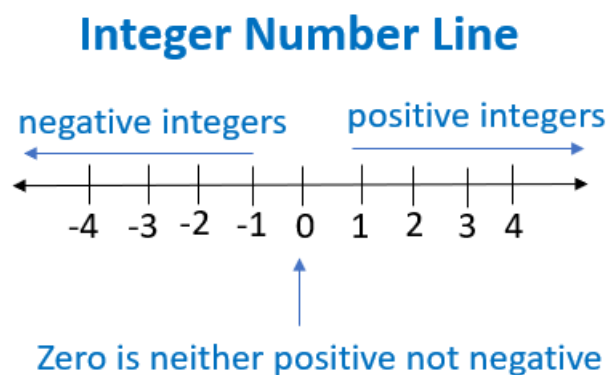


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Advisory Service

Picture, model, or diagram

Non-Example



0.5, -6.2, 81.9
are not **integers**

divisor

$$18 \div 3 = 6$$

3 is the **divisor**



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Picture, model, or diagram

Non-Example

18		
6	6	6

18 divided between **three**
The number of groups is the divisor

$$18 \div 3 = 6$$

18 is the dividend

dividend

18 ÷ 3 = 6
18 is the **dividend**



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Picture, model, or diagram

Non-Example

18		
6	6	6

18 divided between three
The original amount to be divided is the dividend

18 ÷ **3** = 6
3 is the divisor

quotient

$$18 \div 3 = 6$$

6 is the quotient



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Picture, model, or diagram

Non-Example

18		
6	6	6

18 divided between three gives six in each group
The size of the group is the quotient
(The 'answer' to a division calculation)

$$18 \div 3 = 6$$

18 is the dividend

lowest common multiple (LCM)

The **LCM** of 24 and 36 is **72**



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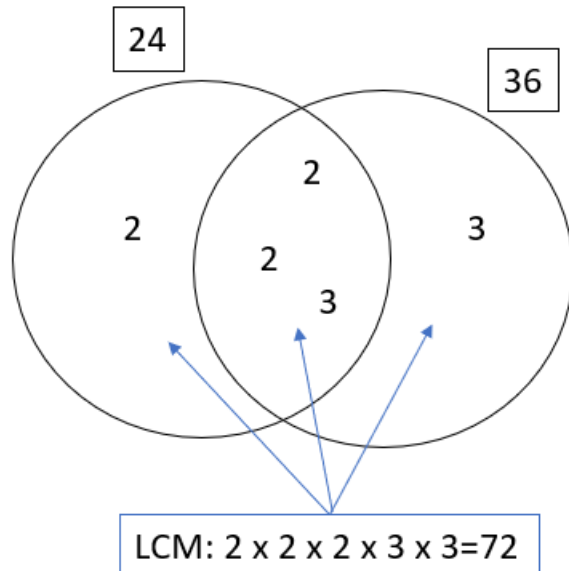
Picture, model, or diagram

Non-Example

Prime Factor Form

$$24 = 2 \times 2 \times 2 \times 3$$

$$36 = 2 \times 2 \times 3 \times 3$$



The HCF of 24 and 36 is **12**
(highest common factor)

highest common factor (HCF)

The **HCF** of 24 and 36 is **12**



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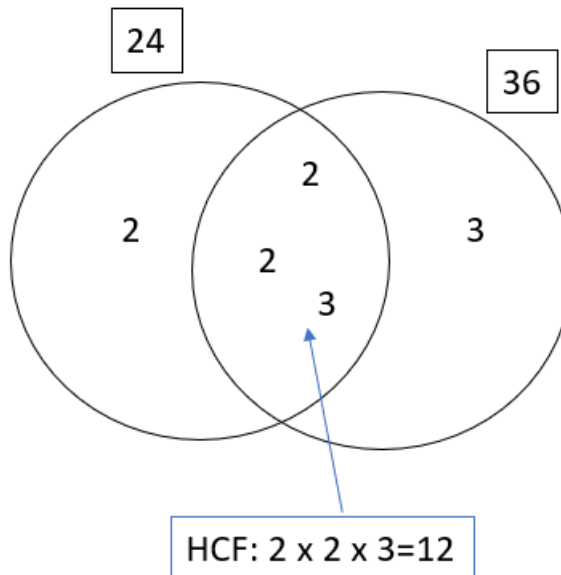
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Picture, model, or diagram

Prime Factor Form

$$24 = 2 \times 2 \times 2 \times 3$$

$$36 = 2 \times 2 \times 3 \times 3$$



Non-Example

The **LCM** of 24 and 36 is **72**
(lowest common multiple)

square root

$$\sqrt{9} = + / - 3$$

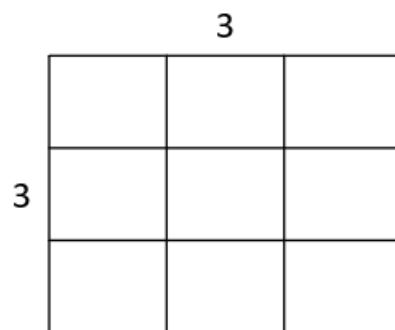
$$\left\{ \begin{array}{l} 3 \times 3 = 9 \\ -3 \times -3 = 9 \end{array} \right\}$$



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Picture, model, or diagram



$$3 \times 3 = 3^2$$

$$3 \times 3 = 9$$

$$\sqrt{3^2} = 3$$

Non-Example

$$3\sqrt{8} = 2$$

$$\{ 2 \times 2 \times 2 = 8 \}$$

cube root

$${}^3\sqrt{8} = 2$$

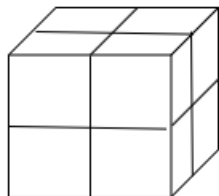
$$\{2 \times 2 \times 2 = 8\}$$



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Picture, model, or diagram



$$2 \times 2 \times 2 = 2^3$$

$$2 \times 2 \times 2 = 8$$

$${}^3\sqrt{8} = 2$$

Non-Example

$$\sqrt{9} = + / - 3$$

$$\begin{cases} 3 \times 3 = 9 \\ -3 \times -3 = 9 \end{cases}$$

product rule (for counting)

The number of different ways 3 hats can be distributed amongst 3 people is $3 \times 2 \times 1 = 6$

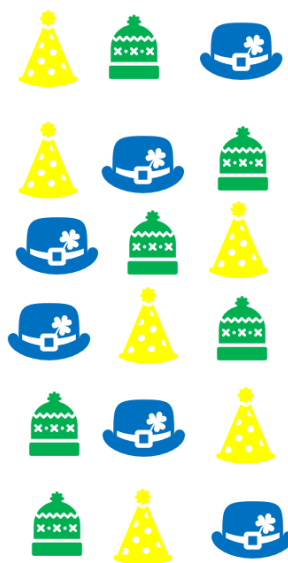


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Picture, model, or diagram

Non-Example



Person 1 has three choices (YGB)
Person 2 then has two choices (GB)
Person 2 then has one choice (B)

The probability of selecting yellow,
then green, then blue is

$$\frac{1}{6}$$

index (indices)

$14^3 = 14 \times 14 \times 14 = 2744$

14 is the base
3 is the index (or power)



$x^1=14$

x^4	x^3	x^2	x^1	x^0
38416	2744	196	14	1

$14 \times 3 = 42$

surd

$$\sqrt{3}$$

{3 is an 'imperfect square'}

A surd is not a whole number. It's decimal equivalent is a non-repeating, non-terminating decimal.
You cannot place it accurately on a number-line.

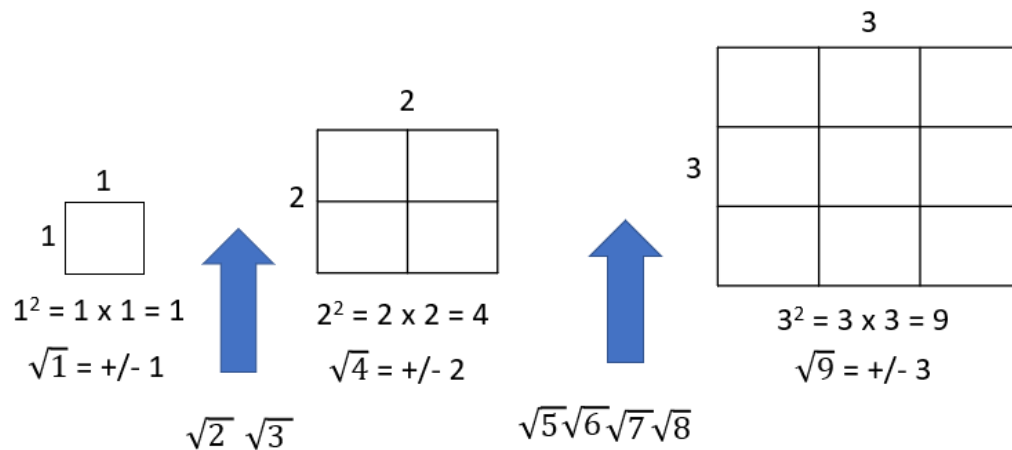


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Picture, model, or diagram

Non-Example



$$\sqrt{4}$$

{4 is a 'perfect square' and can be evaluated as +/- 2 }

rationalise (denominator)

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

3 is a rational number ~ you can place it on a number-line



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Picture, model, or diagram

Non-Example

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

$$\frac{\sqrt{b}}{\sqrt{b}} = 1$$

$$\frac{2}{\sqrt{3}}$$

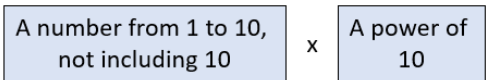
This fraction has an irrational (surd) denominator
A surd is not a whole number. It's decimal equivalent
is a non-repeating, non-terminating decimal. You cannot
place it accurately on a number-line

standard (index) form

3.754×10^5



Numbers in standard form have two parts



Standard index form	Ordinary form
1×10^3	1000
1×10^2	100
1×10^1	10
1×10^0	1
1×10^{-1}	0.1
1×10^{-2}	0.01
1×10^{-3}	0.001

375 400

upper bound

The length of a book is 27.3 cm to one decimal place.
What is the **upper bound** of the book length?

The **upper bound** is +0.05cm

$27.25 \leq \text{book length} < 27.35$

The **upper bound** is 27.35cm



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Picture, model, or diagram

Non-Example

The length of a book is 27.3 cm to one decimal place
What is the longest length the book could be ?

27.25 27.26 27.27 27.28 27.29 27.3 27.31 27.32 27.33 27.34 27.35



Any number above
(and including) this value
rounds to 27.3 to one decimal place

LOWER BOUND

27.25

Any number below
(and not including) this value
rounds to 27.3 to one decimal place

UPPER BOUND

27.35

The lower bound is -0.05cm

$27.25 \leq \text{book length} < 27.35$

The lower bound is 27.25cm

lower bound

The length of a book is 27.3 cm to one decimal place.
What is the **lower bound** of the book length?

The **lower bound** is -0.05cm

$27.25 \leq \text{book length} < 27.35$

The **lower bound** is 27.25cm



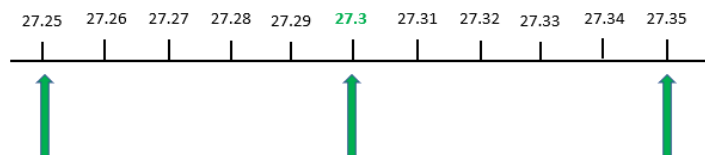
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Picture, model, or diagram

Non-Example

The length of a book is 27.3 cm to one decimal place
What is the longest length the book could be ?



Any number above
(and including) this value
rounds to 27.3 to one decimal place

LOWER BOUND

27.25

Any number below
(and **not** including) this value
rounds to 27.3 to one decimal place

UPPER BOUND

27.35

The upper bound is +0.05cm

$27.25 \leq \text{book length} < 27.35$

The upper bound is 27.35cm

substitute

Evaluate $7x - 10$ when $x = 12$

substitute $x = 12$ into the expression

$$7(12) - 10 = 74$$



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x	x	x	x	x	x	x	-10
12	12	12	12	12	12	12	-10
74							

Find x when $7x - 10 = 74$
Solve for x

$$7x = 84 \quad (\div 7)$$

$$x = 12$$

inequality

$$73 \leq t < 81$$

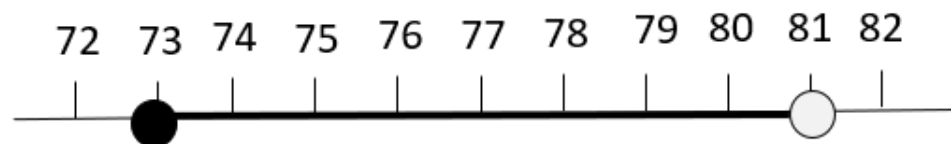


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Picture, model, or diagram

Non-Example



$$t = 80$$

expand (brackets)

$$(x + 3) (x - 5) = x^2 - 2x - 15$$



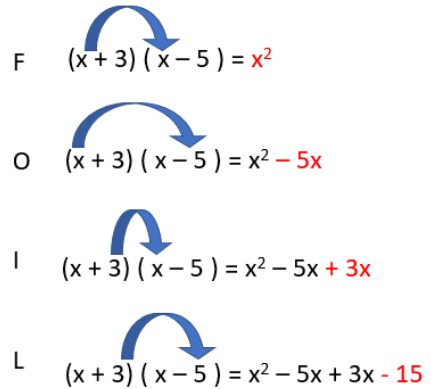
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Picture, model, or diagram

Non-Example

F = first pair
O = outer pair
I = inner pair
L = last pair



	x	+3
x	x^2	$+3x$
-5	$-5x$	-15

$$3 \times 4 + 7 \times 4 = 10 \times 4$$

factorise

$$x^2 - 2x - 15 = (x + 3)(x - 5)$$



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Picture, model, or diagram

Non-Example

Factor pairs to -15

$$-15 = +3 \times -5$$

$$-15 = -3 \times +5$$

$$-15 = -1 \times +15$$

$$-15 = +1 \times -15$$

	x	+3
x	x^2	$+3x$
-5	$-5x$	-15

Which pair also sum to -2
For the x-term?

$$-2 = +3 - 5$$

$$10 \times 4 = 3 \times 4 + 7 \times 4$$

quadratic (function)

$$y = 3x^2 + 5x - 6$$

In general, **quadratic functions** are of the form
 $y = ax^2 + bx - c$

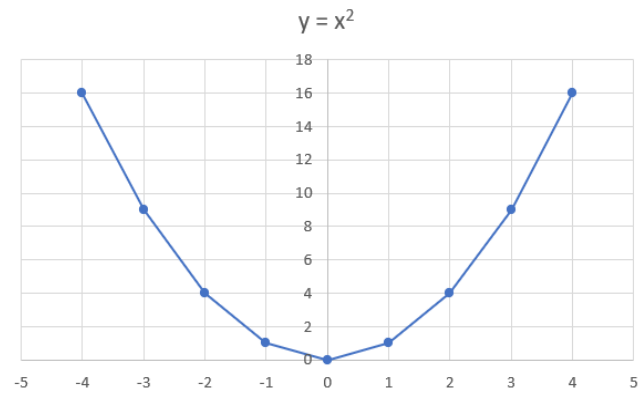


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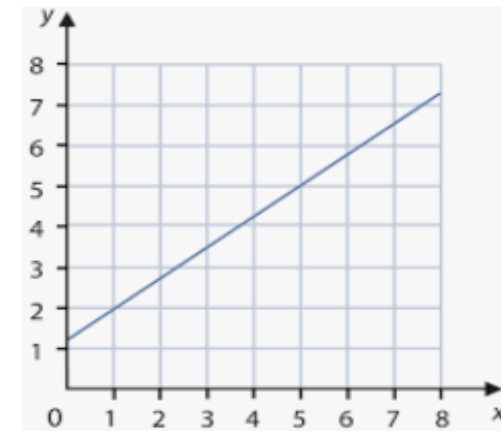
Non-Example



$$y = x^2$$

$$y = ax^2 + bx - c$$

($a = 1$; $b = 0$; $c = 0$)



$$y = x + 1$$

n^{th} term

15, 18, 21, 24, 27 ...

