Hampshire Services
HIAS SCHOOL IMPROVEMENT

Example Problem Pairs
I, We, You

Access and Success for All

Jo Lees
Jo.lees@hants.gov.uk

Hampshire County Council

Learning Episodes and Paired Examples

Access and Success for All

Developing the work of Craig Barton in his book:
'Reflect, Expect , Check, Explain'

Learning Episode ~ A basic model

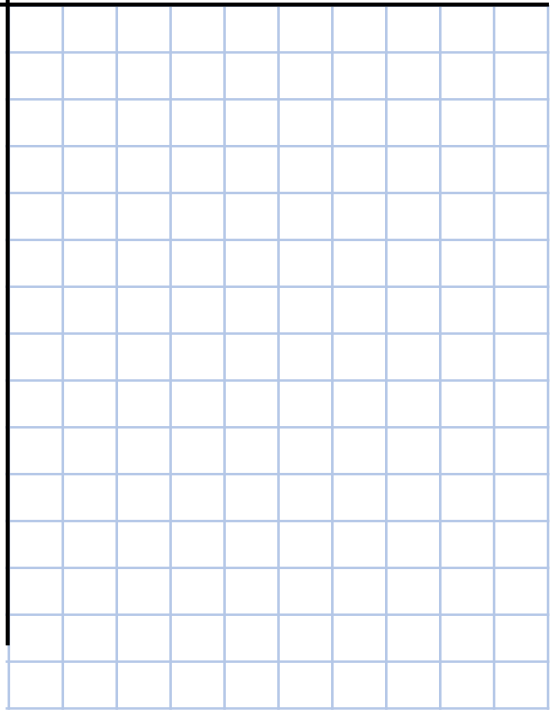
Worked example : The teacher models how to carry out a method

Practice: Pupils work through questions that require that method, gaining confidence and competence

Problem solving: With the basics secure, pupils are challenged to do something more complex involving that method. Perhaps use it in a non-standard way, in a context, in a SATs question, or interleaved with other mathematical concepts

Worked Example → Student Teacher

When modelling an example, it is important to show the underlying mathematical structure by using visual models and more formal calculations side by side

