

SERVICES FOR SCHOOLS

Tackling Educational Disadvantage (TED): Building Blocks for Excellence

Subject Level: Mathematics

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Introduction

Hampshire County Council Children's Services is committed to securing good educational outcomes for all its children and young people. To this end the School Improvement Team within the Hampshire Inspection and Advisory Service (HIAS) works with schools to secure strong, inclusive leadership and high-quality teaching and learning for all children and young people.

Underpinning the work of HIAS in tackling educational disadvantage are the following core principles:

- the entitlement of every child and young person to receive a good education that enables them to maximise opportunity and success in learning and life, irrespective of need, prior attainment, background or circumstance
- strong pathways for all children and young people into further education, training, employment and independent living
- successful and fulfilling participation in society, economic prosperity, good physical and mental health
- access for every child and young person to suitable, high-quality provision, which meets diverse need and diminishes barriers to participation and engagement
- equity of access and onward life chances; those who need something more or something different in order to realise this ambition **do** receive something more or something different
- a rounded education for every child and young person; all should have access to and benefit from a breadth of experience and cultural capital
- a strong commitment to prevention and early intervention
- a happy and memorable childhood for all.

Educational disadvantage is a national issue that HIAS is determined to tackle locally. This guidance has been developed following a year-long project with a small group of Hampshire schools, working with HIAS and Marc Rowland from the Rosendale Research School in London (and initially from the National Education Trust). It is intended to support practitioners in tackling educational disadvantage by understanding and addressing the implications of that in the classroom, through high-quality teaching and learning. Fundamentally the guidance explores strong pedagogy and the key factors which will help make a difference for those vulnerable to underachievement in the classroom.

The focus group when developing this guidance included those categorised as *disadvantaged* by virtue of being eligible for free school meals, being in local authority care or having been adopted from local authority care. It is important to note, however, that the materials are aimed at supporting any pupils vulnerable to underachievement and, therefore, experiencing educational disadvantage.

The building blocks for excellence in tackling educational disadvantage

A year on from the start of the project with Hampshire schools, Marc Rowland formulated a set of *building blocks for excellence* derived from the experiences and learning of those schools alongside HIAS. These building blocks have been used to shape the *Tackling Educational Disadvantage* (TED) guidance and summarise the key factors that contribute to a successful whole-school strategy to improve outcomes for disadvantaged pupils. Schools may also find the building blocks useful to support discussions around strengths and areas for improvement, to inform self-evaluation. Crucially, we believe that any evaluation should have within it robust evidence of impact and outcomes, including a strong voice from children and young people. Talking to them at an appropriate level about their perceptions in relation to the building blocks offers invaluable insight to the school self-evaluation process.

The building blocks for excellence are:

- the school culture is one of high aspiration for all, focusing on attainment and equity of access to a range of provision
- the curriculum offer is wide and varied for all and reflects the importance of cultural capital
- there is a collective, shared vision and ambition for disadvantaged pupils, which recognises that academic attainment is necessary and critical in terms of widening opportunity and life chances, but is not in itself sufficient for success
- all staff understand the school's strategy for tackling educational disadvantage and their role within it; all are accountable for the outcomes of disadvantaged pupils
- there is a unified belief that all pupils, irrespective of background or barriers, can attain well; high expectations for all is a lived and evidenced mantra
- disadvantaged pupils and their families are valued equally to others; there is a clear commitment on the part of the school to work in genuine partnership with families, to achieve best outcomes for pupils
- equal access to high-quality teaching and learning is at the forefront of decisions and organisation; it is recognised as a key to success for disadvantaged learners
- leaders and class teachers target resource at pupils at risk of underachievement; success is measured through the quality and extent of learning, rather than the amount of resource made available or the number of interventions provided
- professional learning for adults within the school is appropriately targeted to pupil need and promoting high-quality learning for all; it is at the heart of the school's strategy

- there is a strong, universal understanding of the barriers that can be faced by disadvantaged pupils, how those barriers present in the classroom and how they can be tackled
- there is a good understanding of the experiences of growing up as a disadvantaged child or young person in the community
- Pupil Premium is targeted effectively to need; it is not focused solely on those with low prior attainment but instead supports pupils of all prior attainment groups to achieve well
- data is used effectively to inform planning at both a strategic level and at the point of delivery in the classroom
- there is recognition that outcomes data offers a picture of the symptoms of educational disadvantage; strategies needed to tackle this must focus on the causes
- careful consideration is given to how groups of pupils and learning are organised and how resources, such as learning assistants, are deployed and equipped to maximise their impact
- the importance of language and vocabulary development is given high status; this forms part of the strategy across all subject areas and curricular experiences
- dialogic talk or *talk for learning* and opportunities for collaborative learning are recognised as powerful tools to support learning and address gaps in understanding/ vocabulary for disadvantaged pupils
- intervention strategies are carefully determined based on individual need, are regularly reviewed for impact and do not compromise exposure to high-quality teaching and learning/other curricular areas; they are additional and extra
- school leaders and teachers recognise and respond to the needs of different cohorts and pupils; they recognise that a strategy that is successful one year may not be successful another year and adjust accordingly
- points of transition in a pupil's education or home life are recognised as potentially negative factors in the continuum of learning and are planned for accordingly, in order to minimise adverse impact
- robust quality assurance processes are in place, internally and externally, including a clear and dedicated focus on disadvantaged pupils, their provision and outcomes, as well as the culture of inclusion more broadly
- school self-evaluation is timely, routine and rigorous; it instigates change where needed and drives further improvement
- school self-evaluation outcomes and research evidence are used in an honest and open way and not used selectively to support existing practices, activities or biases
- the school engages in action research internally and also looks to best practice and research locally, nationally and internationally

- the school recognises the part it can play in system-wide improvement and participates in opportunities to share and disseminate good practice beyond its own boundaries.

How the guidance works

The TED guidance uses the building blocks for excellence to identify four key areas:

- whole-school culture and engagement
- access and equity, strong foundations for learning
- pitch, progression and expectations
- thinking, reflection and motivation.

Each of the four areas explore some key questions and considerations for leaders and practitioners, in the context of their own school or classroom. In this overview guidance, the questions are generic; in each of the guidance booklets for subjects there are also questions specific to that subject. The intention is to prompt reflection, debate and thinking to inform leadership, provision, teaching and learning at all levels: senior leaders, middle leaders and class teachers.

Much of the guidance focuses on strategies, approaches, questions and considerations at a classroom level; however, this will only have the desired maximum impact if the whole-school culture and ethos is aligned to – and driving – the *strong outcomes for all* agenda of inclusivity and equity. A defining factor for successful schools and strong outcomes for all is the quality of leadership, culture and ethos. In these schools there is a set of clearly defined, inclusive values, which are lived out by all and are explicit within the school community. In these schools all staff own and buy into the vision and strategy for tackling educational disadvantage, taking their lead from the most senior staff.

The whole-school culture will determine the success of a school's strategy. Leaders will need to support and promote the pedagogical approaches offered within this guidance in the context of an inclusive culture. It is, therefore, important that all senior leaders within a school are familiar with the contents of the TED materials and what they seek to achieve, in order to support their implementation in the classroom.

A breadth of experience and, therefore, of curriculum offer is a critical part of the inclusion agenda and of raising overall attainment for disadvantaged learners. A narrow curriculum will not provide the richness and diversity needed to promote strong participation, enjoyment of learning, depth of vocabulary and the connections in understanding that support good outcomes for disadvantaged pupils. HIAS has, therefore, ensured that guidance is available for a range of subjects beyond the core, including art, computing and design technology in the first phase of producing the materials. Other subjects will be added in subsequent phases. Each guidance booklet follows the same structure, in order to support a whole-school approach delivered at a subject-specific level.

A word of caution

As with all labels, using the term *disadvantaged* carries with it the risk of stereotyping. It is, of course, important to flag those who are in groups that are typically more prone to underachievement in order that we can be proactive and alert to early signs. In so doing, however, there is the danger of not going beyond the label to determine cause or particular areas of vulnerability and specific needs. Furthermore, labels such as *disadvantaged* can have a negative impact on perceptions of ability, expectations of those pupils and aspirations for their futures. We must guard against this if any strategy to tackle educational disadvantage is to be successful.

“Teachers referring to their ‘low ability pupils’ is commonly heard in schools. Other synonyms are no better, whether it’s circles, red group, snail group or other proxies for the ‘bottom set’. In my experience pupils in such groups are socio-economically disadvantaged learners who haven’t accessed high-quality early years education, have grown up with limited access to language and cultural capital. They may not be achieving optimally at the moment, but labelling them as low ability is doing them a great disservice.”

Learning without Labels, Edited by Marc Rowland (2017)

Whole-school culture and engagement

Access and equity, strong foundations



Thinking, reflection and motivation

Pitch, progression and expectations

Why is important to succeed in mathematics?

We believe that all young people, regardless of background, should have access to great maths teaching in primary and secondary school.

The numeracy skills of workers in England are a bigger predictor of economic returns than in many countries.¹

Attainment in mathematics at school has lifelong consequences. Leaving school with a good GCSE in maths is a prerequisite for progression into the world of employment and further education. Too many young people do not yet make the grade and, as a result, risk social and economic exclusion.

These pupils are disproportionately drawn from disadvantaged homes. Data suggests that at least one in two young people from low-income households in England are not achieving the expected level in mathematics at the age of 16. This not only denies these young people access to a secure financial future, it is also a barrier to social mobility.

In the NFER 2016 analysis report on disadvantaged pupils' outcomes for mathematics², it states: *"The impact of socio-economic background on mathematics performance in England can be seen from the most to least disadvantaged. As socio-economic background of pupils increases, so does average mathematics performance; the gap between the most and the least disadvantaged is equivalent to over three years of schooling"*.

The report examines other countries and the lack of change over time, suggesting that it is very difficult for countries to reduce the impact of socio-economic background on performance but the gap is notably lower in Wales, Northern Ireland and Scotland. If we are to break this link between family income and mathematical attainment, narrowing the gap in mathematics performance between those who are considered disadvantaged and those who are not, we must consider other factors and look to see what is working well.

It was found in the 2016 NFER report² that disadvantaged pupils who perform better than average, given their socio-economic background, tend to be autumn born, are more confident in their abilities and are less likely to truant. This would indicate that there is a need to pay attention to summer-born pupils and offer strategies to ensure that they are not left behind. In addition to this, supporting schools to increase confidence in mathematical ability, through growth mindset and the work of researchers and practitioners such as Jo Boaler (*Fluency without fear*³), would be beneficial when tackling underperformance of disadvantaged pupils.