The n^{th} term of this sequence is

$$3n+12$$

$$U_n = 3n+12$$

$$n=1, U_1 = 3(1) + 12 = 15$$

$$n=2, U_2 = 3(2) + 12 = 18$$

$$n=3, U_3 = 3(3) + 12 = 21$$

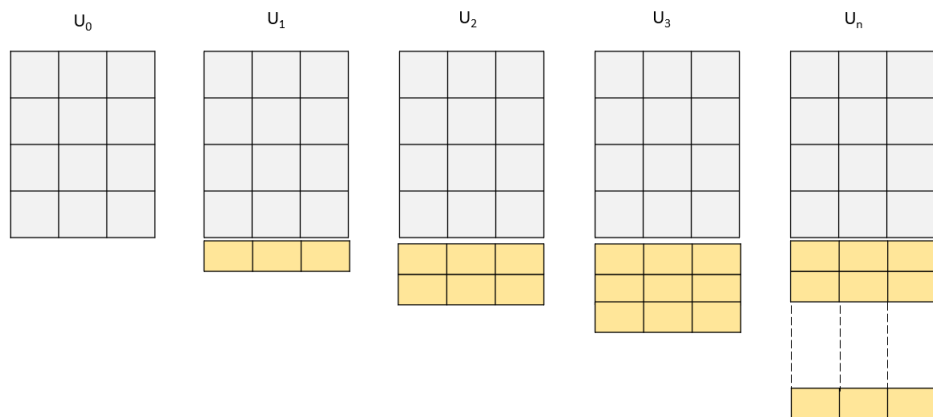


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Picture, model, or diagram

Non-Example



$$U_n = 3n+12$$

15, 18, 21, 24, 27 ...

The term-to-term rule for this sequence is 'add 3'

algebraic fraction

$$\frac{x^2}{x+3}$$



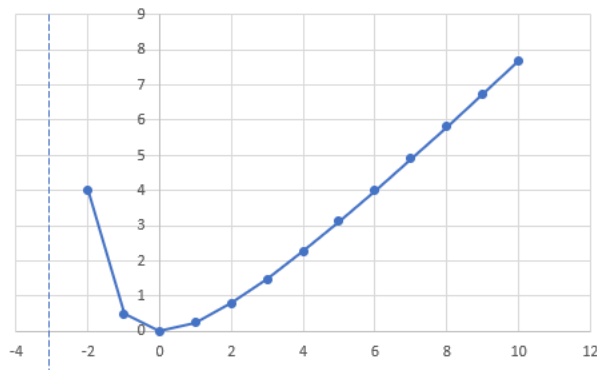
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Picture, model, or diagram

Non-Example

$$y = \frac{x^2}{x+3} \text{ for } x > -3$$



This graph has an asymptote at $x = -3$

$$2x(x+4)$$

identity ($\equiv$)

$$a^2 - b^2 \equiv (a + b)(a - b)$$

$$(a - b)(a + b) = a^2 + ab - ab - b^2$$

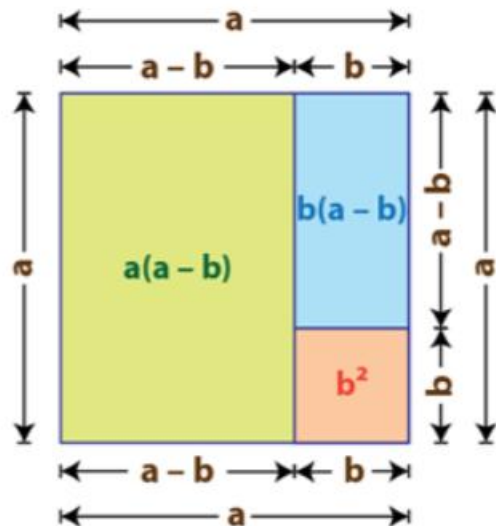


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Picture, model, or diagram

Non-Example



$$ab \approx 17$$

function

$$f(x) = 3x - 12$$



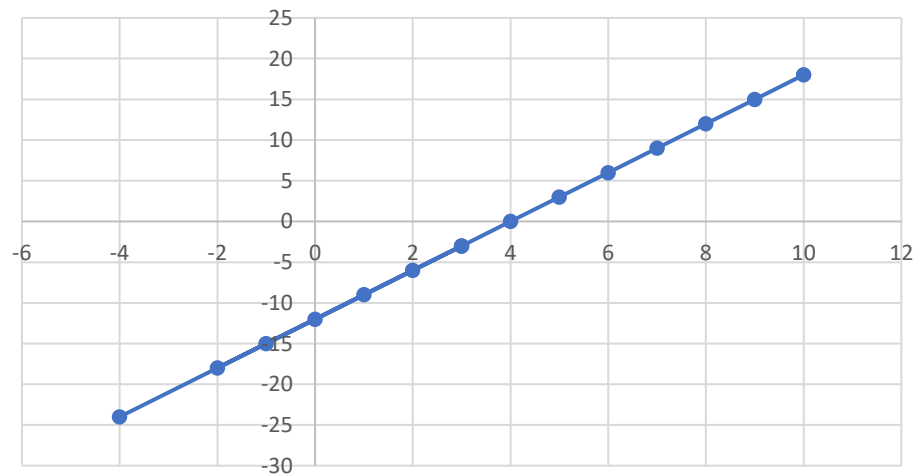
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Picture, model, or diagram

Non-Example

$$f(x) = 3x - 12$$



$$3x - 12$$

inverse function

$$f^{-1}(x)$$

$$f(x) = 3x - 12$$

$$f^{-1}(x) = \frac{x + 12}{3}$$

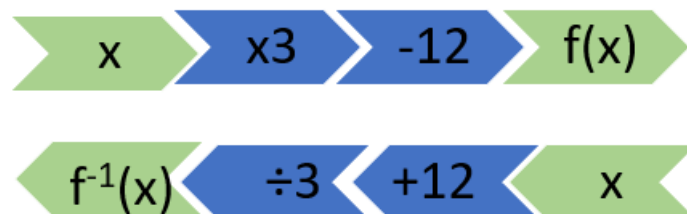


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Picture, model, or diagram

Non-Example



$$3x - 12$$

composite function

$gf(x)$

$$f(x) = x + 4$$

$$g(x) = x^2$$

$$gf(x) = (x + 4)^2$$

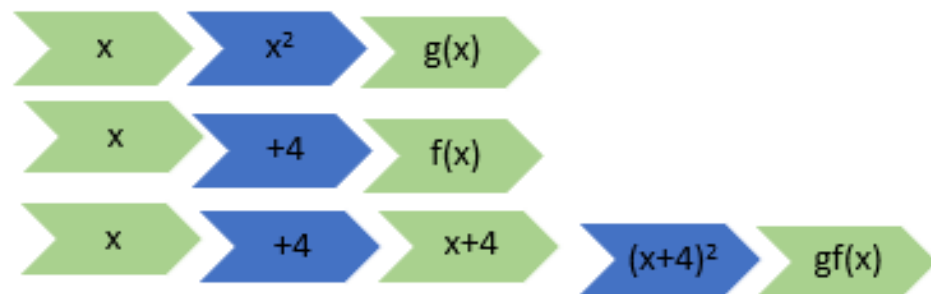


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Picture, model, or diagram

Non-Example



$$g(x) = x^2$$

turning point

$$y = x^2 - 6x + 4$$

Complete the square

$$y = (x-3)^2 - 5$$

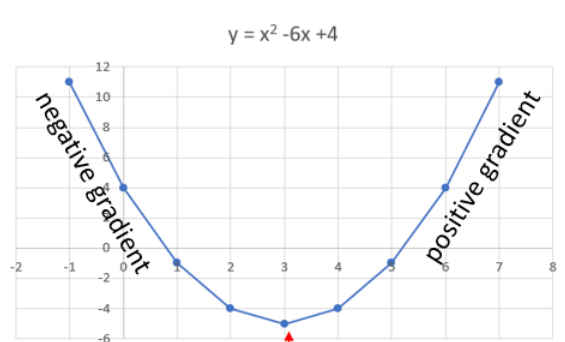
turning point is at (3,-5)



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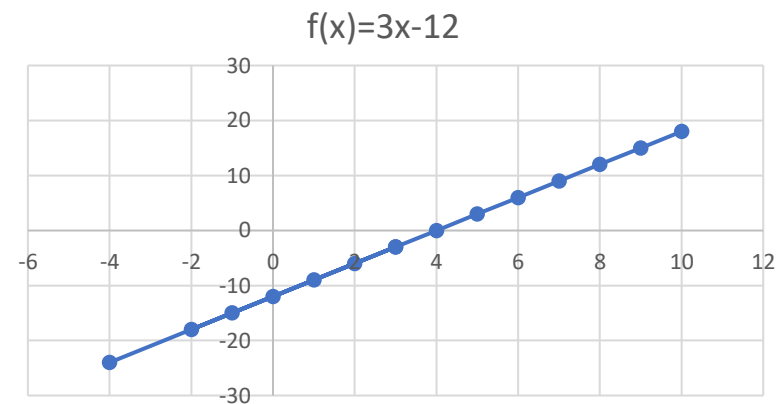
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Picture, model, or diagram



Turning point at (3 , -5)

Non-Example



This linear graph has no turning point

iteration

Use the **iterative formula** $x_{n+1} = 1 + \frac{11}{x_n - 3}$
and the starting value $x_1 = -2$ to find a value for x_4

$$x_2 = 1 + \frac{11}{-2-3} \quad x_2 = -1.2$$

$$x_3 = 1 + \frac{11}{-1.2-3} \quad x_3 = -1.619$$

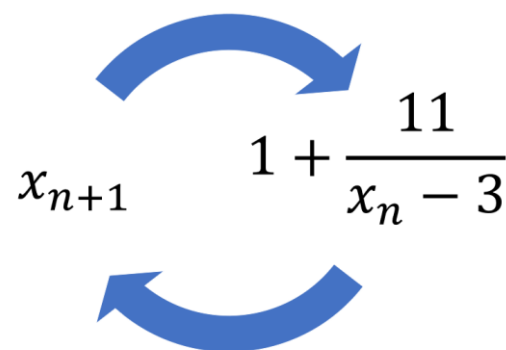
$$x_4 = 1 + \frac{11}{-1.619-3} \quad x_4 = -1.381$$



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Picture, model, or diagram



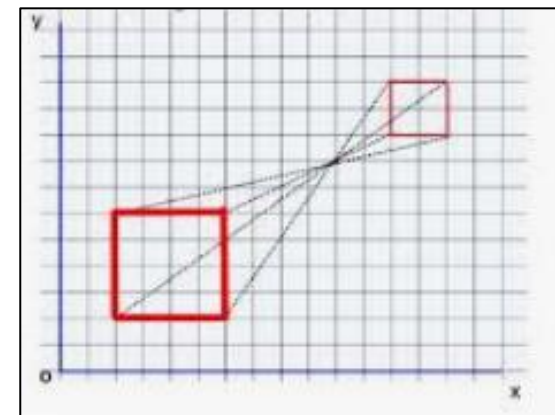
Non-Example

Substitute $x = -2$ into the equation
 $y = 1 + \frac{11}{x-3}$ to find the value of y

$$y = 1 + \frac{11}{-2-3}$$

$$y = -1.2$$

scale factor



This is an enlargement with a **scale factor of -2**

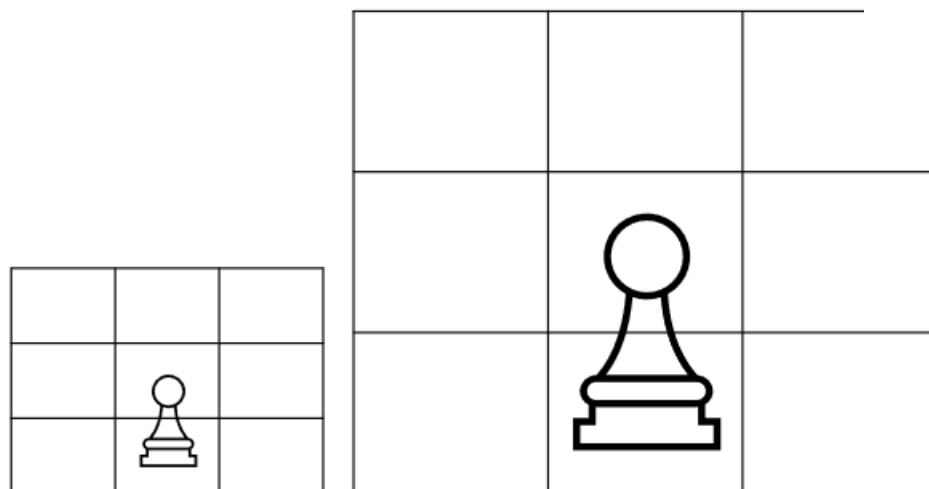


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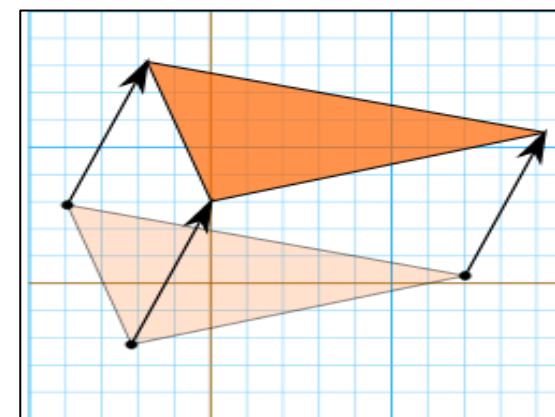
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Non-Example