Model	Calculations
	

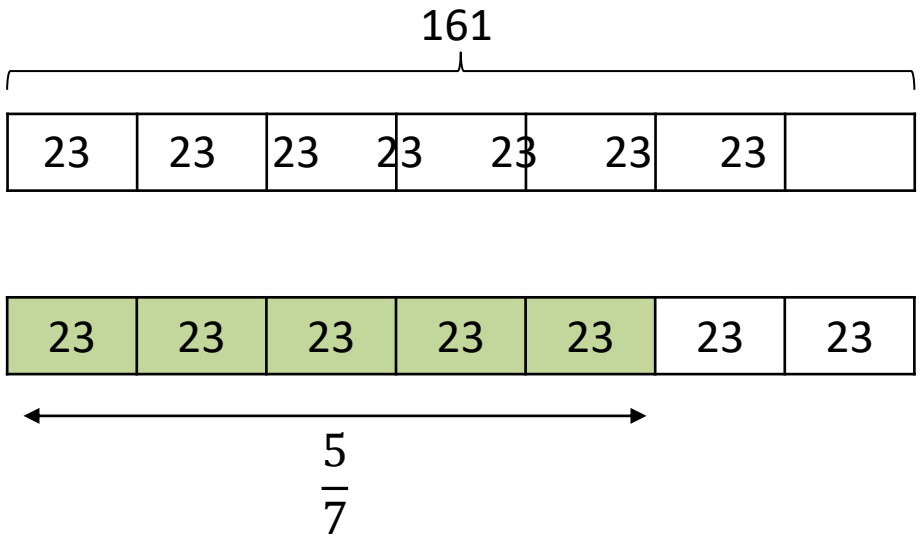
I DO, WE DO, YOU DO

I do, we do , you do

Worked Example → Silent Teacher

Question: Find $\frac{5}{7}$ of 161

Model



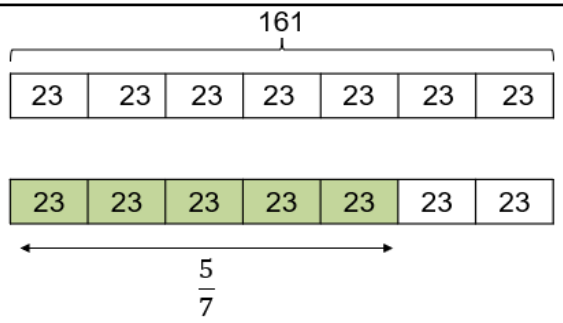
Calculations

161 ÷ 7 = 23

23 x 5 = 115

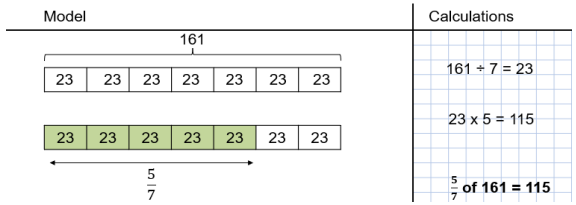
$\frac{5}{7}$ of 161 = 115

I DO, WE DO, YOU DO

Worked example	Thinking
Find $\frac{5}{7}$ of 161 Model  Calculations 161 ÷ 7 = 23 23 x 5 = 115 $\frac{5}{7}$ of 161 = 115	What did I do first ? Why have I divided by 7? What does my division calculation represent? Why did I multiply by 5? What does my multiplication calculation represent? How do I know this is a sensible answer?

I DO, WE DO, YOU DO

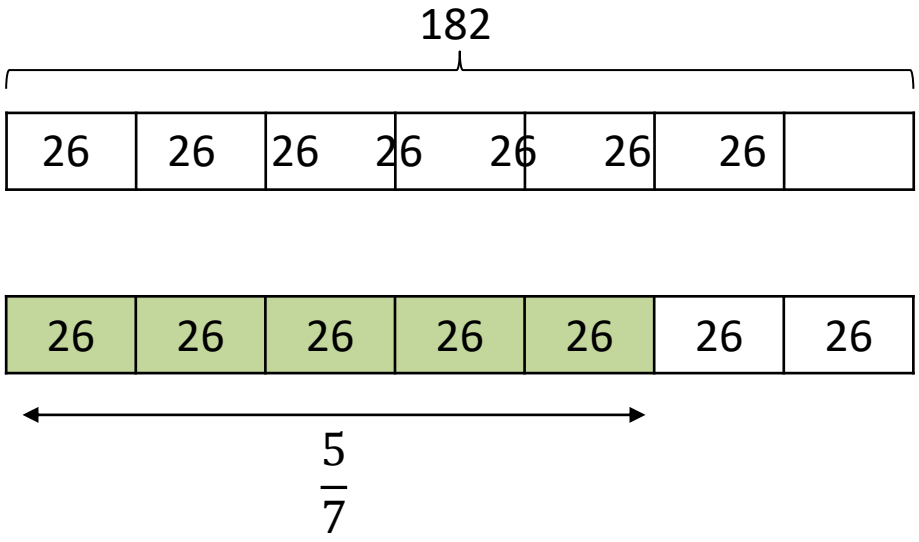
Worked Example -- Silent Teacher -- Problem Pair

Worked example	Thinking	Your turn
Find $\frac{5}{7}$ of 161 	What did I do first ? Why have I divided by 7? What does my division calculation represent? Why did I multiply by 5? What does my multiplication calculation represent? How do I know this is a sensible answer?	Find $\frac{5}{7}$ of 182

I DO, WE DO, YOU DO

Find $\frac{5}{7}$ of 182

Model



Calculations

182 ÷ 7 = 26

26 x 5 = 130

$\frac{5}{7}$ of 182 = 130

I DO, WE DO, YOU DO

Over to you:
Create a pair of examples for your class on something you are currently teaching.

Worked example	Thinking	Your turn

Model	Calculations

Learning Episode ~ A basic model

Worked example : The teacher models how to carry out a method

Practice: Pupils work through questions that require that method, gaining confidence and competence

Problem solving: With the basics secure, pupils are challenged to do something more complex involving that method. Perhaps use it in a non-standard way, in a context, in a SATs question, or interleaved with other mathematical concepts

I DO, WE DO, YOU DO

Practice → Intelligent Practice

Intelligent practice sequences are sequences of questions which enable pupils to gain practice in carrying out a mathematical method, whilst at the same time providing opportunities to think mathematically

The three elements needed to support mathematical thinking

- The role of the task
- The role of the pupil
- The role of the teacher

→ A catalyst for mathematical thinking

→ To behave mathematically:
- ✓ *Reflect*
- ✓ *Expect*
- ✓ *Check*
- ✓ *Explain*

→ The actions that support students to think and behave mathematically:
- ✓ *Modelling*
- ✓ *Planned questions*
- ✓ *Scaffolding for access*
- ✓ *Challenging for depth*
- ✓ *Providing the time to think*
- ✓ *Encouraging and finding the right words to say to the right students at the right time*

The role of the task

A poorly designed task makes it very hard for mathematical thinking to occur.