Since 2011, disadvantaged pupils have benefited from the Pupil Premium (PP) funding. The Government also established an independent charity, the Education Endowment Foundation, in 2011 to provide evidence for schools of what works to raise the attainment of disadvantaged young people. This is intended, in part, to guide the use of the PP funding. In November 2017, The Education Endowment Foundation published a guidance report drawing on the best available evidence regarding the teaching of maths at Key Stages 2 and 3 entitled *Improving mathematics in key stages two and three*⁴. The guidance

report offers eight practical, evidence-based recommendations to support those struggling with their mathematics and outlines some key principles for effective mathematics teaching. This guidance is accompanied by a self-assessment guide to support schools when evaluating provision and the quality of their maths teaching. These recommendations, listed below, will contribute significantly to the structure of this toolkit:

- 1 Use assessment to build on pupils' existing knowledge and understanding
- 2 Use manipulative and representations
- 3 Teach pupils strategies for solving problems
- 4 Enable pupils to develop a rich network of mathematical knowledge
- 5 Develop pupils' independence and motivation
- 6 Use tasks and resources to challenge and support pupils' mathematics
- 7 Use structured interventions to provide additional support
- 8 Support pupils to make successful transition between primary and secondary school.

Tackling educational disadvantage: guidance for mathematics

In each of the following sections you will find information about what each priority might mean in school, any potential problems that could present barriers to successful implementation, prompts, research quotes and references, and questions to support your thinking around each of the four priorities:

- 1 Whole school culture and engagement
- 2 Access, equity and strong foundations
- 3 Pitch, progression and expectations
- 4 Thinking, reflection and motivation.

There is a glossary to define what we mean when we use certain terms in the body of the text.

At the end of this document are four appendices, one for each priority, in which you will find resources and ideas to help you implement or develop each area in your school.

Whole-school culture and engagement

Vision is underpinned by a shared moral purpose to provide the best education and life chances for all, resulting in a culture without limitations on achievement.

- Everyone in the school community takes responsibility for their own learning and invests in the learning of others, believing that ability can be developed and attainment is never fixed. This is reflected in vocabulary used by both pupils and staff.
- Schools are honest and robust in their self-evaluation in order to shape priorities and drive improvement.
- Action planning focuses on the identified areas of need and prioritises actions that will have the most impact. These are embedded, monitored and evaluated for impact and changed in response to findings.
- Schools understand that quality of implementation is as important as the strategies chosen and invest time in the implementation process.
- Schools are confident to change approaches if impact is not evident over time, learning from findings.
- A wide variety of research and information is used to inform thinking, not to justify existing activities or biases.
- The teaching and learning aims of the school are derived from a shared understanding of effective pedagogical approaches.
- Schools design a bespoke curriculum for their pupils which provides the body of knowledge and skills for pupils to succeed, ensuring that it is relevant and responsive to their lives and context.
- Views and ideas from all stakeholders are actively sought and valued, and used to inform actions.
- All pupils and staff feel that they belong in their school; they value the community and their role within it.
- Subject teams innovate subject-specific pedagogy in response to national and local changes. Teachers are experts in their subject.
- The continuing professional development (CPD) offer is focused on improving outcomes for pupils, developing shared approaches to teaching and learning, subject-specific knowledge, pedagogy and understanding of progression. Teachers learn from one another.
- Teachers and pupils share innovative approaches with honesty about successes and failures within the context of a safe learning culture.

- All teachers are committed to addressing barriers to learning for disadvantaged pupils in their own classrooms, believing that improving teaching and learning can have a profound impact on pupils' experiences, outcomes and life chances.

“Sustained, in-depth changes in practice, which target the needs of vulnerable children, require effective CPD, particularly if they are to be owned by practitioners and have the capacity to be taken to scale.”

Coghlan et al (2009); Dyson et al (2010); Higgins (2013); Sharples et al (2011); Cordingley and Bell (2007)

“Professional development programmes must consider both subject knowledge and subject-specific pedagogy in order to achieve their full potential.”

***Developing Great Teaching* (Teacher Development Trust, 2015)**

“I am very hopeful that someone will read this and know what I need and the way I want to learn. I hope you don't throw my [response] paper away and say it's child's talk. Please take what I say seriously as I am hoping that education will improve. This is the first time that someone has listened to what I have to say – Mariam, Year 5.”

Eleanore Hargreaves, *Children's Experiences of the Classroom* (2017)

“Everything works somewhere, and nothing works everywhere.”

Dylan Wiliam, *Inside the Black Box: Raising Standards Through Classroom Assessment* (1998)

“Treat implementation as a process, not as an event.”

***Putting Evidence to Work: a School's Guide to Implementation* (Education Endowment Foundation, 2018)**

“Excellent maths teaching requires good content knowledge, but this is not sufficient. Excellent teachers also know the ways in which pupils learn mathematics and the difficulties they are likely to encounter, and how mathematics can be most effectively taught.”²

***Improving Mathematics in Key Stages 2 and 3: Guidance Report* (Education Endowment Foundation, 2017)**

Potential barriers

- Teachers and staff do not believe high attainment for disadvantaged pupils is possible.
- Staff make negative judgements about pupils and their families, believing that barriers cannot be overcome.

- If staff express these views, pupils and their families may feel that they do not belong in the school and come to believe that they cannot achieve.
- External obligations such as Ofsted drive disadvantaged focus, rather than a moral purpose.
- Teachers are judged and categorised within the school, supporting the view that ability is fixed.
- Teachers feel scared to take risks, so pupils feel the same. A culture of learning for all is not established.
- Staff have little opportunity to learn about what makes the biggest difference to pupils and to develop their professional expertise.
- Work to improve outcomes focuses on monitoring of data rather than effective pedagogy.
- Insufficient importance is given to identifying priorities and exploring a range of possible approaches and actions prior to strategic decision-making.
- Approaches are adopted without considering the context of the school and the readiness of the school to deliver the implementation plan.
- Action becomes the focus; quality of implementation is fundamental to success.

Key questions for teachers of mathematics

- Do you offer opportunities for teachers to develop their subject knowledge and passion for mathematics (eg through opportunities for teachers as mathematicians to engage in research groups, collaborative planning, department time used to explore problem-solving and reasoning and to discuss mathematical pedagogy as both teachers and learners)?
- How do you utilise the strengths of different staff to support colleagues and the wide range of learners?
- Are teachers enabled to observe each other's teaching?
- Do teachers take risks and constantly reflect on teaching and learning?
- Has a shared vision for mathematics within the school been clearly established and is this embodied in your curriculum design?
- How do you share mathematical pedagogy across the whole school? Do all teachers have a clear understanding of ways to support pupils' access to mathematical ideas in other subjects?

- How do you choose mathematical contexts that both reflect and challenge values, attitudes and experiences linked to the local community? Do the contexts you select offer positive role models and inspiration to your pupils? Can they see themselves as mathematicians?
- Are pupils involved in making choices about the strategies they employ when problem-solving? Does the culture of the classroom enable pupils to feel empowered to be proactive in their choices?
- Do teachers ensure access and success for all through the use of a range of models, images and structured resources to support conceptual understanding? Do your curriculum design and schemes of learning promote the concrete-pictorial-abstract approach (CPA)?

Access, equity and strong foundations

- Every school, cohort and pupil is different. Schools have a deep understanding of the factors that influence the learning, progress and attainment of their pupils, addressing barriers and inequalities to create a culture of opportunity for all.
- Schools engage positively with, and therefore understand, pupils, parents and the community, consciously rejecting assumptions and stereotypes.
- The school's vision and strategic plan are based on fundamental understanding of their context and high and equitable aspirations for all pupils; these values are fully evident in curriculum, day-to-day practice and behaviours.
- Robust systems are in place to address gaps in fundamental life skills and experiences, empowering all pupils to access the curriculum and opportunities beyond school.
- Schools build a culture that develops pupils' literacy and numeracy skills. Teachers understand how they can ensure all pupils grasp the basic skills required to access the curriculum.
- Pupils' eyes are opened to the full range of future educational and employment pathways; all staff ensure that pupils believe all pathways are open to them.
- All teachers identify and address gaps in pupils' understanding of the world, enabling their active participation in society.
- The school actively reaches out to support parents' and carers' engagement in their children's education.

“The impact of socio-economic background on mathematics performance in England can be seen from the most to least disadvantaged. As socio-economic background of pupils increases, so does average mathematics performance; the gap between the most and least disadvantaged is equivalent to over three years' of schooling.”

Wheater, R, Durbin, B, McNamara, S and Classick, R, *Is Mathematics Education in England Working for Everyone?* NFER analysis of the PISA performance of disadvantaged pupils (Slough: NFER, 2016)

“Leaving school with a good GCSE in maths is a prerequisite for progressing in quality, apprenticeships and further education.”

Improving Mathematics in Key Stages 2 and 3: Guidance Report
(Education Endowment Foundation, 2017)

“Teachers should develop pupils' numeracy and mathematical reasoning in all subjects so that they understand and appreciate the importance of mathematics.”

The National Curriculum in England (Department for Education, 2013)

“It is clear that a large and rich vocabulary is the hallmark of an educated individual. Indeed, a large vocabulary repertoire facilitates becoming an educated person to the extent that vocabulary knowledge is strongly related to reading proficiency in particular and school achievement in general.”

**Isabel L Beck, Margaret G McKeown and Linda Kucan,
*Bringing Words to Life: Robust Vocabulary Instruction***

“One study found that by the age of three, children from the most prosperous households have heard 30 million more words than children from impoverished households.”

Lost for Words: Poor Literacy, the Hidden Issue in Child Poverty. A policy paper
(National Literacy Trust, July 2013)

Potential barriers

- Common barriers to learning for disadvantaged pupils and less successful learners are not identified and actions not defined, leading to a culture where teachers are driven to raise attainment for disadvantaged pupils, without a shared understanding of how to do this. As a result, teachers feel that the expectation of high attainment for disadvantaged pupils is unachievable.
- The curriculum is designed without listening to these pupils, resulting in a curriculum that is not relevant or engaging for them.
- Teachers stereotype parents and, as a result, do not engage with parents in equal measure.
- Planning does not effectively deal with key gaps in prior learning and pupils, therefore, have rocky foundations.
- Gaps in pupils’ literacy and numeracy are not effectively identified or addressed, and pupils are unable to fully access the curriculum.
- The role of developing literacy/numeracy is viewed as the responsibility of others (perhaps subject specialists in secondary, for example).
- The basic skills required to enable access to the curriculum are not clearly defined. Not all staff have the subject knowledge or understanding of effective pedagogy to close these gaps in learning.
- Pupils are not exposed to the full range of educational and employment pathways, and believe their pathway is fixed.
- Teachers assume contextual understanding and experiences that pupils may not have and learning is, therefore, missed.