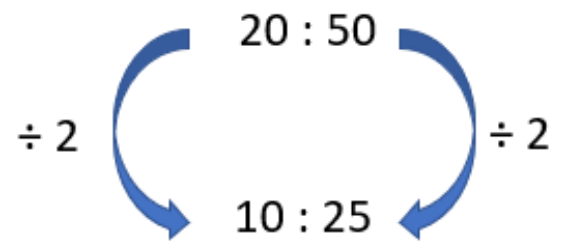
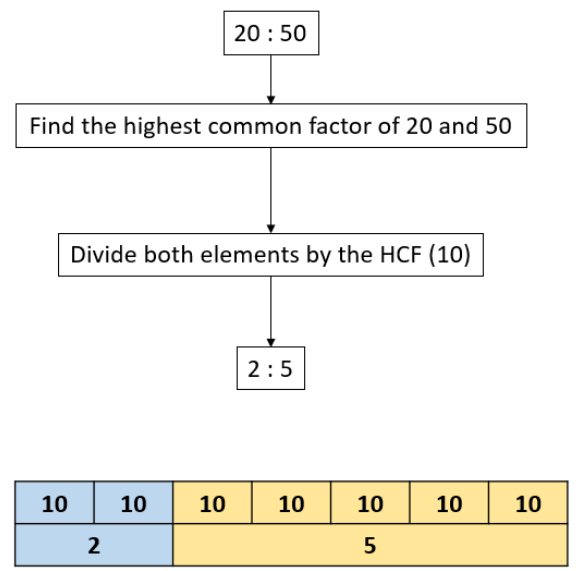
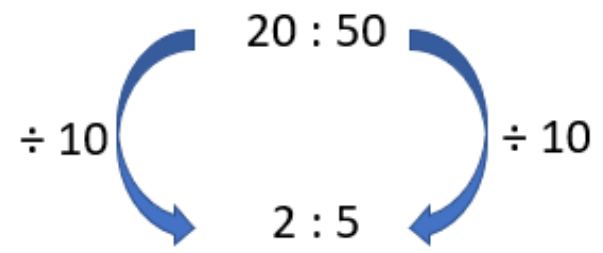


Scale factor 2  corresponding edges are twice as long



This is a translation.

reduce
(to simplest form)



percentage (decrease)

A watch is bought at a car boot sale for £50.
It is later sold in a shop for £25. What is the percentage loss?

The watch has decreased by £25

The **percentage decrease** is
(difference / original) x 100 = $25/50 \times 100 = 50\%$
A 50% loss has been made.



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original cost	
sold price	loss (difference)

The **percentage decrease** is (difference ÷ original) x 100

A watch is bought at a car boot sale for £40.
It is later sold in a shop for £50. What is the percentage profit?

The watch has increased by £10

The **percentage increase** is
(difference / original) x 100 = $10/40 \times 100 = 25\%$
25% profit has been made.

percentage (increase)

A watch is bought at a car boot sale for £40. It is later sold in a shop for £50. What is the percentage profit?

The watch has increased by £10

The **percentage increase** is
(difference / original) x 100 = $10/40 \times 100 = 25\%$
25% profit has been made.



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sold price	
original cost	difference

The **percentage increase** is (difference ÷ original) x 100

A watch is bought at a car boot sale for £50. It is later sold in a shop for £25. What is the percentage loss?

The watch has decreased by £25

The **percentage decrease** is
(difference / original) x 100 = $25/50 \times 100 = 50\%$
A 50% loss has been made.

**compound
(units)**

Calculate the density of aluminium if 20 cm³ has a mass of 54 g.

Density = mass ÷ volume

= 54 ÷ 20

= 2.7 g / cm³ (grams per cubic centimetre)

Density is measured using **compound units**

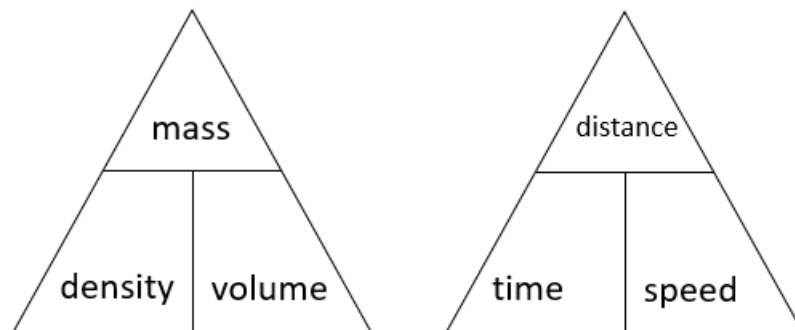


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Picture, model, or diagram

Non-Example



The mass of one apple is 10 grams

Calculate the mass of 5 apples

$$\underline{10\text{g} \times 5 = 50\text{g}}$$

directly proportional

y is directly proportional to x

when $x = 3$, $y = 15$

write an equation for y in terms of x

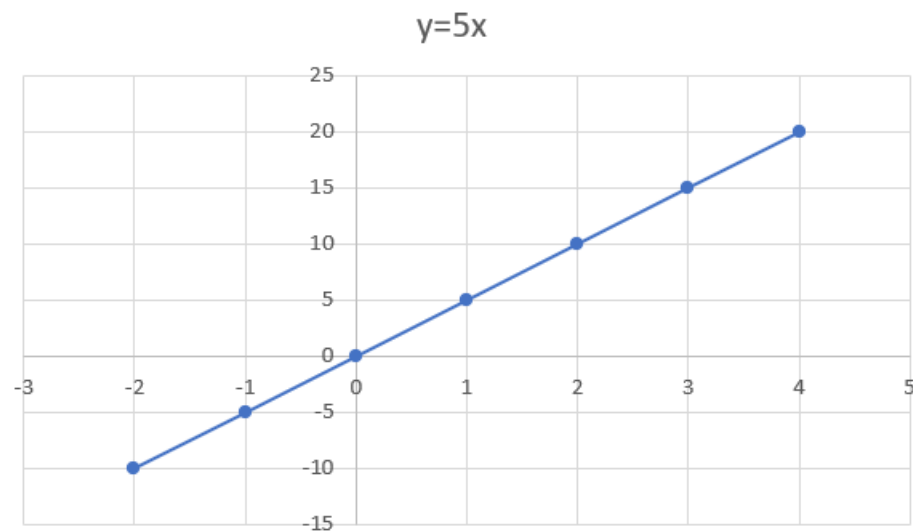
$y \propto x$ $y = kx$	→	$15 = 3k$ $k = 5$ $y = 5x$
---------------------------	---	----------------------------------



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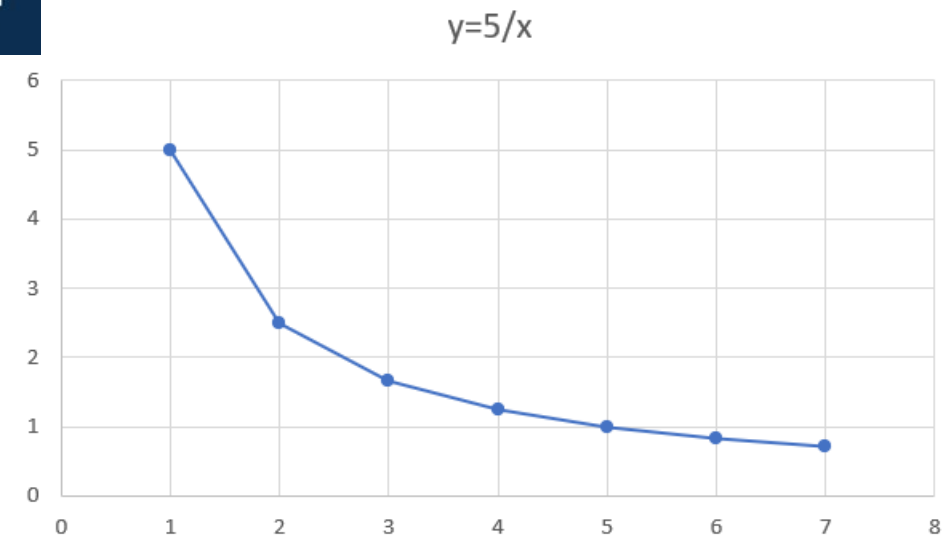
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Picture, model, or diagram



This graph shows direct proportion

Non-Example



This graph shows inverse proportion

inversely proportional

y is **inversely proportional** to x

when $x = 5$, $y = 1$

write an equation for y in terms of x

$$\begin{array}{|l}
 y \propto \frac{1}{x} \\
 y = \frac{k}{x}
 \end{array}
 \longrightarrow
 \begin{array}{|l}
 1 = \frac{k}{5} \\
 k = 5 \\
 y = \frac{5}{x}
 \end{array}$$



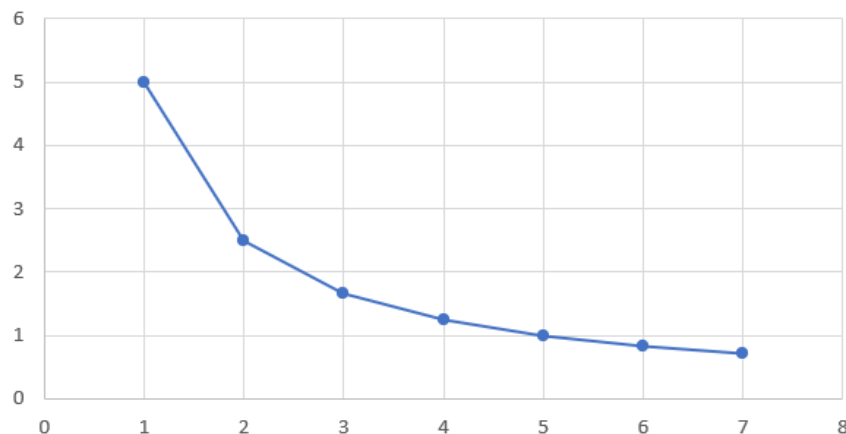
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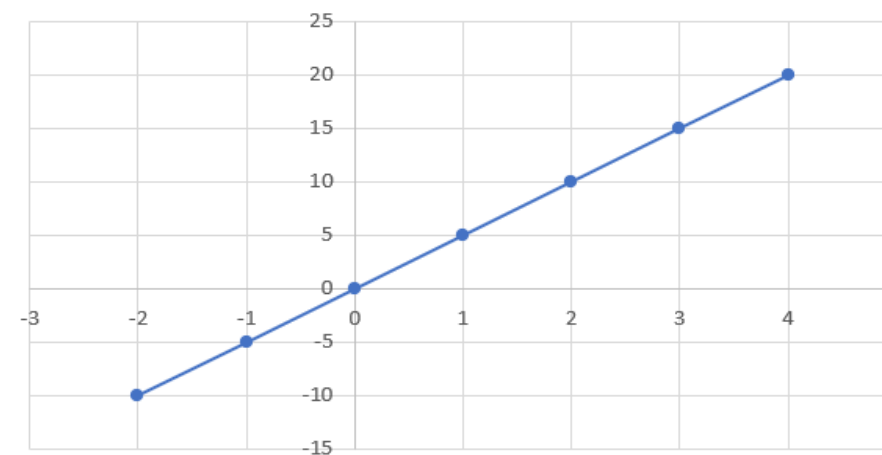
Non-Example

$$y = 5/x$$



This graph shows inverse proportion

$$y = 5x$$



This graph shows direct proportion

trigonometric ratio (trigonometry)

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

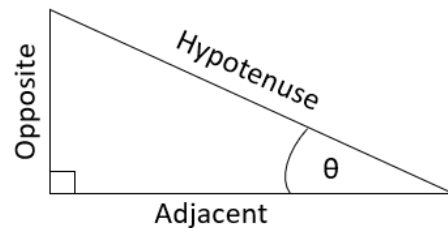


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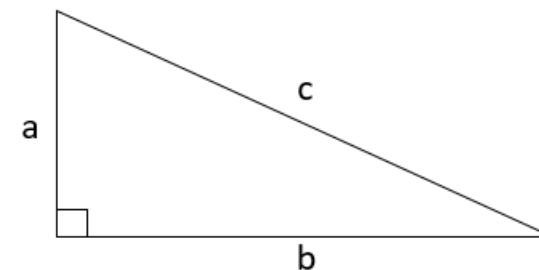
Picture, model, or diagram

Non-Example



SoH CaH ToA

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



$$a^2 + b^2 = c^2$$

exponential growth

A population of rabbits doubles every month.