Careful sequencing gets pupils part of the way to mathematical thinking

Such sequences provide a catalyst for curiosity and a platform to ask *why* ?

Task :

Calculate $\frac{5}{7}$ of 175 using the method we have just learned

(you can use a bar model to help you)

175 ÷ 7 = 25

25 x 5 = 125

$\frac{5}{7}$ of 175 = 125

When you have finished , compare with a partner to check your answer

REFLECT

$\frac{5}{7}$ of 175 = 125

$\frac{5}{7}$ of 350 =

Without telling anyone,
think about what has changed and what has stayed the same

Describe to the person next to you your reflection and listen to theirs. Are they the same ?

EXPECT

$\frac{5}{7}$ of 175 = 125

$\frac{5}{7}$ of 350 =

Given that you know the answer to the first question is 125,
can you now form an expectation about the answer to the second question?

Share your **expectation** and make sure you describe your reasons

CHECK

$\frac{5}{7}$ of 175 = 125

$\frac{5}{7}$ of 350 = 2

Work out the answer to the second question, using the method we have just learned (including a bar model if you wish) , show all your working.
Share your answer.

EXPLAIN

$\frac{5}{7}$ of 175 = 125

$\frac{5}{7}$ of 350 = 250

Look at Q1 with its answer of 125
Look at Q2 with its answer of 250
If this second answer surprises you, can you explain why ?
If this answer does not surprise you, can you think of a way of explaining the relationship that would help someone who doesn't understand it yet?

REFLECT, EXPECT, CHECK, EXPLAIN

Now work through these other examples

How does the process of 'reflect, expect, check, explain' enhance your mathematical thinking alongside your practice of the method

$\frac{5}{7}$ of 175 = 125

$\frac{5}{7}$ of 350 = 250

$\frac{5}{7}$ of 700 =

$\frac{5}{7}$ of 70 =

$\frac{5}{7}$ of 35 =

$\frac{5}{7}$ of 17.5 =

REFLECT, EXPECT, CHECK, EXPLAIN

Intelligent Practice and Task Design

$\frac{3}{7}$ of 175 = 75

$\frac{3}{7}$ of 350 = 150

$\frac{3}{7}$ of 700 = 300

$\frac{3}{7}$ of 70 = 30

$\frac{3}{7}$ of 35 = 15

$\frac{3}{7}$ of 17.5 = 7.5

$\frac{5}{7}$ of 175 = 125

$\frac{5}{7}$ of 350 = 250

$\frac{5}{14}$ of 175 = 62.5

$\frac{5}{14}$ of 350 = 125

$\frac{5}{7}$ of 35 = 25

$\frac{5}{14}$ of 17.5 = 6.25

Over to you:
After completing a sequence of Intelligent Practice questions, consider the following prompts

Prompts

- Choose two things you like
- Choose two things you would change
- Choose a relationship between answers to explain in two ways
- Think of two questions to continue the sequence

Learning Episode ~ A basic model

Worked example : The teacher models how to carry out a method

Practice: Pupils work through questions that require that method, gaining confidence and competence

Problem solving: With the basics secure, pupils are challenged to do something more complex involving that method. Perhaps use it in a non-standard way, in a context, in a SATs question, or interleaved with other mathematical concepts

What is the purpose of Problem Solving ?

To allow pupils to apply the method they have practiced earlier in the Learning Episode in novel ways

Mathematical problems are the unfamiliar, complex and interesting challenges that occur naturally in mathematics.

Which experiences do pupils need in order to develop into creative, resilient problem solvers?

Key Ingredients of Problem Solving
things we want pupils to experience at the end of each Learning Episode

1. Same Surface, Different Depth type problems
2. Goal-Free problems
3. Purposeful Practice

1. Same Surface, Different Depth type problems (using I See Reasoning)

Finish the picture

$\frac{3}{7}$ of ☐ = ☐

Find two different solutions

$\frac{3}{\square}$ of 42 = ☐

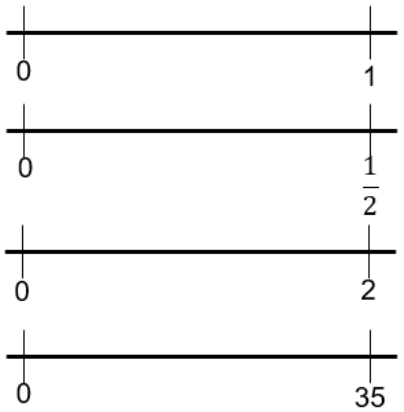
$\frac{3}{\square}$ of 42 = ☐

$\frac{3}{7}$

Which fraction is larger ?
Justify your answer.