Key questions for teachers of mathematics

- Has the mathematics subject leader identified some key barriers to learning for disadvantaged pupils in the school and created an action plan to address these? How are teachers driving through these actions and adapting to meet the needs of their pupils?
- How is the local community understood, so that the curriculum is designed effectively for the school's pupils?
- Do parents feel welcome within the school/Mathematics Department? Is there regular, positive contact between the school and parents?
- What steps do you take to engage parents with the mathematics curriculum? Are there opportunities for parents to explore how to support their children's learning at home (eg support for understanding calculation strategies, the learning of times tables, questioning skills, revision sessions)? Are parents able to enjoy and participate in mathematical activities (eg home/school learning games and quizzes, celebration of pupils' mathematical achievements)?
- What strategies are used to help pupils to develop their mathematical vocabulary? Do teachers consciously create regular and cumulative routines to build vocabulary knowledge?
- What extra-curricular activities and experiences are available to pupils? Are pupils offered opportunities to experience puzzles and problem-solving, eg sudoku and code-breaking competitions; visit locations with a mathematical theme or history, eg buildings designed by Inigo Jones, Bletchley Park, hands-on museums; participate in recreational mathematics workshops, eg Maths on Toast and Maths Inspiration; visit universities to explore degrees in mathematics and related subjects; meet professional mathematicians, eg engineers, accountants and computer programmers; and meet other people in work who use mathematics in their daily life, eg hairdressers, electricians, architects and gardeners?
- When planning learning journeys, how do teachers plan to address gaps in pupils' experiences and contextual knowledge in order to ensure that they are able to access the mathematics?
- How are pupils' skills in discussion and presentation developed, including the use of appropriate mathematical language? How do you build pupils' confidence to speak and contribute to discussion? Are pupils' contributions and questions structured, welcomed, valued and tested?
- How do you encourage pupils to engage in and enjoy mathematics?
- Does your school have a positive culture for mathematics, with a common belief that all can achieve?
- How are pupils supported to complete their homework for mathematics?

Potential, pitch and expectations

Teachers use assessment for, and of, learning effectively to ensure high expectation and challenge for all. Teaching should be pitched to enable pupils to progress in their learning. There is a shared understanding of the 5-16 years' mathematics curriculum so that the learning journey is continuous and consistent, particularly at points of transition.

- High-quality teaching is available for all. The curriculum is ambitious in challenge and equitable. All pupils have access to a wide range of experiences, vocabulary and collaborative opportunities in different peer groups.
- Teaching is responsive and adaptive to enable progression, with teachers utilising a wide repertoire of approaches based on pupils' needs.
- Behaviour is not a limiting factor to the class/grouping a pupil is placed in.
- Formative teacher assessment informs starting points and provides richer information than baseline tests. Formative and summative assessment is used primarily to inform practice and ensure progression.
- Target setting expects that the majority of pupils meet or exceed age-related expectations where prior attainment does not determine future attainment.
- Schools use a broad range of information to understand each child.
- Assessment data provides information so that leaders can track whole-school progress in a meaningful way and ensure that no learner is left behind.
- Schools use academic transition information (in addition to pastoral) to maintain momentum.
- Teachers work with colleagues to understand prior curriculum/pedagogy and assessment in order to teach effectively, secure progression and address gaps in learning.

Assessment should be used not only to track pupils' learning but also to provide teachers with information about what pupils do and do not know."

Improving mathematics in Key Stages 2 and 3: guidance report
(Education Endowment Foundation, 2017)

"There is solid evidence that poor teaching disproportionately disadvantages deprived children. Equally, evidence tells us that excellent teaching disproportionately benefits them. So high-quality teaching must be at the core of all Pupil Premium work."

Sir John Dunford, *Ten Point Plan for Spending the Pupil Premium Successfully* (2014)

“The effects of high-quality teaching are especially significant for pupils from disadvantaged backgrounds: over a school year, these pupils gain 1.5 years’ worth of learning with very effective teachers, compared with 0.5 years with poorly performing teachers. In other words, for poor pupils the difference between a good teacher and a bad teacher is a whole year’s learning.”

Improving the Impact of Teachers on Pupil Achievement (Sutton Trust, 2012)

Potential barriers

- Stereotyping leads to a judgement of ability and expected outcomes, resulting in predictions that cap the pupil’s learning. As a result, teaching is underpitched and lacks pace, meaning pupils fail to catch up/keep up with peers.
- Pupils are identified as *low-ability* rather than *low attaining*, sending the message to pupils, parents and staff that ability is fixed and low attainment acceptable. This leads to low aspirations.
- Low aspirations for pupils result in work that lacks challenge. Access to the curriculum is limited. Pupils are often oversupported and do not engage in sufficiently rich and rigorous mathematical dialogue .
- Pupils may be put into a *bottom set*, a *weak* group within the class, or be too regularly supported by a teaching assistant .This can have a negative impact on learning and progress.
- Disadvantaged pupils are involved in interventions outside the classroom. Teachers are less able to support the pupil through effective assessment for learning. Interventions are not informed by the teacher so that learning feeds back effectively into lessons.
- Transition information focuses on pastoral information, with opportunities for curriculum discussions being less evident. This can give the next teacher an unbalanced view.

Key questions for teachers of mathematics

- How do you ensure that curriculum paths are not predetermined and that there is a flexible, aspirational approach for each learner?
- What format/approach is used to pass on transition information in mathematics between year groups/key stages? Is this designed to identify strengths and weaknesses so that teachers can effectively build on prior learning and address gaps in knowledge, skills and understanding?

- Do teachers primarily use data to inform teaching and learning? Do teachers use a wide range of assessment information when making judgements about a pupil's attainment (eg teaching pupils for a number of lessons rather than baseline testing or using classwork in addition to more formal assessment/test-based outcomes?)
- Are age-related expectations in mathematics clearly identified, defined and exemplified in order to ensure that pupils and teachers alike know what success looks like in each year group? Are these expectations built on a clear understanding of previous key stages/year groups, and not just the next national assessment point? What are the non-negotiable skills for each year group that the majority of pupils will achieve?
- How well do teachers understand the way the curriculum has been taught in previous years? Are they aware of the approaches that have been taught? Do teachers have a shared language to describe key concepts that will help pupils to see how their learning is progressing and to make connections to prior learning?
- When do teachers have transition conversations that enhance the mathematical learning journey of each individual?
- How do you group pupils and how are these decisions made? Do all pupils, across the range of classes, have access to the curriculum, pitch and expectation required to meet age-related expectations, or is pupil grouping a limiting factor?
- Is the quality of teaching consistently good across all classes? For example, do all classes have the same access to rich mathematical talk, collaborative problem-solving and reasoning? If not, how might you move towards a more equitable learning experience?
- How do teachers address specific curriculum gaps and develop further skills, knowledge and understanding so that pupils do not get left behind? What approaches make this possible? Where is this most successful?
- How well are interventions planned and executed so that they address mathematical gaps from earlier years? Do teachers of subsequent year groups know how to teach these skills effectively?
- How is teaching adapted to ensure that pupils are supported to meet age-related expectations? Do teachers offer a wide range of conceptual representations to ensure access and success for all?

Thinking, reflection and motivation

Staff and pupils are reflective and motivated and understand that these characteristics, alongside trust and challenge, are fundamental to effective learning. There is a shared appreciation of and commitment to collaborative thinking and lifelong learning.

- There is a culture of openness that promotes improved outcomes for children by including robust challenge.
- Staff and pupils are expected to be outward looking, engaging with a wide range of people and ideas; everyone's ideas are valued. Ideas are explored and evaluated before deciding which should be pursued.
- Teachers promote and model collaborative thinking, creating a safe environment in the classroom for learners to do likewise. Trial and improvement are valued as part of the learning process where mistakes can enable progress to be made. Pupils are encouraged to reflect on and take responsibility for their learning.
- Feedback has many forms; modelling, grouping pupils based on next steps, pre-teaching, peer teaching, self-evaluation and verbal feedback are valued by all. There is not an over-reliance on written feedback.
- Thinking aloud together is a vital part of the learning process when developing and reflecting on our knowledge, understanding and skills. Dialogic talk for mathematics and self-regulation is modelled and developed at all levels (senior leadership team (SLT), middle leaders (MLs), teachers, pupils).

"In order to help students be metacognitive, teachers must first become more aware of their own thinking."

Israel and Massey, *Metacognitive Think-alouds* (2005)

"Encourage pupils to take responsibility for, and plan an active role in, their own learning. This requires pupils to develop metacognition – the ability to independently plan, monitor and evaluate their thinking and learning."

Improving Mathematics in Key Stages 2 and 3: Guidance Report
(Education Endowment Foundation, 2017)

"Modelling by the teacher is a cornerstone of effective teaching; revealing the thought processes of an expert learner helps to develop pupils' metacognitive skills."

Metacognition and Self-Regulated Learning Guidance Report
(Education Endowment Foundation, 2018)

Potential barriers

- Teachers are challenged on attainment and progress of disadvantaged pupils without the support and explorative discussion to understand how to buck the trend. Challenge feels unfair and results feel unattainable.
- The wide range of research to support schools in meeting the needs of disadvantaged pupils are not filtered down throughout the school to engage staff in thinking and reflection.
- Pupils spend considerable time with the teacher or teaching assistant, accessing less peer-to-peer dialogue. As a result, pupils are less independent in their thinking and less aware of their own and others' metacognition.
- Teachers are required to focus on marking, rather than immediate, good quality feedback (written or verbal) to pupils in a way that has most impact for that learner.
- A lack of adult subject knowledge prevents questioning for greater depth and rich mathematical discussion happening in the classroom. This can contribute to the cycle of poor behaviour and low attainment

Key questions for teachers of mathematics

- Do teachers plan collaboratively in order to create a shared understanding of the learning journey? Do teachers discuss pupils' learning along the way, reflecting on pedagogy and impact?
- Do pupils and teachers recognise the wide range of approaches to feedback or is written marking the main form of feedback? Do teachers seek out disadvantaged pupils to ensure that they understand the feedback given and know how to act upon it? Is assessment of disadvantaged pupils' work specifically used to inform planning and next teaching?
- Are there opportunities for collaborative book looks, marking, and moderation of outcomes in order to discuss pupil needs and next steps for teaching?
- How is the effectiveness of planning and teaching in mathematics evaluated? Is assessment used to refine planning for the future, as well as to identify next steps for pupils?
- How do teachers model the thinking process behind mathematical problem-solving in order to make the reasoning visible?
- Do pupils collaboratively engage in mathematics so that they can think aloud with one another? What other opportunities do you create for collaborative working?
- Is dialogic talk an integral element of your teaching and learning repertoire? How do you develop pupils' talk for learning?

- What teaching strategies do you use to support pupils with the evaluating and editing of each iteration of a mathematics problem? Do you give learners opportunities to adapt what they have found?
- How precise is your diagnostic assessment? Do you use diagnostic marking to identify patterns in errors (in calculation, for example) and plan to address these through responsive teaching?
- Do you support learners through pre-teaching to reduce the need for post-teaching booster groups? Keep up not catch up.
- How do you celebrate excellence in pupils' work?
- Does your task design pay attention to *low threshold, high ceiling* so that all learners can access and be challenged by the task. Does the task allow for multiple routes and representations?
- Have you planned for, and included, probing and challenging questions, anticipating misconceptions and access to greater depth.