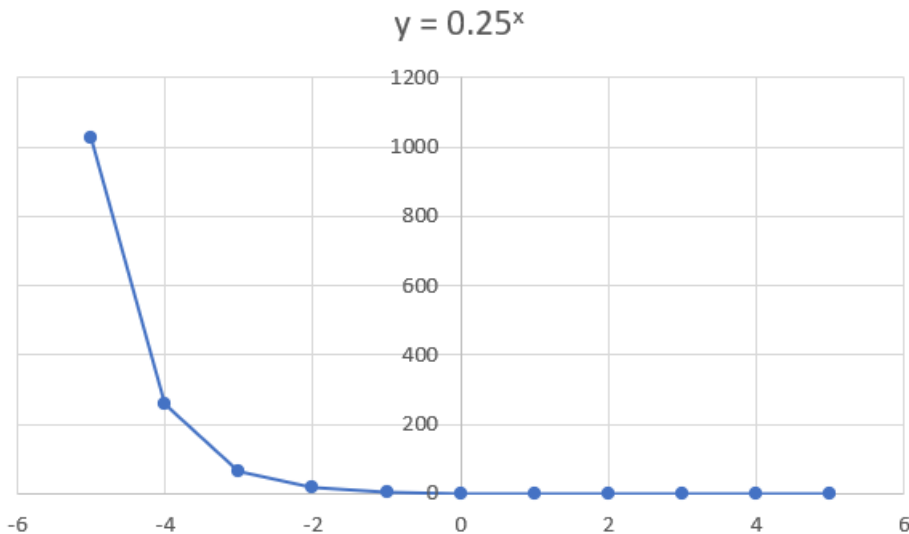
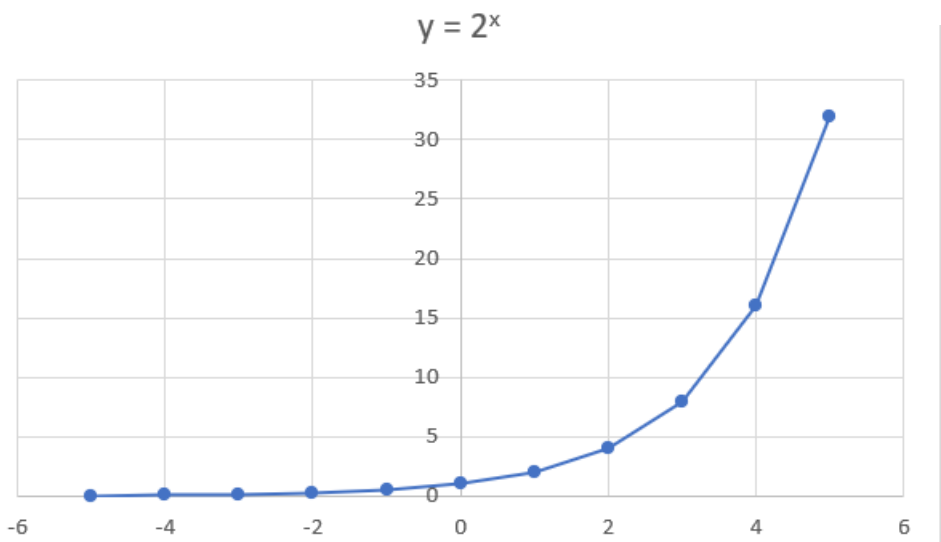
The population is growing exponentially.

Month	Number of rabbits
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$
5	$32 = 2^5$
6	$64 = 2^6$
n	2^n



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exponential decay

Year	Grams of salt
1	2500000
2	1250000
3	625000
4	312500
5	156250
6	78125
7	39063
8	19531
9	9766
10	4883
11	2441
12	1221
13	610
14	305
15	153

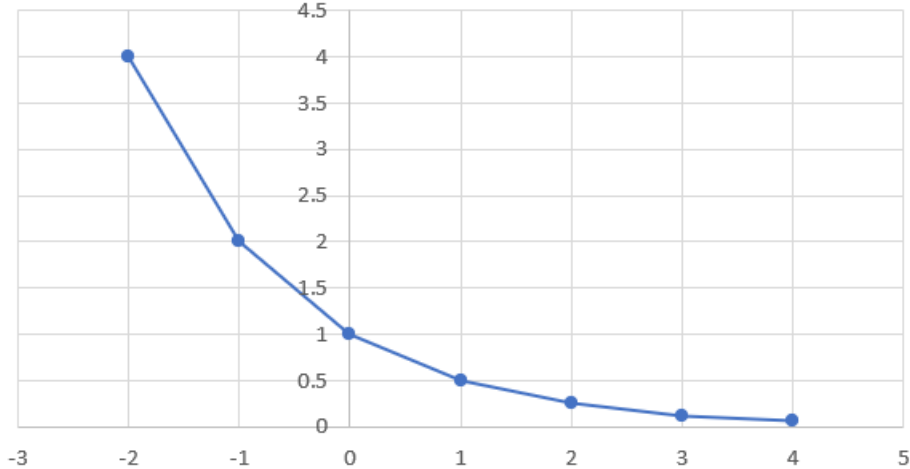
A restaurant decreases its salt use by 50% per year. In year 0, the restaurant uses 5,000,000 grams of salt. Salt usage is **decaying exponentially**.



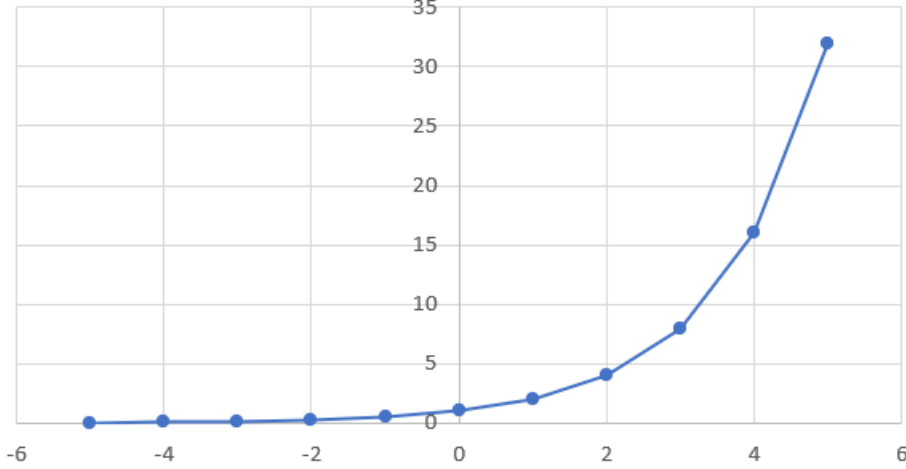
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$y=0.5^x$



$y = 2^x$



trapezium (-a)

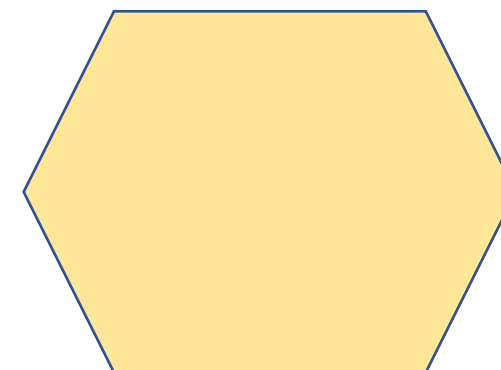
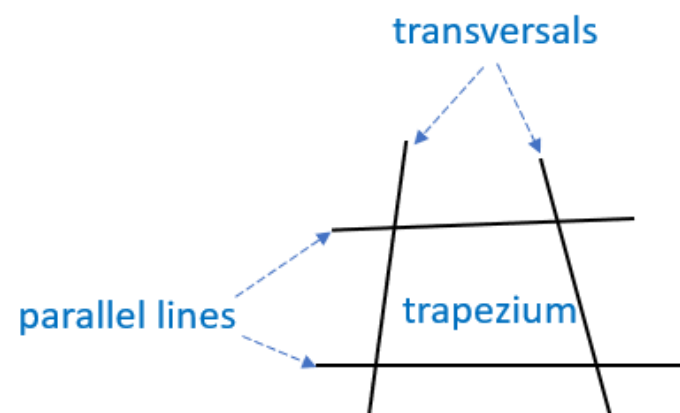


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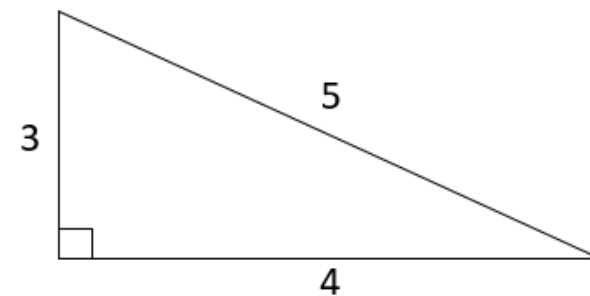
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Picture, model, or diagram

Non-Example



Pythagoras' Theorem



$$3^2 + 4^2 = 5^2$$

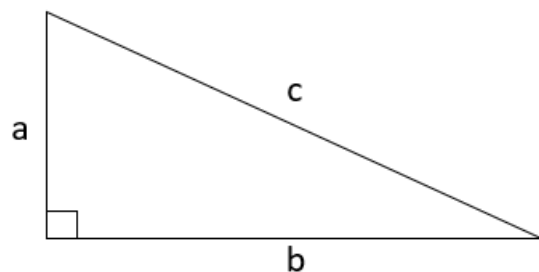
$$9 + 16 = 25$$



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Picture, model, or diagram

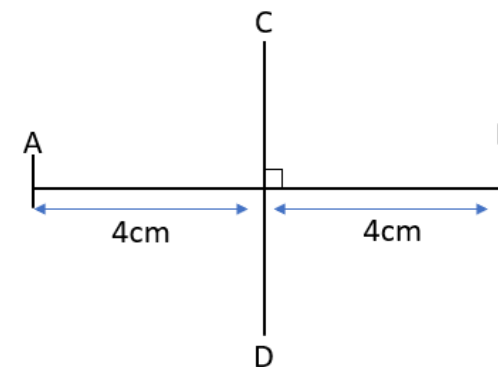


$$a^2 + b^2 = c^2$$

Non-Example

$$\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

perpendicular bisector



$AB = 8\text{cm}$

CD bisects AB at 90°

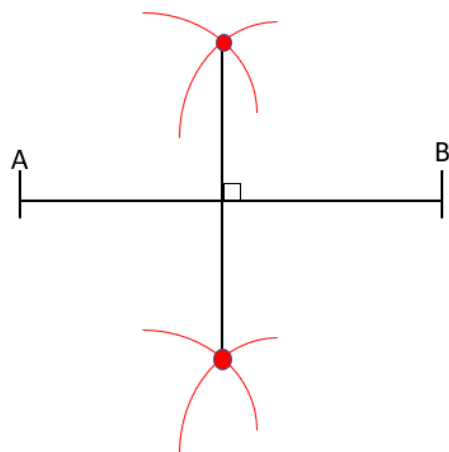


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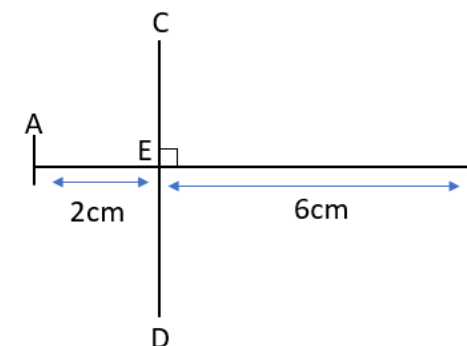
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Picture, model, or diagram

Non-Example



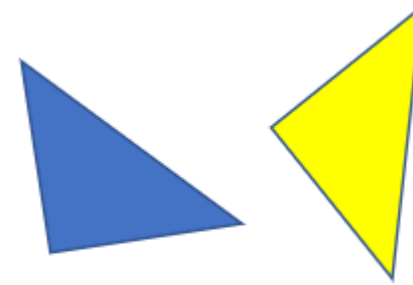
Use a pair of compasses.
Place the point on A and open the
compasses to over half way
Make an arc above and below the line.
Repeat from B
Join the arcs



$AB = 8\text{cm}$

CD intersects AB at 90° at E

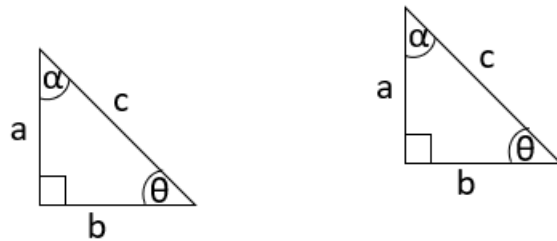
congruent



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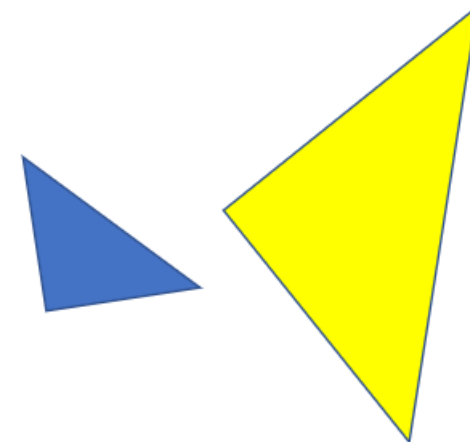
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Picture, model, or diagram

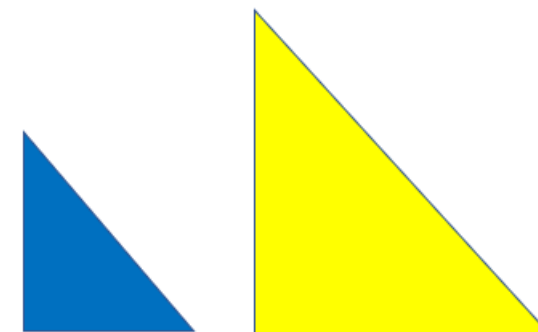


These triangles are congruent
All corresponding angles are the same size
All corresponding sides are the same length

Non-Example



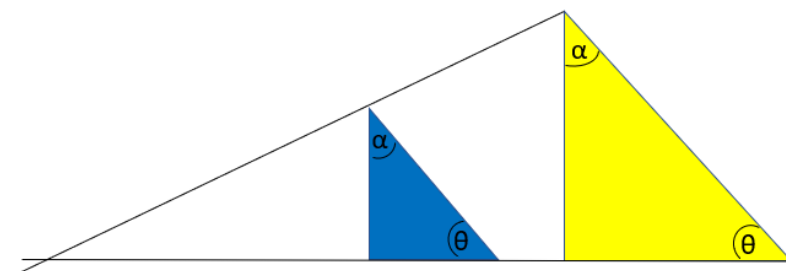
similar



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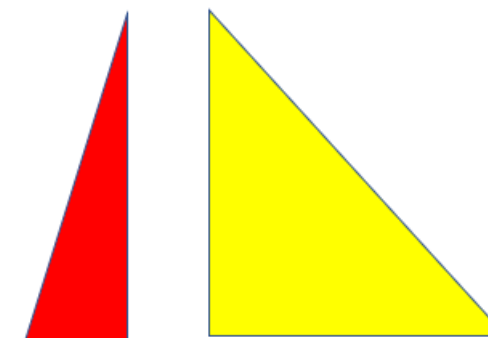
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Picture, model, or diagram



These triangles are similar
All corresponding angles are the same
All corresponding sides are proportional