$\frac{3}{7}$ or $\frac{2}{3}$

Draw arrows to show the position of $\frac{3}{7}$ on each number line

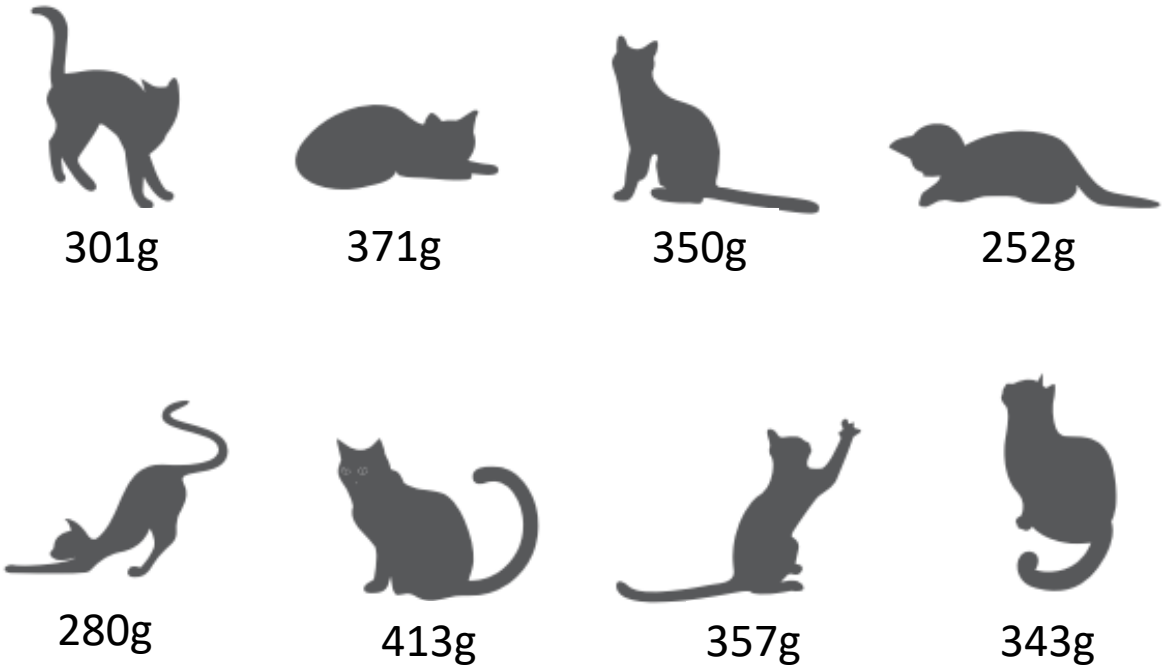


2. Goal-Free problem using a KS2 SATs question

This picture shows the masses of eight kittens at 4 weeks old

They should each have a mass of 490g at 6 weeks old

What can you work out?



(adapted from P3, Q7, 2019)

3. Purposeful Practice
– thinking hard
– getting out of the comfort zone

If I know that $\frac{3}{7}$ of a number is 42, what is the number ?

If $\frac{3}{7}$ of a number is greater than 47 , what is the smallest possible value for this number ?

If $\frac{3}{7}$ of a number is less than 47 , what is the largest possible value for this number ?

If $\frac{3}{7}$ of a number is the same as $\frac{5}{7}$ of 84, what is the number ?

The Learning Episode

How would this work in your classroom / school ?

What are the barriers to success when implementing this?

What will your colleagues like / do well ?

What will you need to support / provide CPD for ?

Hampshire Services
HIAS SCHOOL IMPROVEMENT

HIAS MATHS TEAM

Jo.Lees@hants.gov.uk

For further details on the full range of services available please contact us using the following email:

htlcdev@hants.gov.uk

Hampshire Services
HIAS SCHOOL IMPROVEMENT

Upcoming courses

Keep up-to-date with our learning opportunities for each subject through our Upcoming Course pages linked below. To browse the full catalogue of learning offers, visit our new Learning Zone. Full details of how to access the site to make a booking are provided here.

- English
- Maths
- Science
- Geography
- RE
- History
- Leadership
- Computing
- Art
- D&T
- Assessment
- Support Staff
- SEN