Glossary

Accuracy	Mathematical precision that depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results.
Concept	The underlying idea of a particular piece of mathematics that enables an individual to use and apply in other areas of connected mathematics.
Content	The mathematics curricula strands of number, algebra, geometry and statistics and how these relate to the <i>big ideas</i> in maths, namely: quantity; space and shape; change and relationships and uncertainty and data.
Context	The aspect of an individual's world in which problems are placed.
Cultural capital	Non-financial social assets that promote social mobility beyond economic means. In a school context, cultural capital includes parents' education, occupation, knowledge, and the cultural <i>consumption</i> and practices within the household.
Economic capital	A family's income. This is an important component of overall educational capital. Educational outcomes and poverty are directly connected.
Educational capital	The combination of three different types of capital: economic, cultural and social.
Efficiency	An efficient strategy is one that can be carried out easily, keeping track of sub-problems and making use of intermediate results to solve the problem.
Flexibility	The knowledge and use of more than one approach to solving a particular kind of problem, such as two-digit multiplication. Mathematicians need to be flexible in order to choose an appropriate strategy for the numbers involved, and also be able to use one method to solve a problem and another method to check the results.
Fluency	The ability to combine mathematical efficiency, accuracy and flexibility.
Manipulatives	A physical object that pupils or teachers can touch and move, used to support the teaching and learning of mathematics. Popular manipulatives include Cuisenaire rods and Dienes blocks.
Mathematical literacy	An individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens.

Problem-solving

- Seeking solutions not just memorising procedures.
- Exploring patterns not just memorising formulas.
- Formulating conjectures, not just doing exercises.

Polya (1945)

Procedures

Processes and algorithms that enable you to find answers to problems according to set rules.

Reasoning

The critical skill that enables us to make use of all other mathematical skills. An understanding of mathematical structure is needed to make sense of the mathematics. Reasoning is linked to systematic thinking.

Representations

Representation refers to a particular form in which the mathematics is presented. Examples include:

- two fractions could be represented on a number line
- a quadratic function could be expressed algebraically or present visually as a graph
- a probability distribution could be presented in a table or represented on a histogram.

Social capital

Defined by the Organisation for Economic Co-operation and Development (OECD) as “*networks together with shared norms, values and understandings that facilitate co-operation within or among groups*” (OECD, 2001) .In the school context, this links to how well family members can relate and interact with each other, the community and schools. Families that experience domestic violence; migrant families that experience discrimination or are not familiar with the host-country’s social context; and single-parent families can all be seen as disadvantaged in terms of social capital.

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- ³ Boaler, J, *Fluency Without Fear*. www.youcubed.org/evidence/fluency-without-fear/
- ⁴ Henderson, P, Hodgen, J, Foster, C and Kuchermann, D, *Improving Mathematics in Key Stages Two and Three, Guidance Report* (Education Endowment Fund, 2017) (<https://educationendowmentfoundation.org.uk/>)
- ⁵ Boaler, J, *How Complex Instruction Led to High and Equitable Achievement: the Case of Railside School* (The University of Sussex) (<https://nrich.maths.org/content/id/7011/nrich%20paper.pdf>)
- ⁶ Boaler, J, Altendorff, L, Kent, G, *Complex Instruction in England – the Journey, the New Schools, and Initial Results* (The University of Sussex) (https://nrich.maths.org/content/id/7011/CI_Schools_in_UK2.pdf)
- ⁷ Jerrim, J, Greany, T, Perera, N, *Educational Disadvantage: How Does England Compare?* (Education Policy Institute/UCL, April 2018)
- ⁸ Quigley, A, Muijs, D, Stringer, E, *Metacognition and Self-regulated Learning, Guidance Report* (Education Endowment Fund, 2017) (<https://educationendowmentfoundation.org.uk/tools/guidance-reports/metacognition-and-self-regulated-learning>)
- ⁹ *Improving Outcomes for Disadvantaged Learners 2017-18* (www.rosendale.researchschool.org.uk and www3.hants.gov.uk/education/hias.htm)
- ¹⁰ Gifford, S, *Early Years Mathematics: How to Create a Nation of Mathematics Lovers?* (2015) (<https://nrich.maths.org/11441>)

Appendix A:

Whole-school culture and engagement

In this appendix you will find ideas, articles, resources and examples of good practice to support effective whole-school culture and engagement.

Do you have a shared vision for mathematics in your school?

We believe that when mathematics is taught, it should be in order to promote understanding rather than just to gain knowledge.

We aim to provide the highest standard of teaching for all pupils in a positive, well-managed environment. Learning is at the heart of our Mathematics Department, and each teacher contributes to the continual improvement of teaching and learning across the department.

Pupils are our priority and we wish to ensure access and success for all, so that learners are well equipped to go out into the world of work and further education with a strong foundation in mathematics, whatever they decide to do in life.

Our vision for mathematics

Mathematics is essential for everyday life and understanding our world. It is also essential to science, technology and engineering, and the advances in these fields on which our economic future depends. It is, therefore, fundamentally important to ensure that all pupils have the best possible mathematics education. They need to understand the mathematics they learn so they can be creative in solving problems, as well as being confident and fluent in developing and using the mathematical skills so valued in the world of industry and higher education.

To set or not to set?

How does this affect motivation and progress?

“One of the most difficult challenges faced by teachers of maths is the wide range of pupils they teach. Even when taught in sets, maths classes often include students with low motivation and weak knowledge alongside others with advanced understanding and high motivation. Not surprisingly many teachers are supporters of ability grouping as it seems too hard to teach very mixed groups. In two different research studies I have conducted, in

England and the US, I have followed students through secondary schools that teach and group students differently, investigating the impact of the different teaching and grouping methods upon achievement and enjoyment. In both studies the schools that used mixed-ability approaches resulted in extremely impressive outcomes, including higher overall attainment and more equitable outcomes (Boaler, 2009). But in both cases the maths departments that achieved such goals used particular methods to make the mixed-ability teaching effective.”

How Complex Instruction Led to High and Equitable Achievement: the Case of Railside School⁵, Jo Boaler, The University of Sussex

Web: <https://nrich.maths.org/content/id/7011/nrich%20paper.pdf>

“Among the five schools that took on the complex instruction approach, some had already been teaching students in mixed achievement groups, with an emphasis on problem-solving and investigating, whilst others changed from a traditional approach with students working in sets. In our classroom observations we observed students working in groups, discussing complex problems, with the teacher circulating the room to help and scaffold students’ thinking. Many of the teachers, particularly those who had previously taught students in sets, reported that a much wider range of students were participating in whole-class discussions, with many examples of students who would have been in low groups, being the ones to solve problems and offer important insights. Many of the teachers indicated that they had been surprised by the quality of the work produced by ‘low ability’ students and they were pleased with the change in approach and grouping. In our observations we saw students working extremely well together and were particularly impressed with instances of, for example, high-achieving students working with very low-achieving students with special needs, enjoying the opportunity to help and support them. In our interviews with teachers they also reported moving from ‘talking to the students from the front’ to an approach whereby they engaged students in collaboration and discussion around complex tasks. The teachers have reported that this shift allows them new access into the thinking of their students. Also, instead of determining the path of student thinking in advance, teachers have been able to listen to student thinking as they work then intervene strategically as appropriate.”

Complex Instruction in England – the Journey, the New Schools, and Initial Results, Jo Boaler, Lori Altendorff and Geoff Kent, The University of Sussex⁶

Web: https://nrich.maths.org/content/id/7011/CI_Schools_in_UK2.pdf

“Everyone in the school community takes responsibility for their own learning and invests in the learning of others, believing that ability can be developed and attainment is never fixed. Jo Boaler, Professor of Mathematics Education, Stanford University, who has worked alongside Carol Dweck, uses research evidence to support pupils in learning mathematics. Jo Boaler looks at how we can not only develop Growth Mindsets, but also how we can teach mathematics so that pupils develop a Mathematical Mindset.”

Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts, Boaler, J

Web: www.youcubed.org



The 2018 report on educational disadvantage from the Education Policy Institute also examines evidence around setting. This report comments on whether, and how, students are streamed or grouped by ability within individual schools.

“Such practices are often adopted by schools as a way of addressing within school variation ... on the basis that such setting allows for differentiated forms of teaching and curricula for different ability levels ... There is good evidence that while such practices can benefit higher attaining students, they tend to impact negatively on middle- and lower-attaining students.”

Educational Disadvantage: How Does England Compare?, Education Policy Institute⁷

Key findings from this report

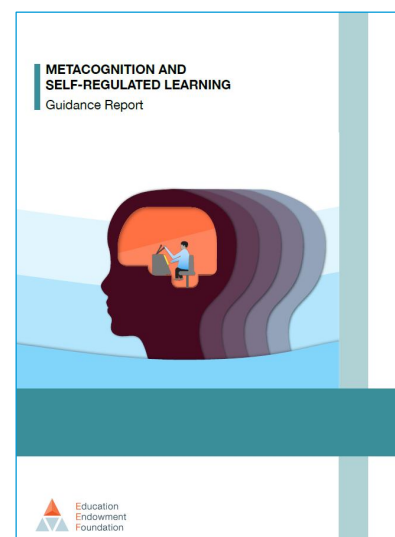
Performance in mathematics

- The average maths GCSE grade of disadvantaged pupils in England is around 3.8. This is lower than a *pass* under the new GCSE arrangements and, on this measure, England is positioned 25th of the 44 nations in the report.
- This is around a third of a grade lower than many other Western nations, including Estonia, Canada, the Netherlands and Ireland, and more than half a grade lower than in the leading Asian nations of Macao, Singapore, Hong Kong, Taiwan and Japan.
- The gap between disadvantaged pupils and their peers in England is equivalent to one whole GCSE grade. This places England at 27 out of 44 jurisdictions in terms of the size of the socio-economic gap.