Non-Example



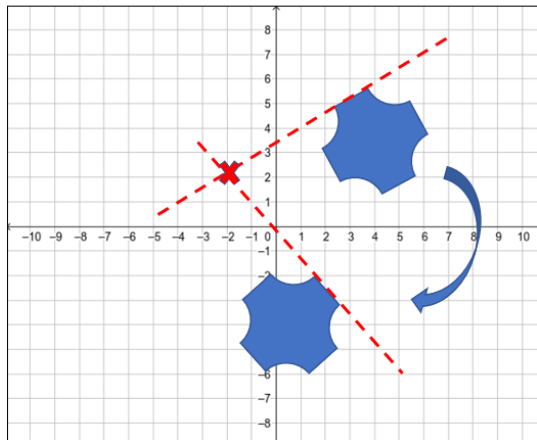
rotation



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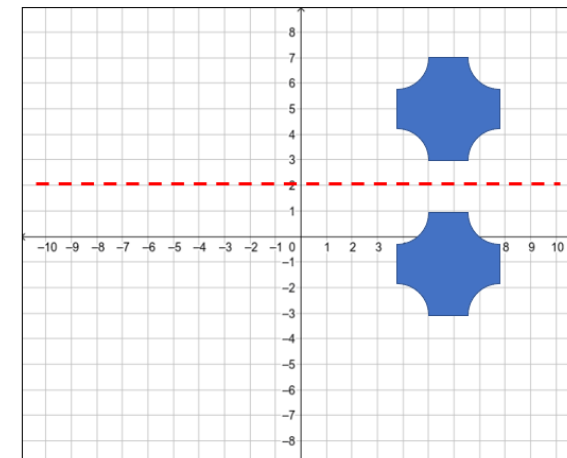
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Picture, model, or diagram



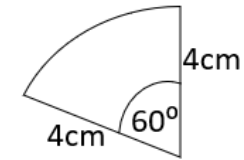
This is a rotation of 90° clockwise about the point $(-2, 2)$

Non-Example



sector

Calculate the area of this **sector** to 1 decimal place



Sector area is $\frac{60}{360}$ of the circle area

$$\text{Sector area} = \frac{1}{6} \times \pi \times 4 \times 4$$

$$\text{Sector area} \approx 8.4 \text{ cm}^2$$

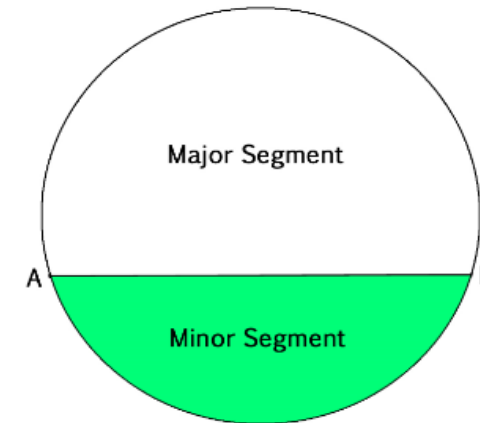
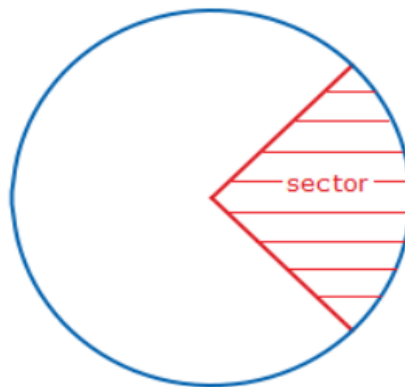


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Picture, model, or diagram

Non-Example



segment

A circle has a radius of length 3cm
 The area of a major **segment** is twice the area of the minor **segment**.
 What is the area of the minor **segment**?

$$\text{Area of circle} = \pi r^2$$

$$\text{Area} = 9\pi \text{ cm}^2$$

$$\text{Area}_{\text{minor}} : \text{Area}_{\text{major}}$$

$$1 : 2$$

$$3 : 6$$

$$\text{Area of minor segment} = 3\pi \text{ cm}^2$$

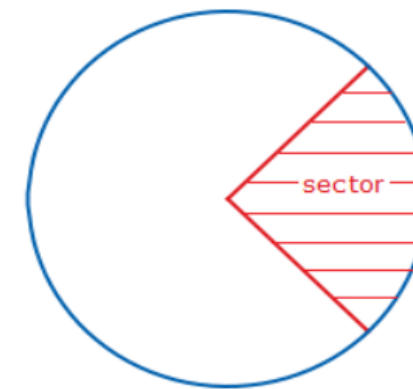
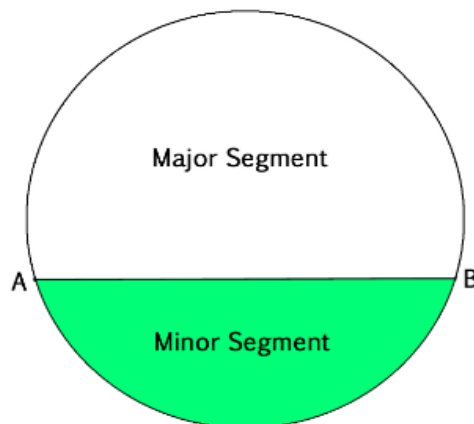


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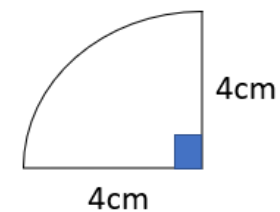
Picture, model, or diagram

Non-Example



arc

Calculate the **arc** length to 2 decimal places



Arc length is 0.25 of the circumference

Arc length = $0.25 \times \pi \times 8$

Arc length $\approx 6.28\text{cm}$

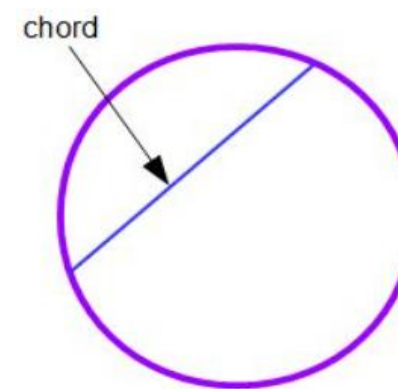
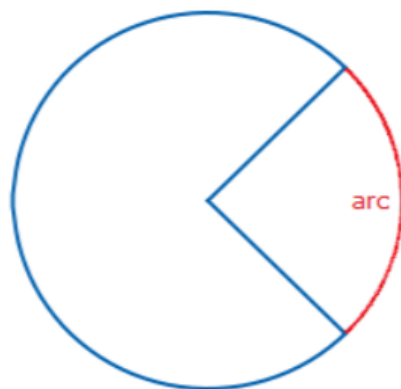


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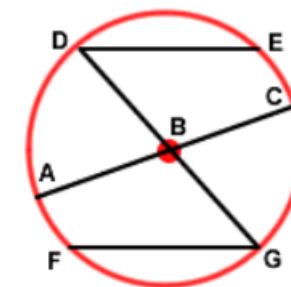
Picture, model, or diagram

Non-Example



chord

All these lines are **chords**
DG and AC are also diameters



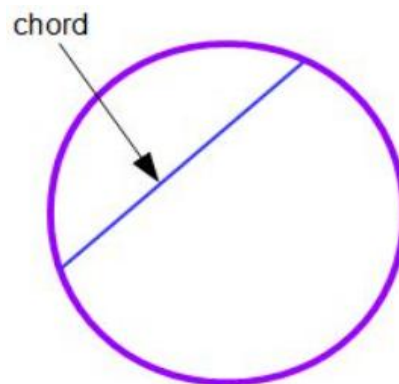
The diameter is the longest possible **chord** in a circle



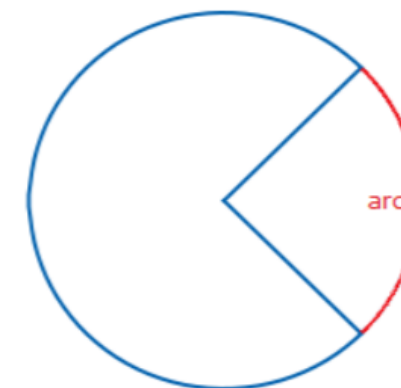
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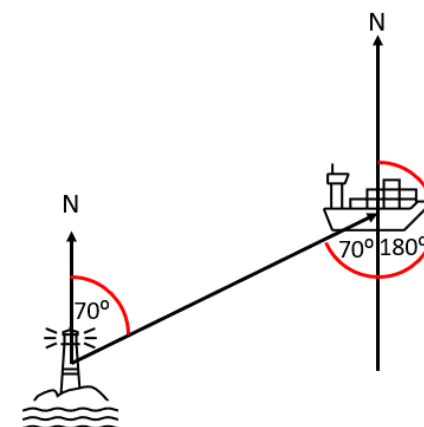
Picture, model, or diagram



Non-Example



bearing



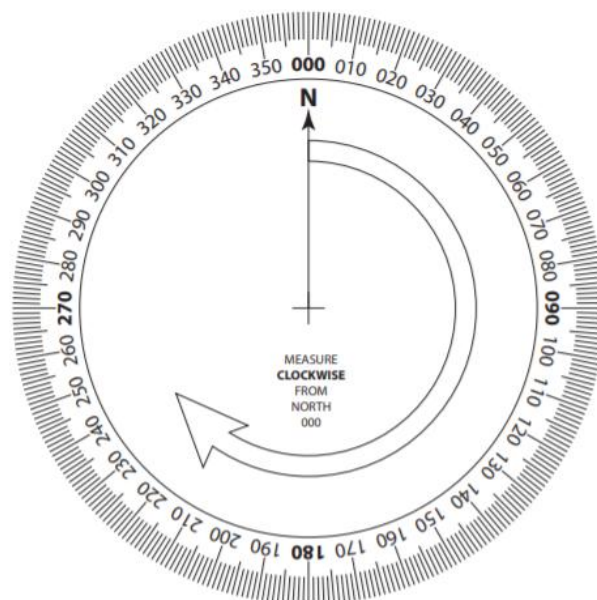
The bearing of the lighthouse from the ship is 250°



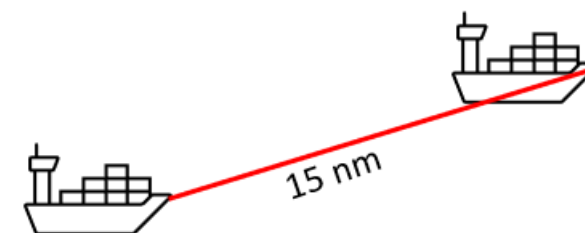
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Picture, model, or diagram

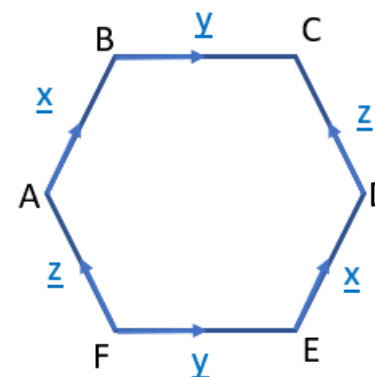


Non-Example



These ships are 15 nautical miles apart

vector



ABCDEF is a regular hexagon

$$\overrightarrow{AB} = \overrightarrow{ED} = \underline{x}$$

$$\overrightarrow{BC} = \overrightarrow{FE} = \underline{y}$$

$$\overrightarrow{DC} = \overrightarrow{FA} = \underline{z}$$

Find $\overrightarrow{BE}$

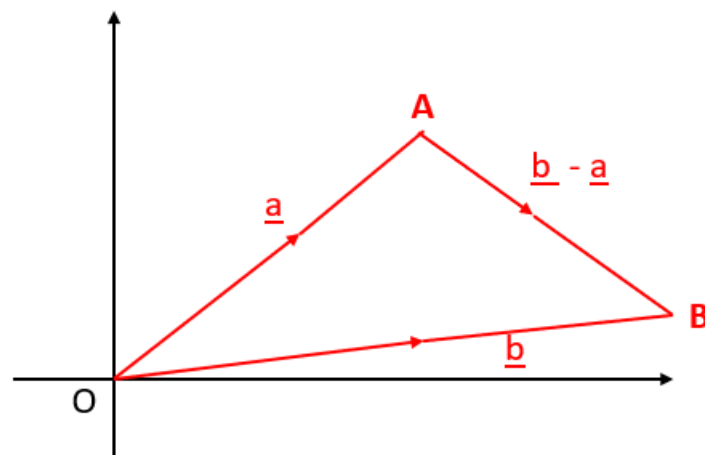
$$\overrightarrow{BE} = \overrightarrow{BC} - \overrightarrow{DC} - \overrightarrow{ED} = \underline{y} - \underline{z} - \underline{x}$$



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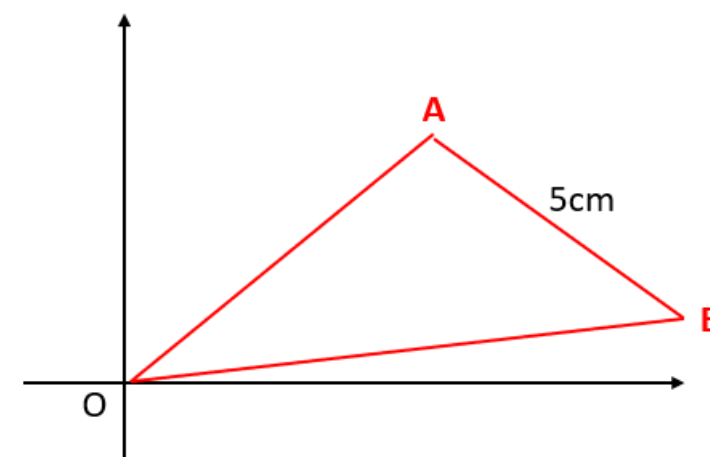
Picture, model, or diagram



$$\overrightarrow{AB} = \underline{b} - \underline{a}$$

A vector has magnitude and direction

Non-Example



$$AB = 5\text{cm}$$

A length has magnitude but no direction

relative frequency

In a probability experiment, coloured counters were taken from a bag without looking and then replaced

Colour	Frequency	Relative Frequency
Purple	7	0.35
Blue	3	0.15
Pink	5	0.25
Orange	5	0.25
<i>Total</i>	<i>20</i>	<i>1.00</i>

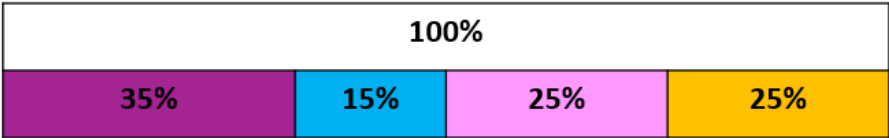
The relative frequency of the event ‘select blue’ is 0.15



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Picture, model, or diagram



The relative frequency shows the proportion of the total for each event occurring.
It can be represented as a fraction , a decimal or a percentage.