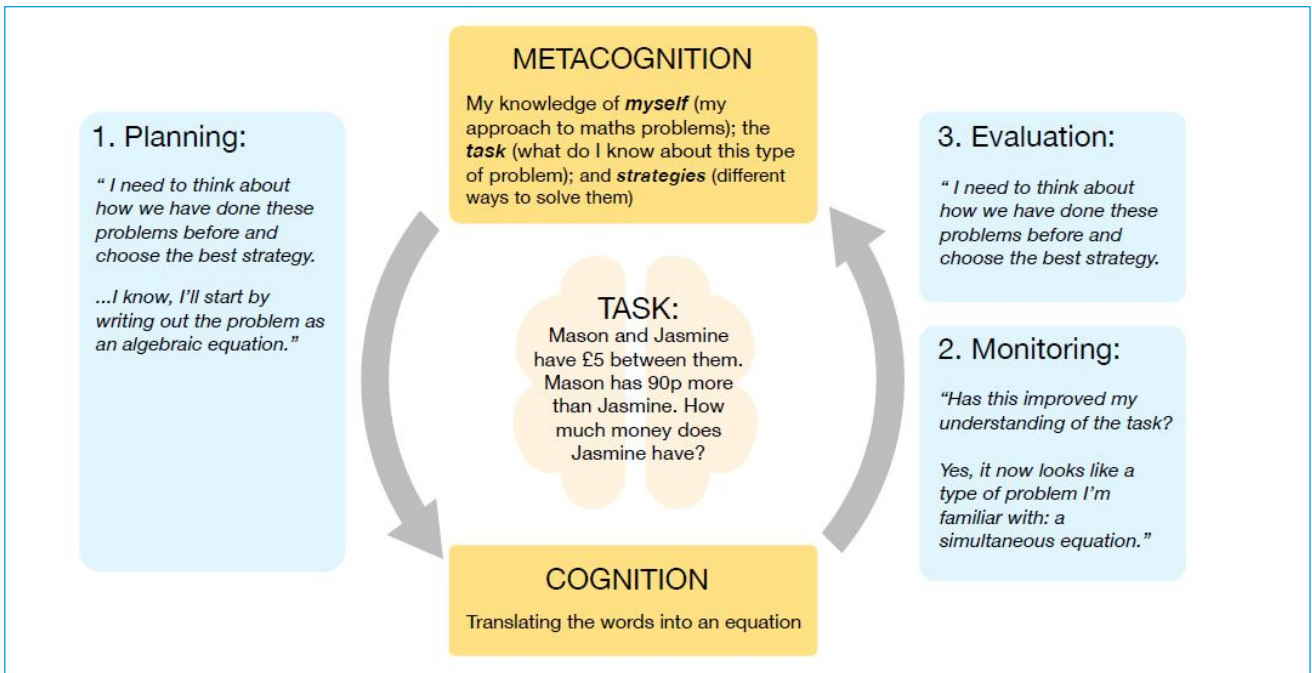
The Education and Endowment Fund guidance report on Metacognition and self-regulated learning⁸

This report defines metacognition and self-regulated learning.

It provides seven recommendations to support schools to implement high-impact, low-cost initiatives that are powerful levers to boost learning.



This is an extract from the report that models the idea of a continuous cycle of planning, evaluating and monitoring in the context of a maths problem.



The full report can be found here:

<https://educationendowmentfoundation.org.uk/>

Extract from *Improving Mathematics in Key Stages Two and Three, Guidance Report*, Education Endowment Fund (2017)

Web: www.educationendowmentfund.org.uk

See next page.

5

DEVELOP PUPILS' INDEPENDENCE AND MOTIVATION

Teachers should encourage pupils to take responsibility for, and play an active role in, their own learning.²⁶ This will require pupils to develop metacognition (the ability to independently plan, monitor, and evaluate their thinking and learning) and motivation towards learning maths.

EVIDENCE SUMMARY

- The review identified six relevant meta-analyses concerned with approaches focused on metacognition and/or self-regulation, which provide moderate evidence for these approaches.
- This recommendation is also informed by an evidence review conducted for the EEF's upcoming guidance report on metacognition and self-regulation.
- The review identified one relevant meta-analysis concerned with worked examples, providing some weak evidence to support the use of worked examples.

DEVELOP PUPILS' METACOGNITION THROUGH STRUCTURED REFLECTION ON THEIR LEARNING

Developing metacognition—often thought of as pupils' ability to think about their own thinking and learning—can help them to become more effective and independent mathematicians.* It is often thought of as pupils' ability to think about their own thinking and learning. Examples of this ability include:

- examining existing knowledge to inform the selection of a particular approach to solving a mathematical task;
- monitoring whether the chosen approach has been successful; and then
- deliberately changing or continuing the approach based on that evidence.

Ultimately the aim is for pupils to be able to do this automatically and independently, without needing support from the teacher or their peers, however, these are complex skills which will initially require explicit teaching and support. Teachers should model metacognition (see example in box F) by simultaneously describing their own thinking or asking questions of their pupils as they complete a task.²⁷ Worked examples could be usefully employed by the teacher to make their thinking explicit.²⁸ Teachers should carefully increase their expectations regarding pupils' independence as the pupils gain competence and fluency. Teachers can provide regular opportunities for pupils to develop independent metacognition through:

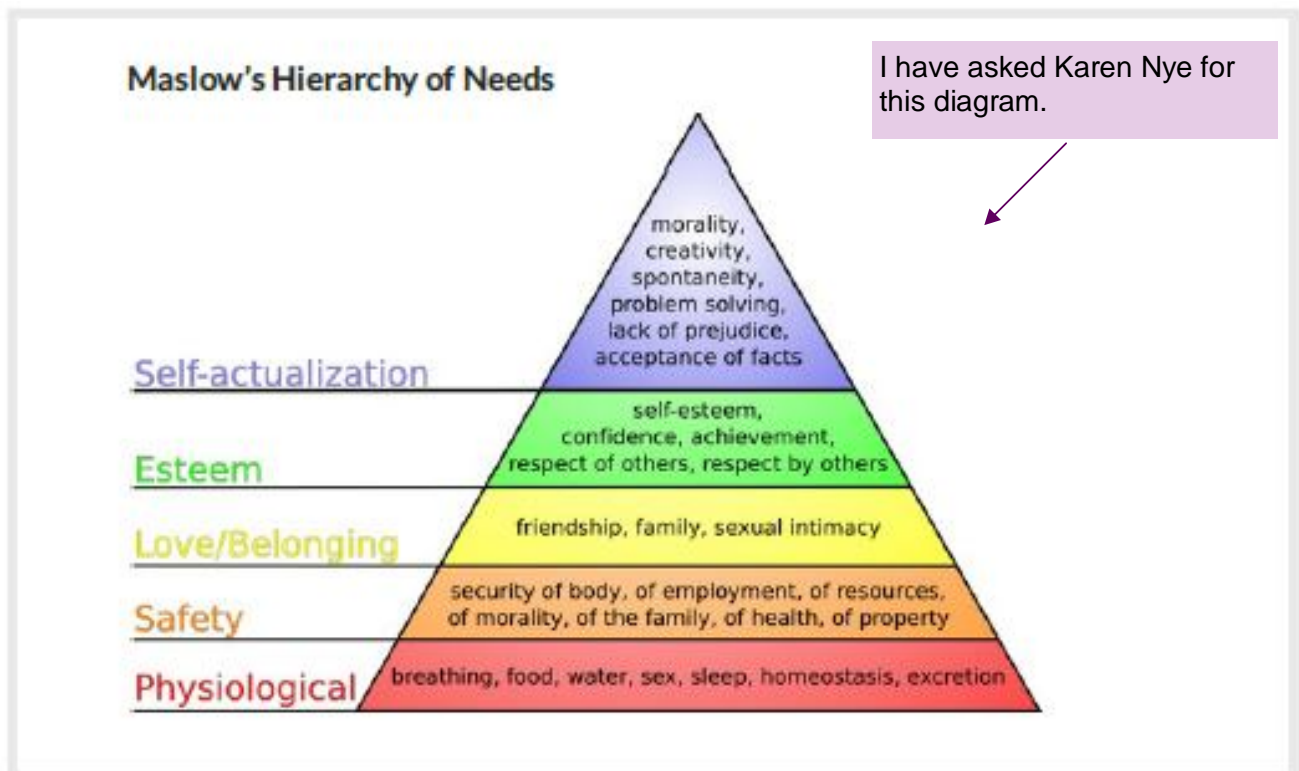
- encouraging self-explanation—pupils explaining to themselves how they planned, monitored, and evaluated their completion of a task; and
- encouraging pupils to explain their metacognitive thinking to the teacher and other pupils.²⁹

* The EEF will publish a guidance report on self-regulation and metacognition in 2018.

The NET project

In 2017, 11 secondary schools worked with the Hampshire Inspection and Advisory Service (HIAS) and the Rosendale Research School to produce case studies sharing their work on tackling educational disadvantage. The report produced also includes the Building Blocks for Excellence upon which this guidance is based.

Warblington School share how they believe that attainment is directly linked with securing a foundation of needs in every student. Maslow's Hierarchy of Needs (1943) was looked at and has been directly linked to the allocation of the Pupil Premium funding provision map.



"All teachers are committed to addressing barriers to learning for disadvantaged pupils in their own classrooms, believing that improving teaching and learning can have a profound impact on pupils' experiences, outcomes and life chances."

Hampshire leading mathematics teachers

The Hampshire leading mathematics teachers were asked to consider how to support vulnerable pupils in their classroom, linked to the *Personal Education Plan (PEP) Toolkit*. They produced case studies as well as a table of suggestions.

The full table and case studies can be found at: <http://maths.hias.hants.gov.uk/course/view.php?id=89>.

- The continuing professional development (CPD) offer is focused on improving outcomes for pupils, developing shared approaches to teaching and learning, subject specific knowledge, pedagogy and understanding of progression. Teachers learn from one another.

- Teachers and pupils share innovative approaches with honesty about successes and failures within the context of a safe learning culture.

Maths-specific problem	Strategies
Not having the correct answer	<ul style="list-style-type: none"> • Using mini-whiteboards to allow students to record a non-permanent answer. • Allow students access to answers of questions they may be working on. This could be through a mark scheme or a selection of answers to sort through. • Make <i>hints</i> and <i>prompts</i> available to students to support them completing the task without having to ask for support. • Give students completed questions with workings and get students to explain how to work out the questions.
Being put on the spot to contribute	<ul style="list-style-type: none"> • Give students red, amber and green cards to place on their desk to indicate if they are happy to contribute. • Use mini-whiteboards with the whole class so all students have to contribute and there is no perception of <i>singling out</i>. • Give a minute warning to students and tell them what you are going to ask them in a minute to allow them to consider and prepare. • Ask students to talk to each other in response to a question and the teacher can then summarise any valuable learning points made.
Recording work in exercise book	<ul style="list-style-type: none"> • Allow students to use mini-whiteboards to draft what they are going to put in their book. • Give printouts of key notes and facts to ensure information is recorded accurately.
Getting started or transitioning from tasks	<ul style="list-style-type: none"> • Present students with laminated checklists of what they need to do at the start of a lesson and offer rewards. • Give a warning when going to move on or finish a task “<i>Three minutes more</i>”, etc.

The value of working together across institutions

If we want to change the culture in our classrooms to one where enquiry is celebrated and it is *safe* to learn, it is always good to have the opportunity to see what others are doing.

The Hampshire Secondary Heads of Department Network Groups and the Primary Mathematics Core Provision Networks are a place where collaboration, discussion, professional development and the sharing of ideas regularly take place. With

approximately 500 Hampshire schools engaging in these, and associated groups, the Hampshire maths networks are one vehicle for change.

Mathematics subject leader meetings and conferences

“I wanted to drop you a line to express my appreciation for the conference you put on this July. The material was excellent and has transformed my summer in ways that I would never have anticipated. I have pursued a number of the topics that were exposed on the day through my own reading (following advice on texts from colleagues at the conference – networking being another great benefit).”

**Chris Messenger, Head of Mathematics at The Hurst Community College,
regarding Hampshire Head of Mathematics Conference 2017**

“It was lovely to share with other maths subject leaders how we felt; our concerns; what grade boundaries might look like. There was empathy and support amongst us for the difficult position we found ourselves in and how we felt the weight of accountability.”

Val Baird, Head of Mathematics, Crestwood Community School

“I think what I have found most useful is sharing our feelings and experiences with each other as we work towards the big unknown that is this summer’s GCSE. It has been reassuring to hear how other schools are doing and to know that we all seem to be in a similar position with regards to how difficult the students are finding the papers.”

Katie Smith, Head of Mathematics, Wildern School

Appendix B

Access, equity and strong foundations

In this appendix you will find ideas, articles, resources and examples of good practice to support effective access, equity and strong foundations.

Have a Maths Day

An extended version of this article appeared in Hantsmaths, Summer 2018.

A Maths Day can be a memorable experience for pupils. Here is one school's account of their day.

This was to be a whole-school event with the purpose of creating excitement about maths with a focus on mathematical reasoning. We wanted it to be a day to remember.



Problem 2a
Is it always, sometimes or never true that...

if you add a number to an even number, the answer is even?

The Maths Day was based on a treasure hunt.

We wanted the children to believe that a number of maths puzzles were hidden in gold envelopes around the parks of Basingstoke and we planned to link *live* to each park to talk to teachers who were on location. In reality, this was all filmed in advance, which was risky as the links would have to run seamlessly so the children didn't become suspicious!

Park tell us that we had to open the first gold envelope and solve the clues that would tell them where in the park to hunt for the second envelope. Off the children went, back to their classes with the first problem to solve. The last stage of solving this problem was to write an explanation of their answer and one child was chosen in each class as the winner.

We started with an assembly in the hall watching the deputy head in the War Memorial

The children all returned to the hall and the chosen child from each class would be given a letter. These letters formed an anagram of the location of the next gold envelope. The whole school puzzled over the anagram and, once solved, it was time for the next *live* broadcast to the park to tell the deputy where to look for the next gold envelope! This was repeated three times and, as you can see from the video

Problem 3
We are going to create a table tennis match between 4 children. If each child plays each child once, how many games will there be?
What if only 2 children are playing?
What if 3 children are playing?
What if 5 children are playing?
What if 10 children are playing?
What if 100 children are playing?



frames, resulted in members of staff climbing on park apparatus, much to the delight of the children who were shouting frantically at the screen to direct them!

The last puzzle resulted in the headteacher finding a special box in our public address system storage unit and rushing it back to school to reveal the prize for the winning class. All of the classes had been asked to send their best examples of reasoning to be judged and the winning class was chosen from these. The special

box was returned to assembly, opened and the winning class was allowed an extra playtime, which was a very popular, yet easy, prize!

The day generated opportunities for the whole school to engage in mathematical puzzles and present their reasoning. The children loved shouting at the teachers in the parks, telling them where to hunt for the next clue, and the day as a whole created the *buzz* that I was seeking.

The Maths Day gave every child the opportunity to be involved, work collaboratively and feel valued as part of a whole class and whole-school team.

Thanks to Fairfield's Primary School.



Keep a whole-school focus on multi-representations that enable access and success for all and build strong foundations for mathematical understanding: here are two schools' accounts of their journey

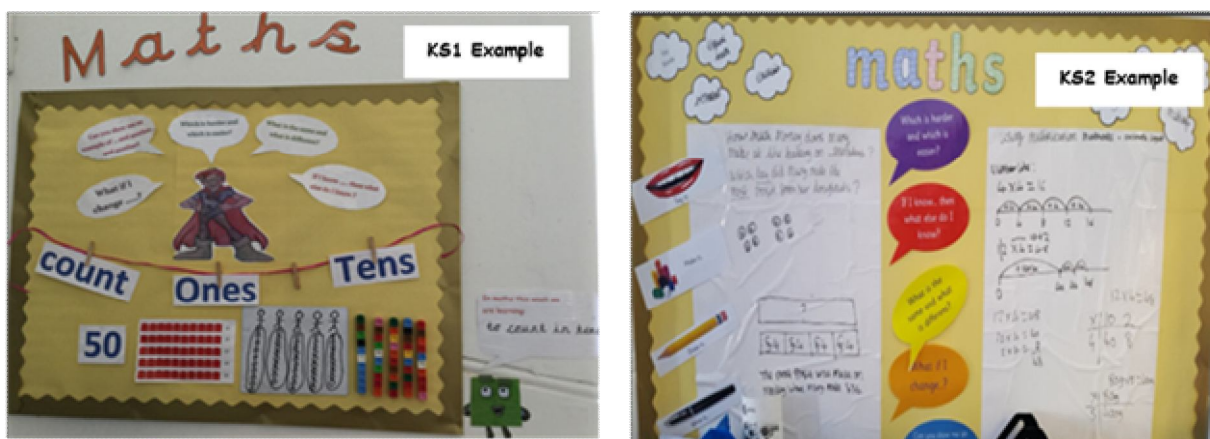
This article originally appeared in Hantsmaths, Summer 2018.

Our journey towards mastering *mastery*

Our school wanted to develop the reasoning and problem-solving aspects of our maths curriculum to ensure that all children had the opportunity to achieve age-related expectations (ARE). It was clear that both these aspects were already being explored within learning, but there was no clear reasoning and problem-solving thread that was consistent and coherent across the key stages and year groups. We addressed this requirement by developing co-ordinated working walls, unit learning journeys and our concrete-pictorial-abstract (CPA) approach.

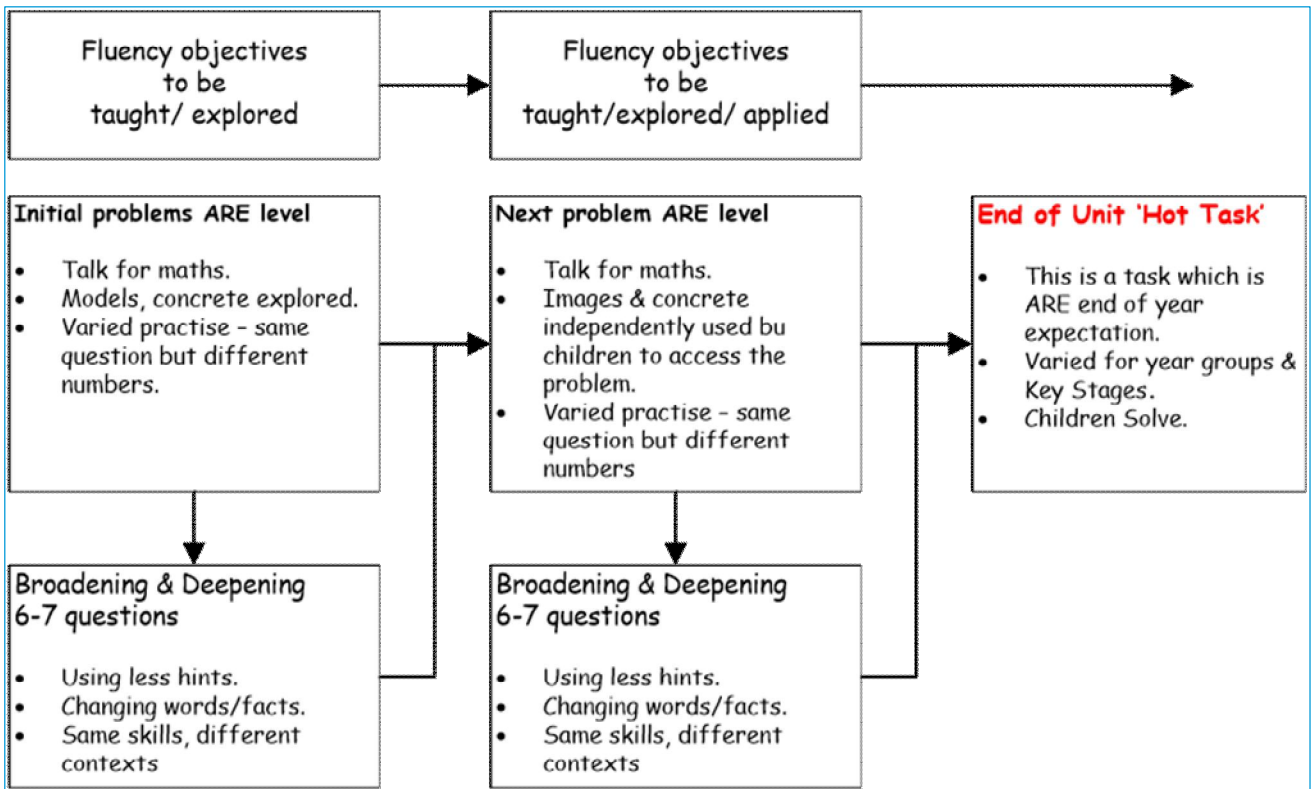
The first aspect to develop was our working walls. This was a quick fix where we co-ordinated across the Early Years Foundation Stage (EYFS), Key Stage 1 (KS1) and

KS2, a set of simple, useful walls for the current unit/ topic that would be explored and learned. Key words, models and images that would be used throughout the units were selected to be obvious and evident to the children. Learning walks showed that pupils and teachers were referring to the working walls as they showed the learning journey to the children. By the end of the unit, children had a *suitcase* of images and resources to apply to problems. There was a consistent identifying colour used as the background to the working wall and it was specifically chosen to be a yellow pastel, so the maths would not be lost on it. Each key stage adapted this to be their own and examples are shown.



The second aspect was to develop children’s learning journeys, which included adapting our lesson planning. It was clear that the children needed to *do less*, but think more deeply about problems. This would help pupils who did not enjoy lots of recording and allow them to work more collaboratively or in a practical, visual way. As a result, we changed our unit planning, so that the journeys were the same for all children unless they were special educational needs and disabilities (SEND). All units and lessons were pitched at ARE and the images and models would be used to help children achieve this, as well as extend the greater depth standard (GDS).

Teaching sessions are run across two or more lessons, where the children explore the concrete-pictorial-abstract alongside each other, using our school mantra of “*Say it, make it, draw it and write it*”. To broaden and deepen, we use question stickers, which the children access independently, that maintains a challenging regime associated with the same skill but encourages deeper reasoning and problem-solving using multi-step problems. The diagram below demonstrates the learning journey.





Recently we have been developing our learning progressions using the concrete-pictorial-abstract approach. We are now at a point where this is being used across almost all lessons, concepts and units. The children are showing a stronger understanding of what they have learnt and are able to use these resources and images in approaching problems. Here is an example of how a Year 3 class was working through multiplication using this approach.