Non-Example

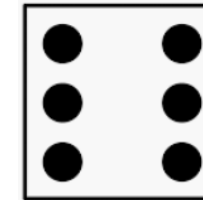
In a probability experiment, coloured counters were taken from a bag without looking and then replaced.
This was repeated twenty times.

Colour	Frequency
Purple	7
Blue	3
Pink	5
Orange	5
<i>Total</i>	<i>20</i>

‘Blue’ was selected three times

outcome

Roll a fair 1-6 die

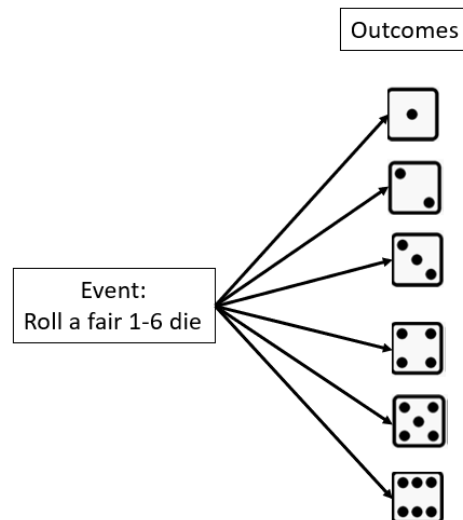


'6' is one possible outcome

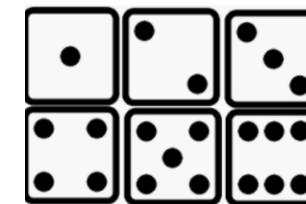


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Roll a fair 1-6 die



'7' is impossible and so is **not** an outcome

random (variable)



A discrete **random variable**:

The number of marbles in a jar

A continuous **random variable**:

The number of seconds taken to complete a race



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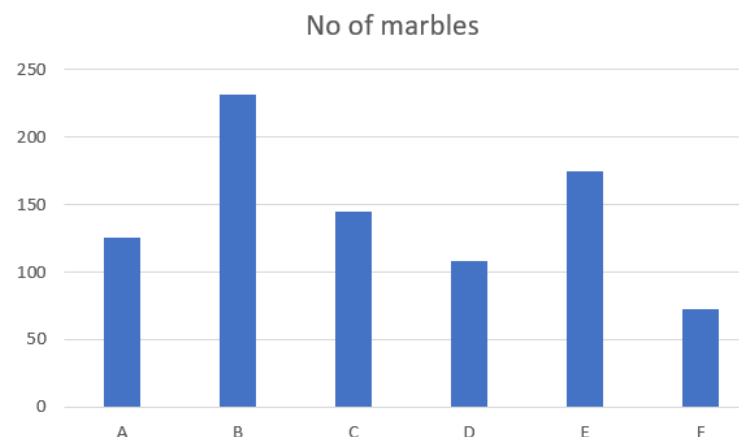
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Picture, model, or diagram

Non-Example

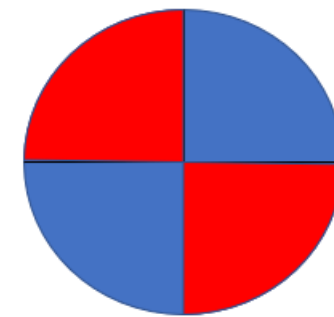
A variable that can take on different values

Marble Jar	Number of marbles
A	126
B	232
C	145
D	108
E	175
F	73



**A sample of exactly 10
marbles from each jar**

fair

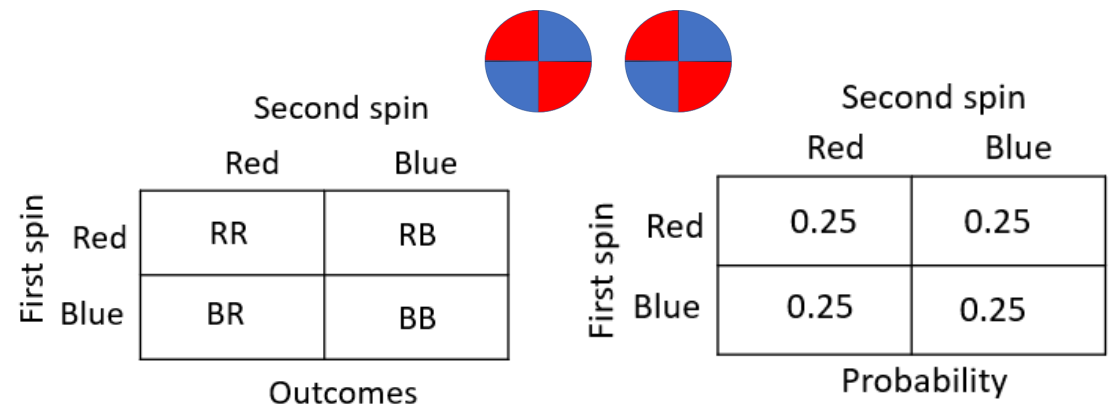


This spinner is fair.
There is an equal chance of 'blue' and 'red'.

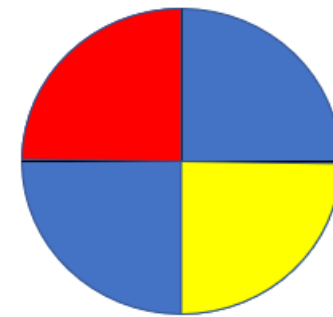


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When using two fair spinners,
the probability of each outcome is the same (0.25)



This spinner is unfair.
There is a greater chance of 'blue' than 'red' or 'yellow'

probability scale (0-1)



Event: Toss a coin

The probability of getting a 'head' is 0.5

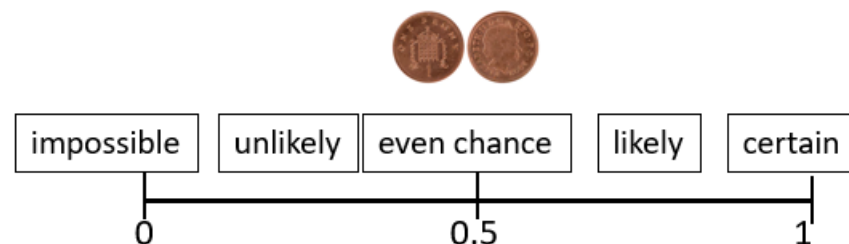
The probability of getting a 'tail' is 0.5



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Picture, model, or diagram



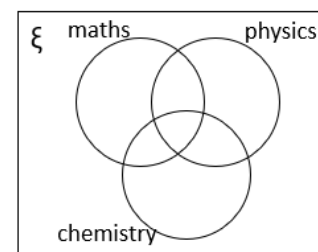
Non-Example



Event: Toss a coin

A 'head' was the result 53 times

Venn diagram



In a year group of 142 students:
 23 study only maths
 18 study only physics
 60 study physics
 45 study only chemistry
 32 study physics and chemistry only
 8 study none of these subjects
 7 study all three subjects

How many study mathematics and chemistry only?

Step one: Fill all the information into the diagram
 Step two: Subtract the number of students accounted for from 142
Solution: 6 students studied maths and chemistry only

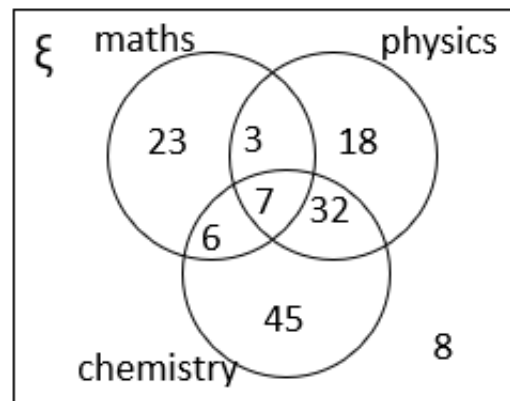
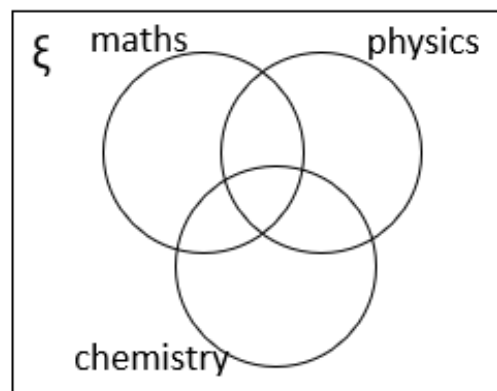


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Picture, model, or diagram

Non-Example



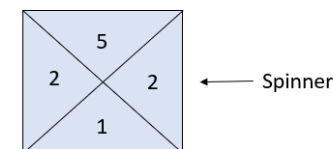
Subject	No of students
Maths	39
Physics	60
Chemistry	90

This table shows the number of students studying each subject

sample space

The spinner is spun **twice** and the score is **added** and recorded in the sample space table below

+	1	2	2	5
1				
2				
2				
5				



Use the sample space to record all possible outcomes and hence work out the probability of scoring **more than 4**



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+	1	2	2	5
1	2	3	3	6
2	3	4	4	7
2	3	4	4	7
5	6	8	7	10

$$P(\text{more than 4}) = \frac{7}{16}$$

Non-Example

Colour	Frequency
Purple	7
Blue	3
Pink	5
Orange	5
<i>Total</i>	<i>20</i>

exhaustive set

If a coin is tossed, there are two possible outcomes **Heads** or **Tails**



The probability of getting a head or a tail is 100%

'Heads and Tails' are an **exhaustive set**

An **exhaustive set** contains **all** possible outcomes



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Non-Example

Event: Roll a fair 1-6 die

100%					
1	2	3	4	5	6

Event: Toss a coin

100%	
H	T

If a fair six-sided (1-6) die is rolled ten times and the outcomes are:

6,6,4,4,3,4,3,5,6,2

The set of actual outcomes is not exhaustive, since 1 has not appeared

empirical

The theoretical probability of rolling a 6 on a fair 1-6-sided die is $\frac{1}{6}$

We can carry out a number of trials to gather **empirical** data to test this.



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Picture, model, or diagram

Non-Example

Event: Roll a fair 1-6 die

Result	Frequency
1	45
2	42
3	39
4	47
5	38
6	44

A probability experiment
collects **empirical data**

The theoretical probability of rolling a 6 on a fair

1-6-sided die is $\frac{1}{6}$

We can use this **theoretical** probability to work out the probability of rolling a 4



theoretical (probability)

Event: Roll a fair 1-6 die

What is the **theoretical probability** of rolling a 4?

Result	<i>number of each outcome number of possible outcomes</i>
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
4	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$



$$P(4) = \frac{1}{6}$$

For a fair die, each number has an equal chance in **theory**



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Event: Roll a fair 1-6 die

1	2	3	4	5	6
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Event: Roll a fair 1-6 die

Result	Frequency
1	45
2	42
3	39
4	47
5	38
6	44

A probability experiment
collects **empirical data**

independent

Landing on heads after tossing a coin
AND
rolling a 5 on a single 6-sided die
are examples of **independent** events.

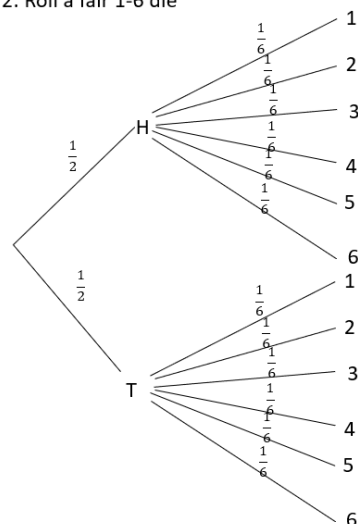


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Picture, model, or diagram

Event 1: Toss a coin
Event 2: Roll a fair 1-6 die



The probability of Event 2 is **independent** of Event 1

Non-Example

A card is chosen at random from a standard deck of 52 playing cards.
Without replacing it, a second card is chosen.

What is the probability that the first card chosen is a Queen
and the second card chosen is a Jack?

$$P(\text{Queen}) = 4 / 52 ; P(\text{Jack}) = 4 / 51$$

$$P(\text{Queen and a Jack}) = 4 / 52 \times 4 / 51 = 16 / 2652$$

$$P(\text{Queen and a Jack}) = 4 / 663$$

The probability of the Jack is dependent on the probability of the Queen.

dependent

A card is chosen at random from a standard deck of 52 playing cards.

Without replacing it, a second card is chosen.

What is the probability that the first card chosen is a queen and the second card chosen is a jack?

$$P(\text{Queen}) = 4 / 52 ; P(\text{Jack}) = 4 / 51$$

$$P(\text{Queen and a Jack}) = 4 / 52 \times 4 / 51 = 16 / 2652$$

$$P(\text{Queen and a Jack}) = 4 / 663$$

The probability of the Jack is **dependent** on the probability of the Queen.

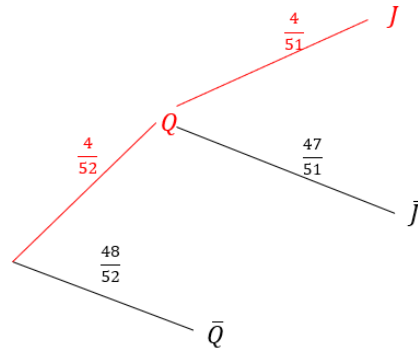


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Event 1: Draw a card, do **not** replace
Event 2: Draw a card



The probability of drawing a Jack second is **dependent** on whether or not a Queen was drawn first

Non-Example

Landing on heads after tossing a coin
AND
rolling a 5 on a single 6-sided die
are examples of **independent** events.

conditional

	Have pets	Do not have pets	Total
Male	0.41	0.08	0.49
Female	0.45	0.06	0.51
Total	0.86	0.14	1

What is the (conditional) probability that a randomly selected person is male, given that they have a pet?