Alisha' car measures 3m.
 Bill's van measures 4m.
 Ben's lorry measures 5m.

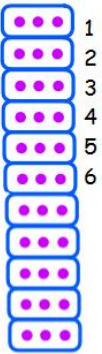
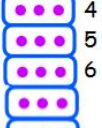
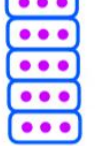
They are travelling on the Isle of Wight ferry which is 100m long. All the cars park in one lane. All the vans park in another lane and all the lorries park in the last lane.

How many cars, vans and lorries can fit on the ferry?





Sticker 2: If campervans measured 7m, buses measured 8m and a limousine measured 9m BUT they were each given 2 lanes, how many would fit on the ferry?

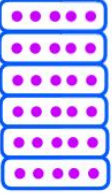


The Ferry Master records it this way.


Cars	<input type="text"/>	x	3m =	
Vans	<input type="text"/>	x	4m =	
Lorries	<input type="text"/>	x	5m =	

each rod is worth 3m



The Ferry Master records it this way.

Cars	<input type="text"/>	x	3m =	
Vans	<input type="text"/>	x	4m =	
Lorries	<input type="text"/>	x	5m =	



We are still in the early stages of our journey but we are determined to ensure access and success for all our learners. We have already seen an impact in mid-year assessments showing that our children are particularly positive in their approach to problem-solving and they understand how to reason with maths as they can always rely on the models and images that they have explored. This means that our classrooms have a can-do approach that values all learning preferences and allows everyone to achieve.

Thanks to Norwood Primary School.

Securing strong foundations with models and images in maths

We wanted to make sure that all learners were offered their mathematics in a way that worked for them. We identified the use of models and images as an area to focus on. Firstly, we had some staff in-service training (INSET) to refresh the use of models and images and introduce how bar modelling can be used to represent mathematical concepts and *unlock* understanding.

Key Stage 1

At the end of the Autumn Term, KS1 began to introduce bar modelling.

To start the children on their bar modelling journey we began to show the children the bar model and reference it along with other key models and images when introducing some maths topics. The children found it particularly useful when trying to understand a word problem or a missing number question and the bar model began to reveal what was being asked of them.

By modelling it ourselves during inputs the children became familiar at first with this image.

After some time, we began to ask the children to make a bar model using cubes to help them represent a question.

It was really important that we emphasised that the bar model wasn't there to solve a question but functioned in a different way: it supported our thinking and provided the children with a way to begin to show their understanding of what was happening or being asked of them.

When we began to show the children the links between addition and subtraction, and provided them with wider exploration questions, again the bar model was a way for the children to prove their thinking and show their reasoning.

Recap - addition and subtraction sum with big focus on subtraction.

13

+ = 13 + = 13

13 - = 13 - =

★

10

$10 \div 5 = 2 \checkmark$

15

$15 \div 3 = 5 \checkmark$

6

$6 \div 3 = 2 \checkmark$

20

$20 \div 4 = 5 \checkmark$

20

Have a go at this one

$20 \div 2 = 10$

$10 \times 2 = 20$

$2 \times 10 = 20$

The children in KS1 are still in the early days of their learning journey for bar modelling but already it is proving to be a powerful tool that speaks universally. As the children and staff's confidence has grown, more and more ways to involve bar modelling into everyday maths has arisen. Year 2 recently used bar modelling to help show their thinking behind division and related multiplications.

Key Stage 2

Like KS1, in KS2 we already use a range of models and images to support understanding in maths (Dienes, place value charts and counters, etc). Year groups have been introducing the bar model alongside some of these so children can see the connections between the different representations and also make some independent decision about which they prefer to use or which might suit the question they are tackling.

3. $4 \times 3 = 12$, $12 \div 3 = 4$ ✓
 4. $3 \times 4 = 12$, $12 \div 4 = 3$ ✓
 5. $5 \times 3 = 15$, $15 \div 5 = 3$ ✓
 6. $6 \times 3 = 18$, $18 \div 6 = 3$ ✓

★★
 Tony has shown his workings for the bar model. Is he correct? Explain your workings.

Answer: Tony is wrong. ✓
 Prove: $55 \div 5 = 11$ ✓
 Explain: His answer is correct, but he has split his bar model into 4 parts, instead of 5. ✓

In this example from Year 3, children are showing their understanding of division using the bar, but also using and then drawing Dienes alongside as a way to support calculation. Again, it has been important to emphasise to the children that the bar model is simply a way to show the problem or the concept – it is not a method for solving.

It has, however, been very useful in supporting reasoning. For example at the bottom, the bar provides visual proof that Tony is incorrect.

One issue has been a feeling that using visual representation, like the bar, seems to *slow down* the learning. The children do not seem to be doing as much *work*. My argument, however, is that in order to lay secure foundations of understanding for later, it is better to do less but to really deepen the learning. To back this up there have been some real breakthroughs with *stuck* children suddenly having that lightbulb moment.

Year 4 have been exploring how a variety of models and images can show understanding and accelerate learning. Here we can see some work on multiplication.

At the top is a one-to-one representation of the calculation, then a more refined, open array showing the partitioned calculations and, finally, a bar model showing the concept in another way.

Our books are becoming a much clearer record of children's understanding and misconceptions. Were we just to see, for example $16 \times 10 = 610$, we might not know what misconception led to the error. However, by getting children to show their understanding using these various methods, it can clearly show where their thinking has gone wrong. More often than not, though, simply the process of drawing a model or image helps children to see and correct errors as they work.

$6 \times 10 = 60$

Open array

bar model

Mrs Roberts checking

Super arrays to show understanding!
 (1+1)P 😊

In Year 5 children have used place value charts to show their understanding of division alongside the more formal algorithm.

The purpose of this is to avoid the assumption that children know what is happening in the formal calculation.

One issue, however, has been encouraging more confident children to show their understanding. Often they are reluctant to do this as they feel they can get to the answer more quickly. We have been working on moving focus away from not only getting the answer, but celebrating children showing their understanding as well.

REACH for Success!
L.O. To divide 4 digit by 1 digit numbers

I can use counters / pictures to help visualise the division	😊
I can line up each digit correctly	😊
I can divide each number by starting on the left	😊
I can use my place value knowledge to borrow if needed	😊

Confidence: Not very confident Start → Extremely confident Finish

$4848 \div 4 = 1212$
 $7227 \div 9 = 803$
 $4892 \div 4 = 1223$
 $7266 \div 7 = 1038$

Would you rather read
6/8 of the pages in a 320 page book or
7/10 of the pages in a 400 page book? Why?

320
 40 40 40 40 40 40 40 40
 $2 = 240$ $320 \div 8 = 40$
 $40 \times 6 = 240$ ✓

400
 40 40 40 40 40 40 40 40 40 40
 $2 = 280$ $400 \div 10 = 40$
 $40 \times 7 = 280$ ✓

I would rather read 280 pages because I like books.

The bar model is very useful in helping children to master tricky topics like fractions. In Year 6 we have checked children can show their understanding of fractions of amounts using a bar model alongside more formal jottings. Then, once confident they understand, we quickly move them on to the more efficient recording.

The bar model has also been very helpful in enabling children to compare fractions with different denominators.

Would you rather
2/5 of £25 or
4/10 of £20? Why?

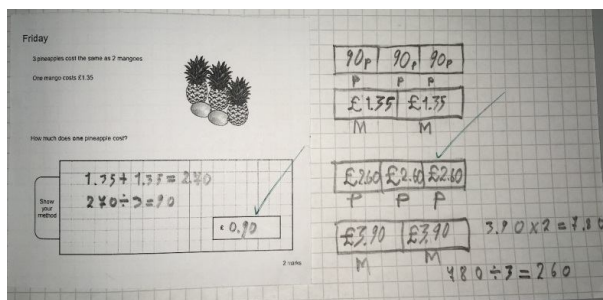
$25 \div 5 = 5$
 $5 \times 2 = 10$
 $20 \div 10 = 2$
 $2 \times 4 = 8$ I would have $2/5$ of £25 because I would have more money.

which is biggest $3/4$ or $4/5$

$\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$ ✓
 $\frac{4}{5} \times \frac{4}{4} = \frac{16}{20}$ ✓

As mentioned before, it has been difficult introducing bar modelling to support understanding in more confident children. The example here, however, demonstrates that the bar can unlock trickier, non-routine problems. For many children, without the bar this problem and similar ones would appear impossible.

The journey in models and images at our school is definitely a work in progress. We are continually reflecting on which representation would best suit the learning. For example, we have found that sometimes just a simple number line is a much more efficient way to show addition or subtraction through the tens barrier than using Dienes. Regular staff meetings, where models and images are explored and discussed, are a great way to keep everyone up to date and constantly reflecting on their practice.



Thanks to The Crescent Primary School.

Improving mathematics in Key Stages 2 and 3: a self-assessment guide

Recommendation 5: Develop pupils' independence and motivation

<p>! INEFFECTIVE</p> <p>Pupils have limited opportunities to practise and develop metacognition.</p> <p>Teachers struggle to orchestrate productive classroom discussions.</p> <p>Pupils are often unmotivated and disengaged when learning mathematics. Pupils demonstrate limited persistence and resilience in their learning.</p> <p>There is a general perception among staff and pupils that some people are naturally 'good' at maths, whilst others are not. Staff often complain about their own difficulties with maths.</p>	<p>🔄 IMPROVING</p> <p>Teachers provide frequent opportunities to practise and develop metacognition. Pupils are taught to plan, monitor and evaluate the approaches they take to mathematics.</p> <p>Teachers can sometimes struggle to orchestrate productive classroom discussions. Some pupils refrain from participating in discussion or actively listening to other pupils' ideas.</p> <p>Pupils demonstrate increasing persistence and resilience. A minority of pupils struggle with motivation.</p> <p>Maths teachers model confidence and interest in maths, but many other staff do not. Some staff and pupils believe that some people are naturally 'good' at maths, whilst others are not.</p>	<p>✓ EXEMPLARY</p> <p>Teachers provide extensive opportunities to practise and develop metacognition. This includes regular opportunities for pupils to explain their approaches to mathematical tasks to themselves, the teacher and other pupils. Teachers carefully increase their expectations regarding pupils' independence as the pupils gain competence and fluency.</p> <p>Teachers are able to orchestrate productive classroom discussions. Pupils actively take part in discussions.</p> <p>Nearly all pupils are motivated and engaged during lessons. Pupils get enjoyment and satisfaction from learning mathematics.</p> <p>All staff, regardless of their subject area specialism or level of responsibility, model confidence and interest in maths. Staff and pupils believe that everyone can succeed in maths if they work hard.</p>
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An extract from the self-assessment guidance materials that support *Improving mathematics in Key Stages Two and Three, guidance report*, Education Endowment Foundation.