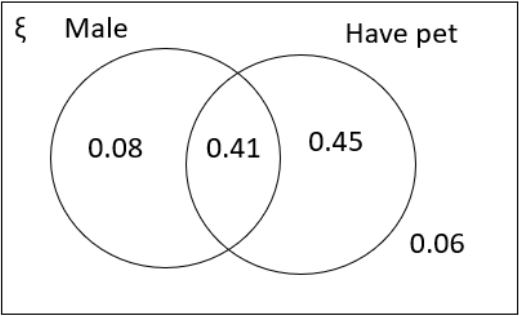
$P(\text{male and have a pet}) = 0.41$
 $P(\text{have a pet}) = 0.86$
 $P(\text{male} \mid \text{have a pet}) = 0.41 / 0.86 = 0.477$



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$$P(\text{male} \mid \text{have a pet}) = \frac{P(\text{male} \cap \text{have a pet})}{P(\text{have a pet})}$$



Event	H	T
H	HH	HT
T	TH	TT

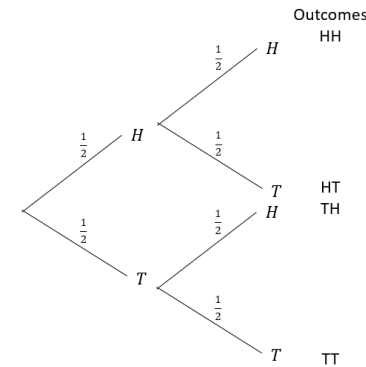
There are four possible outcomes when a fair coin is tossed twice

tree diagram

Event 1: Toss a coin

Event 2: Toss a coin

Use a **tree diagram** to work out the probability of HH



$$P(HH) = \frac{1}{4}$$

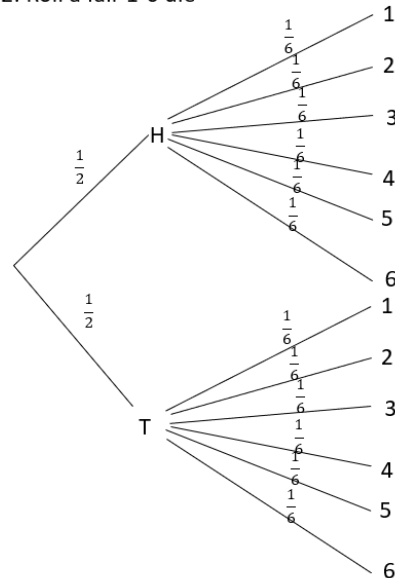


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Picture, model, or diagram

Event 1: Toss a coin
Event 2: Roll a fair 1-6 die



Non-Example

Event	H	T
H	HH	HT
T	TH	TT

grouped (data)

Observed data arising from counts and grouped into non-overlapping intervals is called **grouped data**.

The length of feet of 25 planks of wood were measured.

The lengths were grouped into classes of width 10 feet.



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Length (feet)	Frequency (f)
$0 \leq \text{ft} < 10$	2
$10 \leq \text{ft} < 20$	6
$20 \leq \text{ft} < 30$	9
$30 \leq \text{ft} < 40$	5
$40 \leq \text{ft} < 50$	3

Non-Example

The number of different colour smarties in a pack of 25

It is not possible to group the data as each colour is separate

colour	Frequency (f)
Green	2
Orange	6
Blue	9
Yellow	5
Purple	3

mean ($\bar{x}$)

The **mean** average of 6, 11, 16 is 11

$$(6 + 11 + 16) \div 3 = 11$$

$$\bar{x} = 11$$

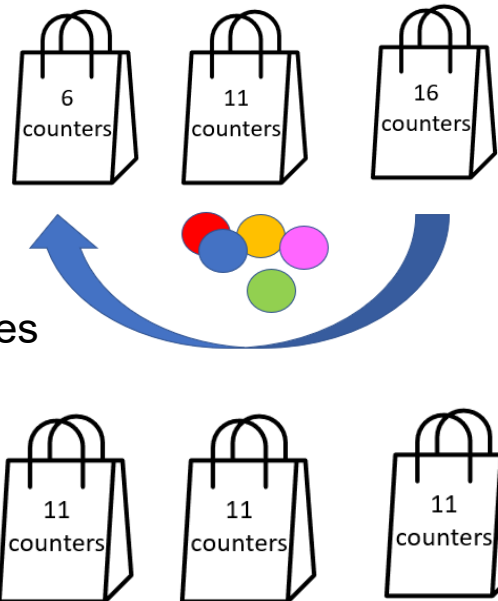


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Non-Example



The mean ($\bar{x}$) can be
thought of as 'equal' shares

The **median** of 12, 6, 3, 5, 8 is 6

3, 5, 6, 8, 12

median

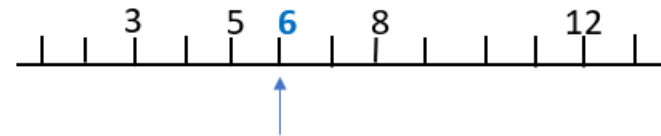
The **median** of 12, 6, 3, 5, 8 is 6

3, 5, 6, 8, 12



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Middle value of this data set is 6

The mode of 8, 5, 6, 8, 9, 8 is 8

5, 6, 8, 8, 8, 9

mode

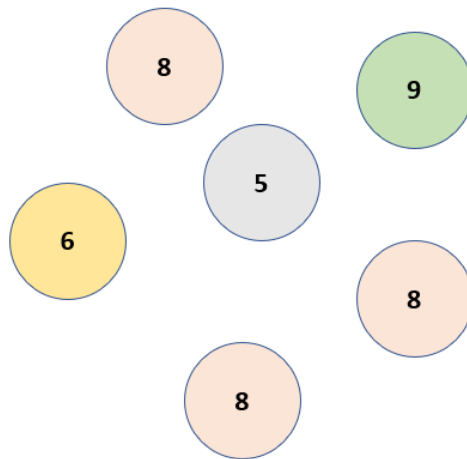
The **mode** of 8, 5, 6, 8, 9, 8 is 8

5, 6, 8, 8, 8, 9



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'8' occurs most frequently

The **median** of 12, 6, 3, 5, 8 is 6

3, 5, 6, 8, 12

range

The **range** of 12, 6, 3, 5, 8 is 9

It is the difference between the maximum (12) and the minimum (3) value in the set.

3, 5, 6, 8, 12

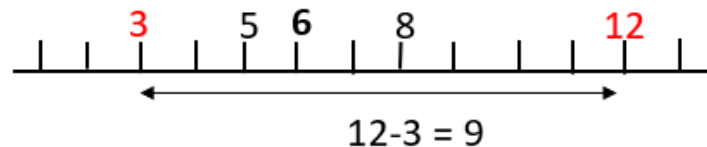
$$12 - 3 = \underline{9}$$



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The **range** of this data set is 9

Non-Example

The **median** of 12, 6, 3, 5, 8 is 6

3, 5, 6, 8, 12

outlier

In a maths test, the following marks were scored:
25, 29, **3**, 32, **85**, 33, 27, 28
Both 3 and 85 are **outliers**.
They lie outside the main cluster of scores.

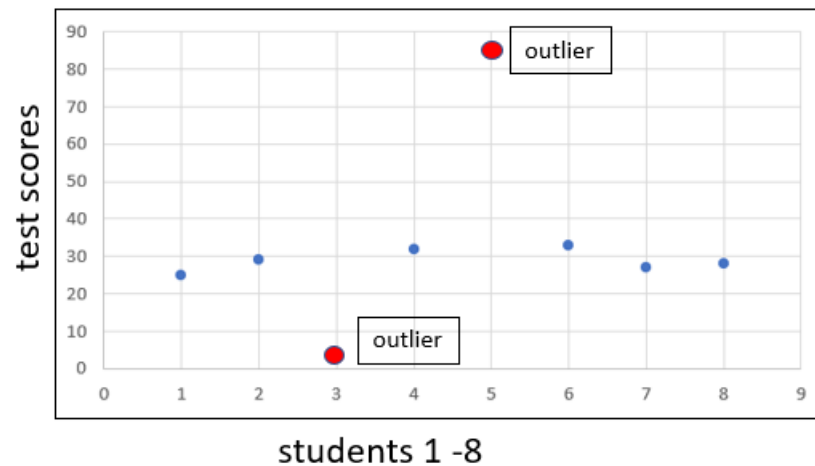


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Non-Example



In a maths test, the following marks were scored:
25, 29, 3, 32, 85, 33, 27, 28
The range of scores is 82

$$85 - 3 = 82$$

bivariate (data)

Ice cream sales versus the temperature on that day.
The two variables are 'ice cream sales' and 'temperature'.

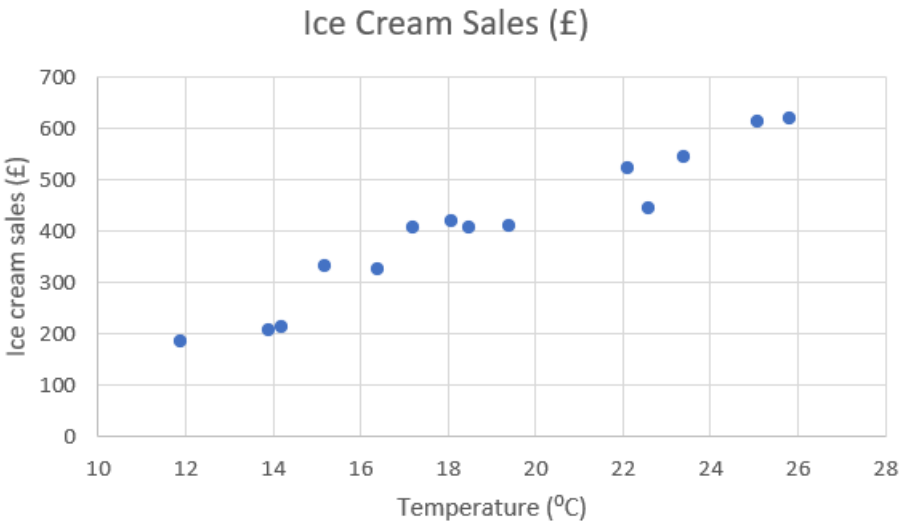
The warmer the temperature, the more ice creams are sold.
'Temperature' and 'ice cream sales' are **bivariate**.

Temperature (°C)	Ice Cream Sales (£)
14.2	215
16.4	325
11.9	185
15.2	332
18.5	406
22.1	522
19.4	412
25.1	614
23.4	544
18.1	421
22.6	445
17.2	408
13.9	207
25.8	620



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With **bivariate data** we have **two** sets of related data we want to **compare**.

Univariate means one variable (one type of data)

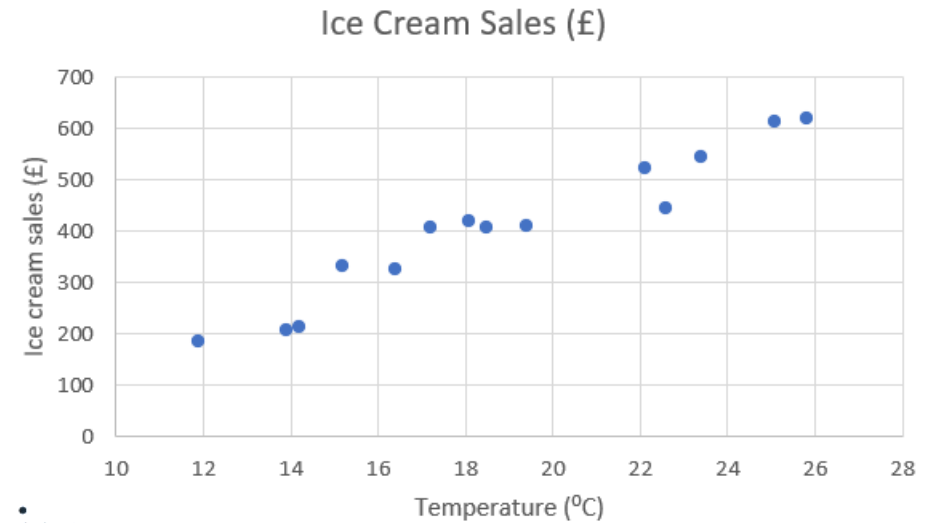
Example: travel time (minutes):

15, 29, 8, 42, 35, 21, 18, 42, 26

The variable is **travel time**

scatter graphs

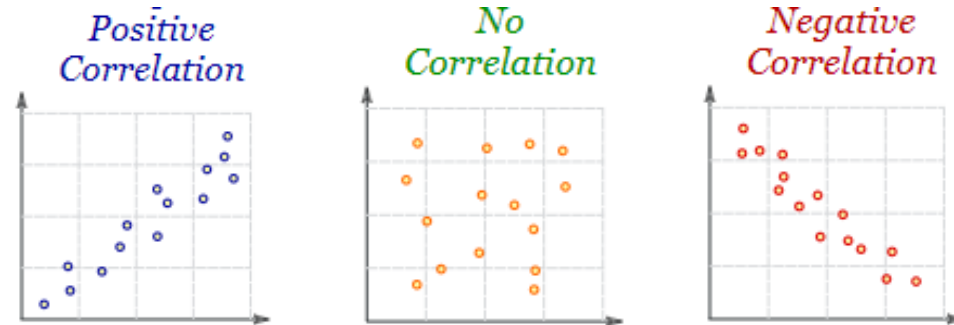
Bivariate data



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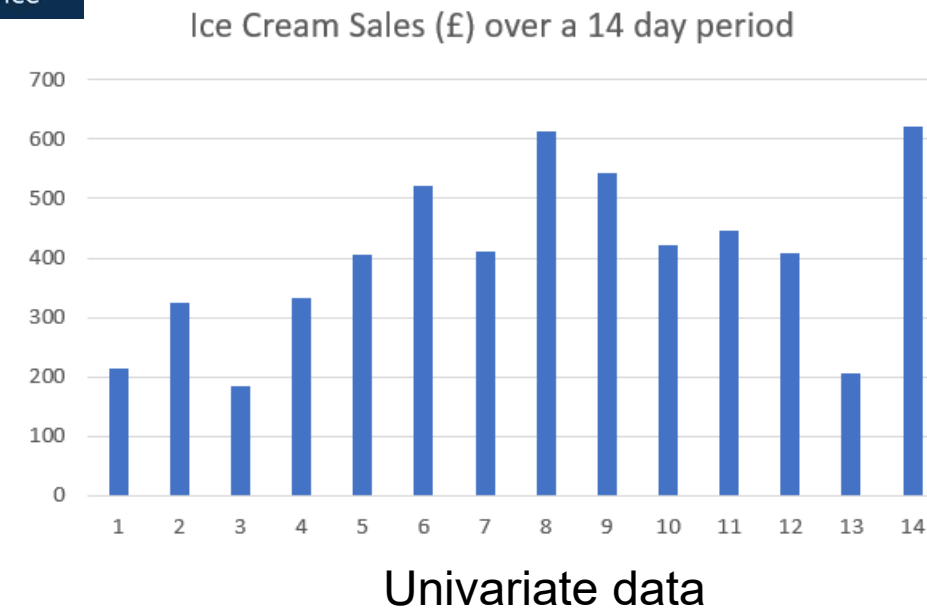
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Picture, model, or diagram



Scatter graphs are used to compare two sets of data.
 Positive correlation means that as one variable increases, so does the other.
 No correlation means that one variable does not have an effect on the other.
 Negative correlations means that as one variable increase, the other decreases.

Non-Example



histogram

Histograms look like bar charts but the area of the bar represents the frequency, not the height.
The class widths can be unequal in a **histogram**.

The ages of 28 children on a school trip

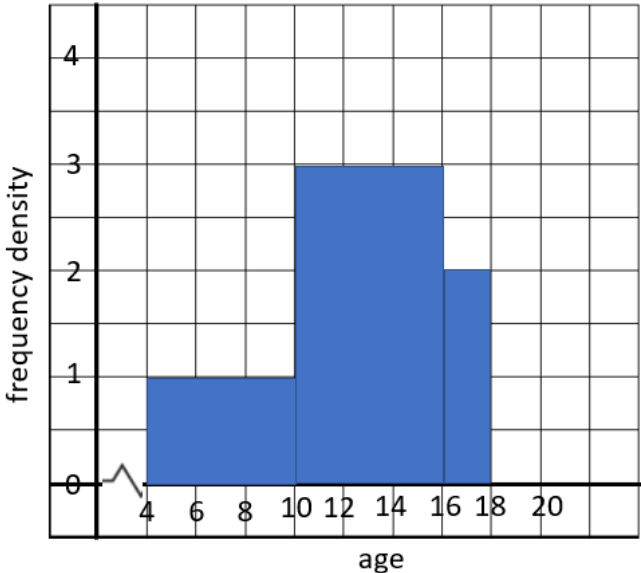
Age	Frequency	Class width	Frequency density
4-9	6	6	$6 \div 6 = 1$
10-15	18	6	$18 \div 6 = 3$
16-17	4	2	$4 \div 2 = 2$



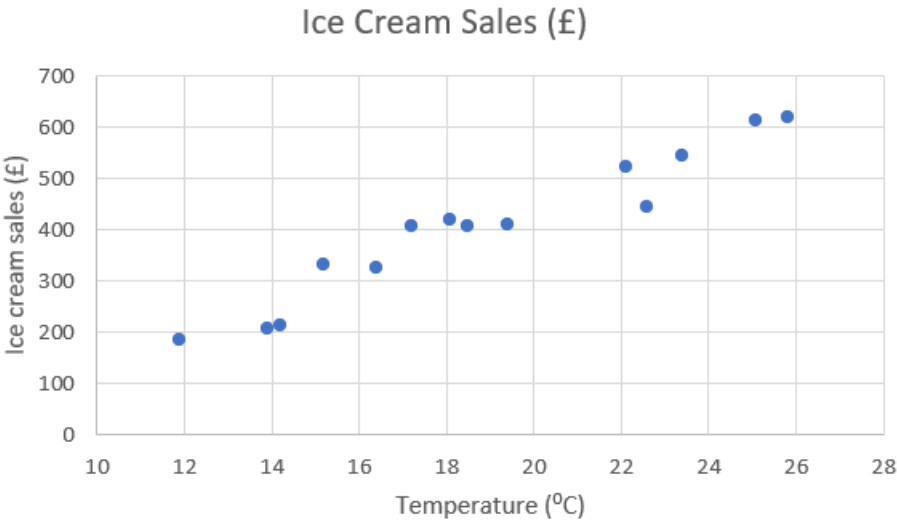
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Non-Example



sample

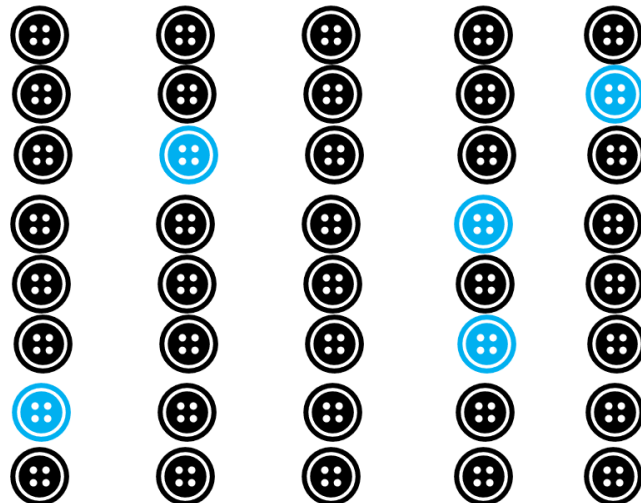
Sample: a collection of data from **part** of the population.

Example: a random selection of five buttons from a box of forty buttons.



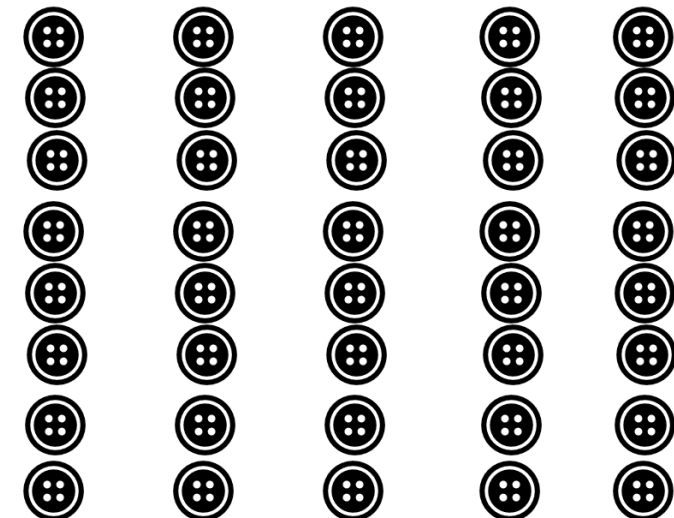
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The blue buttons are a random **sample** from the population

Non-Example



This is the population ~ all the 40 buttons in the box

population

Population: the whole group we are interested in.

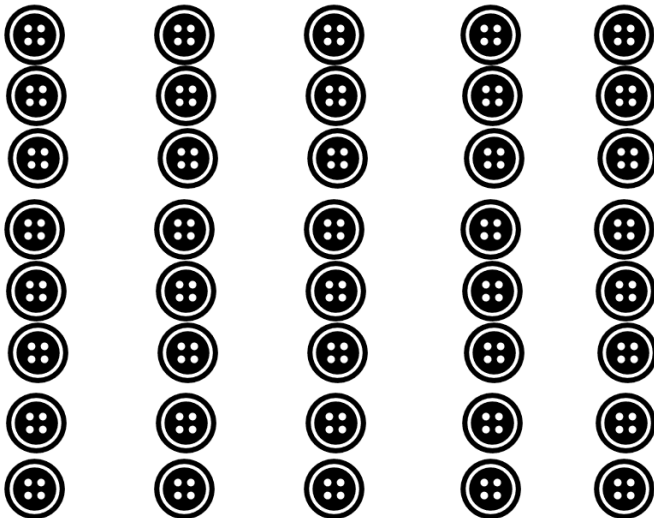
Example: all the buttons in a box of 40 buttons.



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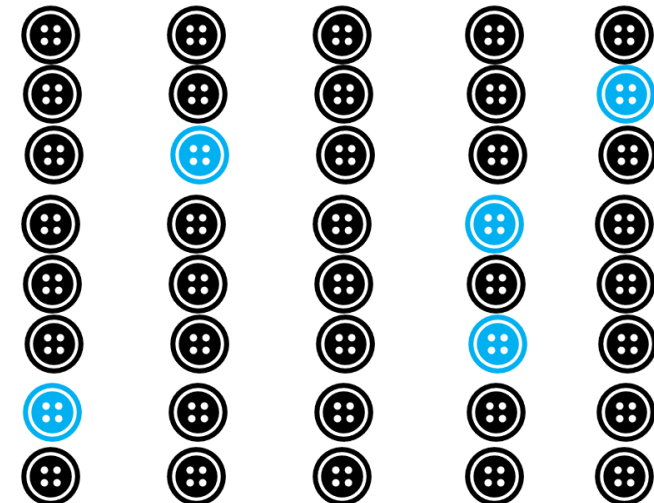
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This is the **population** ~ all the 40 buttons in the box

Non-Example



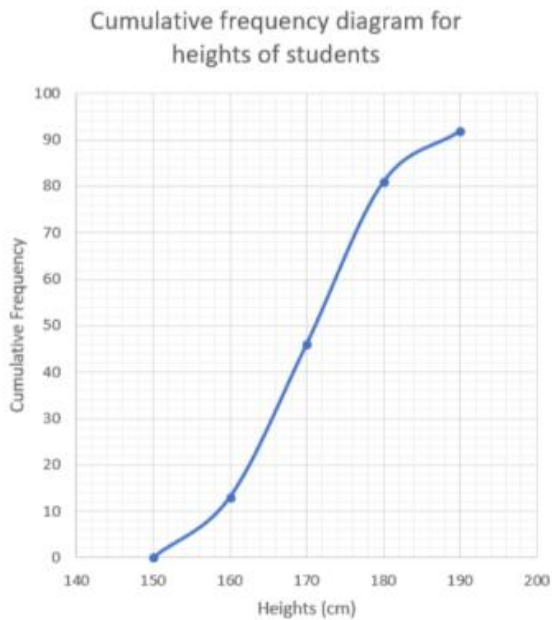
The blue buttons are a random sample from the population

cumulative frequency

Heights, h (cm)	Frequency	Cumulative frequency
150 < h ≤ 160	13	13
150 < h ≤ 160	33	46
150 < h ≤ 160	35	81
150 < h ≤ 160	11	92



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Heights, h (cm)	Frequency
150 < h ≤ 160	13
150 < h ≤ 160	33
150 < h ≤ 160	35
150 < h ≤ 160	11

This data shows the ages of 11 people on a boat trip

9, 10, 10, 12, 13, 14, 17, 18, 19, 21, 21

The data is ordered and the median, quartiles, maximum and minimum values identified.

This is then plotted on a horizontal axis as a box for the middle 50% of the data, with 'whiskers' to show the first and last 25%.

box plot (box and whisker diagram)

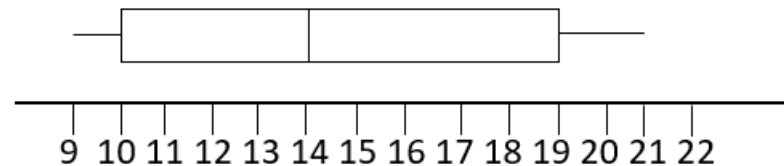


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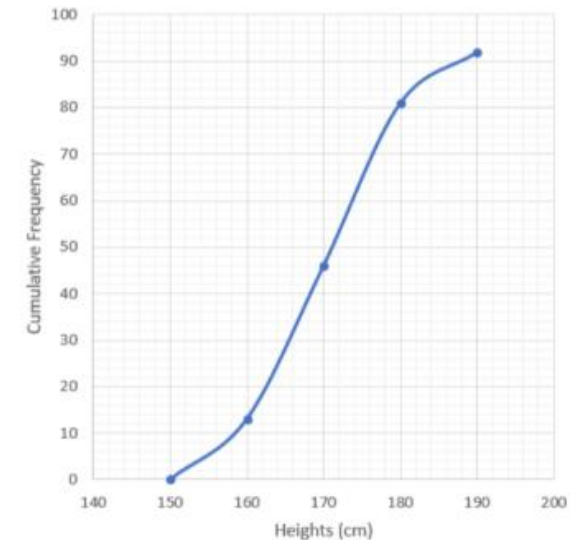
Picture, model, or diagram

9, 10, 10, 12, 13, 14, 17, 18, 19, 21, 21



Non-Example

Cumulative frequency diagram for
heights of students



lower quartile

Quartiles are the values that divide a list of numbers into quarters

Example: 5,7,4,4,6,2,8

Order: 2,4,4,5,6,7,8

Quarter the list:

Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

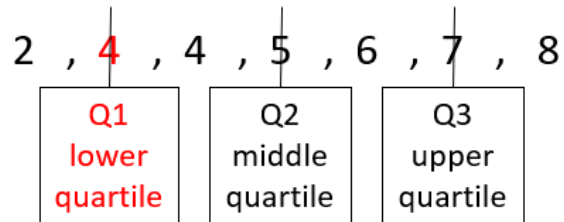


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Picture, model, or diagram

Non-Example



Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

The mode of 8, 5, 6, 8, 9, 8 is 8

5, 6, 8, 8, 8, 9

upper quartile

Quartiles are the values that divide a list of numbers into quarters

Example: 5,7,4,4,6,2,8

Order: 2,4,4,5,6,7,8

Quarter the list:

Lower quartile (Q1) = 4

Median (Q2) = 5

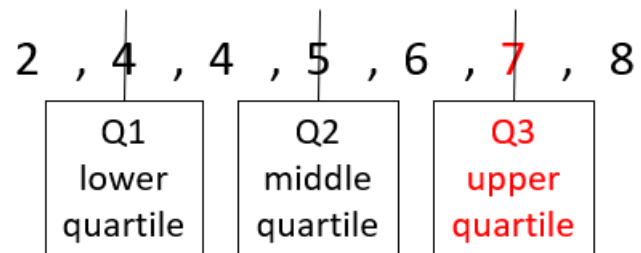
Upper quartile (Q3) = 7



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Picture, model, or diagram



Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

Non-Example

The mode of 8, 5, 6, 8, 9, 8 is 8

5, 6, 8, 8, 8, 9

inter quartile range

Quartiles are the values that divide a list of numbers into quarters

Example: 5,7,4,4,6,2,8

Order: 2,4,4,5,6,7,8

Quarter the list and find the difference between the upper and lower quartiles

Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

Inter quartile range (IQR) = 3

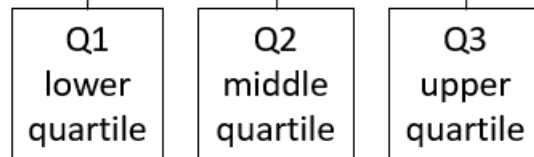


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Picture, model, or diagram

2 , 4 , 4 , 5 , 6 , 7 , 8



Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

Inter quartile range (IQR) = 7 - 4 = 3

Non-Example

2 , 4 , 4 , 5 , 6 , 7 , 8

The range for this data set is $8 - 2 = 6$

Word	Example
Picture, model, or diagram	Non-Example



Word	Example
Picture, model, or diagram	Non-Example