These materials support teachers with self-assessment of their own practice, managing change and improving outcomes for all pupils.

The full report can be found at: <https://educationendowmentfoundation.org.uk/>.

Parent and Carer Guide to NRich can be found at: <https://nrich.maths.org/8514>.

Appendix C

Pitch, progression and expectation

Ideas to promote pitch, progression and expectation in mathematics

What are the barriers to learning?

Barriers to learning can be broadly broken down into three categories:

- pupils and families (for example, vocabulary, oral language, access to resources, broader background subject knowledge)
- community (for example, transport, housing, quality of Early Years provision, quality of employment opportunities)
- in-school (for example, access to high-quality teaching, levels of expectations, streaming and labelling, negative perceptions of families).

Extract taken from 2017-2018 joint HIAS/Rosendale Research School report: *Improving outcomes for disadvantaged learners*⁹. This report includes case studies from 11 Hampshire secondary schools and the *Building blocks for excellence: successful school strategy to improve school outcomes for disadvantaged pupils*: www3.hants.gov.uk/education/hias.htm and www.rosendale.researchschool.org.uk.

Early Years ...

What predicts success in mathematics?

We actually know a lot from research about building mathematical success in the Early Years and we could do this more effectively than at present. A large-scale study of pre-school experiences (Sammons et al, 2002; Siraj-Blatchford et al, 2002) found two key factors which predict progress:

- **parents providing a home learning environment** where, for instance, children were encouraged to paint, draw and play with letters and numbers
- **pre-school settings providing adult-led mathematics focused activities**, such as number rhymes and games, alongside independent play.

How to produce children with mathematics difficulties

Research has also shown us how to create young children with negative attitudes to mathematics and we seem to be going the right way about it:

- **creating mathematics anxiety blocks working memory space** and prevents learning (Maloney et al, 2013). If children have not developed secure number concepts they are likely to be anxious about arithmetic.
- **children with *fixed mindsets*** who believe they are naturally no good at mathematics are less successful than those who have a *growth mindset* and believe they can learn through effort (Dweck, 2006). However, by grouping children in Reception classes, we effectively tell some they are of *low ability* for mathematics (Boaler, 2013).

It is interesting that high-performing jurisdictions avoid both of these, by having a later school starting age, giving children more time to consolidate basic number understandings and by not *ability grouping* children (OECD, 2012).

Effective and appropriate Early Years' mathematics pedagogy

The good news is that we know a lot about this: it involves approaches which are common in Early Years settings (Gifford, 2005)¹⁰:

- playing and playfulness, eg blockplay, number rhymes
- games and activities indoors and out, eg cooking, goal scoring
- routines, eg snacktime, tidying up.

Two important aspects for practitioners to develop are:

- subitising, or recognising number patterns as on a dice: this develops familiarity with number combinations, eg seeing six as double three
- problem-solving and *sustained shared thinking* (Siraj-Blatchford et al, 2002).

The characteristics of effective learning from the EYFS (Department for Education, 2012) could provide a useful basis for exemplification:

- playing and exploring
- active learning
- creating and thinking critically.

The full article, together with references, can be found at: <https://nrich.maths.org/11441>.

Making mathematics accessible and relevant to all

To support pupils to develop their concept of number, the use of concrete resources, models and images is essential in the Early Years curriculum. This should then be built on throughout the 5-16 years mathematical journey. At all ages and stages, it is important that learners engage in a wide variety of representations. The use of the concrete-pictorial-abstract, or CPA, approach allows pupils with different learning preferences and styles to see the structure of the mathematics.

If you cannot see how it works, you will not be able to explain why it works.

Working with parents to share ideas to promote number talk at home will be beneficial to young pupils, supporting number knowledge, familiarity and confidence with number. Examples to share with parents include: singing number rhymes; opportunities to count (stairs, toys, sticks, etc); dice games (moving along a number track, the person who gets to the end wins); finding a particular page number in a book. Engaging parents in maths sessions or workshops can also support attitudes to maths and parental support at home.

Schools should promote the role of mathematics in different roles in the community, for example: nurses, doctors, builders, electricians, shop assistants, mechanics, small business, farmers, self-employed adults. This can be extended to consider employment opportunities that require higher-level mathematics qualifications, for example teachers, vets, doctors, air traffic controllers, architects, computer programmers.

High-quality teaching

Strong and secure subject knowledge from all mathematics teachers is needed to enable insightful questioning to take place in the classroom.

The following ideas have been shared with schools across the 5-16 years spectrum, allowing common structures to be used and adapted to solve increasingly complex mathematical problems.

Key questions to support mathematical thinking

- If you know ... then what else do you know?
- Can you give me an example of ... and another ... and another ...?
- What if you change ...?
- Which is harder and which is easier?
- What is the same and what is different?

Supporting problem-solving and the CPA approach

Teachers can ask learners: “Can you:

- *say it*
- *make it*
- *draw it*
- *write it*
- *explain it?”*

Extracts from the self-assessment guidance materials that support *Improving mathematics in Key Stages Two and Three, guidance report*, Education Endowment Foundation.

Improving mathematics in Key Stages 2 and 3: a self-assessment guide

Web: <https://educationendowmentfoundation.org.uk/>

Recommendation 1: Use assessment to build on pupils' existing knowledge and understanding

! INEFFECTIVE

Assessments are often set without careful consideration of their intended purpose.

Teachers collect summative data but rarely use assessment to collect information about pupils' mathematical strengths and weaknesses. Teaching does not respond to pupils' developing understanding.

Many teachers are not knowledgeable of the common misconceptions in mathematics. This has not been a focus of CPD.

When planning future lessons, teachers do not consider the misconceptions that are likely to arise.

Teachers' feedback is not specific, accurate or clear. It is often limited to empty praise.

Feedback is inefficient and creates a large workload for teachers. Teachers spend a large amount of time marking work. This is potentially distracting teachers from more beneficial activities.

📈 IMPROVING

Assessments are sometimes set with consideration of their purpose, but this is inconsistent. Not all teachers are confident users of assessment for different purposes.

Teachers are able to achieve a good understanding of pupils' strengths and weaknesses, using a variety of data sources, but they do not adapt their teaching in response.

Teachers' knowledge of common misconceptions is patchy. Some teachers need support to improve their knowledge.

Some teachers plan to address likely misconceptions but this practice is not consistent throughout the school.

Some teachers can confidently and consistently provide effective feedback, but others do not.

There is a recognition that marking workload is a problem, but there is still work to be done to minimise the burden of marking.

✓ EXEMPLARY

Careful consideration is given to how the results of an assessment will be used before an appropriate assessment is selected.

Teachers use a variety of types of assessment, as appropriate, to collect information about strengths and weaknesses. They adapt their teaching in response and use assessment information to inform planning.

Teachers have a good knowledge of the common misconceptions in maths and why they arise. They use this knowledge to inform their assessment.

Teachers use their knowledge of common misconceptions to plan future lessons.

Feedback is effective and generally resembles the principles outlined in the guidance.

Feedback is efficient and does not create a large workload for teachers. There is a healthy balance between oral and written feedback.

Recommendation 2: Using manipulatives and representations

! INEFFECTIVE

Manipulatives are rarely or never used to teach maths.

Manipulatives are only used with younger children and when teaching simpler mathematics.

Pupils often become reliant on manipulatives to do a type of task or question. Teaching can tend to focus on 'getting them to the right answer' to a specific problem instead of developing understanding.

Number lines do not feature in teaching.

Teachers rarely introduce pupils to multiple representations.

📈 IMPROVING

Manipulatives are often used, but without a clear rationale for how they will develop more sophisticated mathematics.

Manipulatives are used across the school. However, with older children they are only used as a tool in catch up interventions or to teach simpler mathematics.

Teachers' use of manipulatives to develop independent understanding is patchy. Some teachers do this consistently, but others do not.

Number lines are used, but teachers need more CPD to use them confidently and competently.

Teachers and pupils compare and discuss different representations. However, this discussion is not carefully orchestrated to introduce more abstract, diagrammatic representations. Teachers might introduce too many representations at once, causing confusion.

✓ EXEMPLARY

Teachers use manipulatives appropriately, and with a clear rationale for why the manipulative will support pupils to understand mathematics.

Manipulatives are used across the different year groups in the school. The decision to remove a manipulative is made in response to the pupils' improved knowledge and understanding, not their age.

Teachers enable pupils to understand the links between the manipulatives and the mathematical ideas they represent. Teachers use manipulatives to develop pupils' independent understanding of the mathematics.

Teachers are confident and competent in their use of number lines.

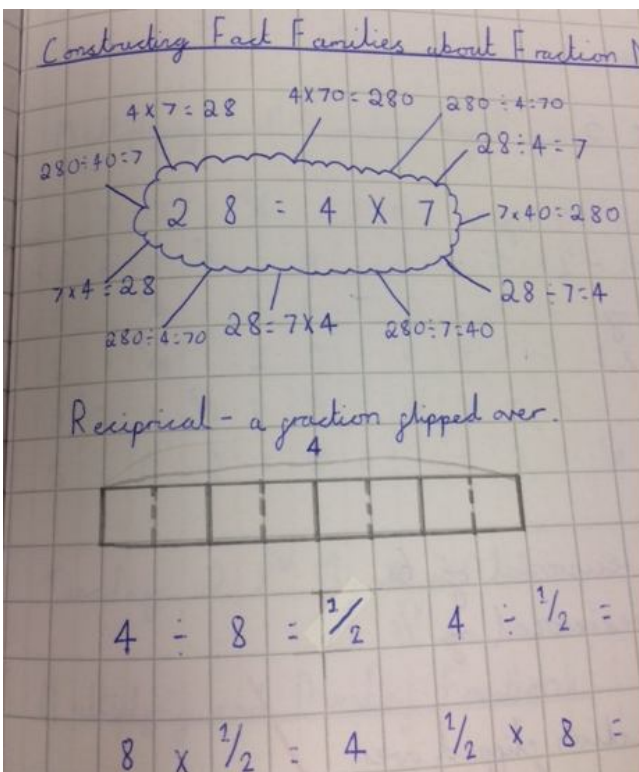
Teachers and pupils compare and discuss different representations. Teachers aim to support pupils to develop more abstract, diagrammatic representations. Teachers are careful to not overload pupils with too many representations at once.

Appendix D: Thinking, reflection and motivation

Some ideas to support the development of thinking, reflection and motivation in schools

To support teachers in promoting collaborative thinking in mathematics offer frameworks to develop problem-solving and reasoning, such as five key questions:

- What is the same and what is different?
- Can you show me an example of ... and another ... and another?
- What if I change ...?
- If I know ... then what else do I know?
- Which is harder and which is easier?



An example of a KS2 pupil's thinking and multiplicative reasoning.

Why developing problem-solving is important

If pupils lack a well-rehearsed and readily available method to solve a problem they need to draw on problem-solving strategies to make sense of the unfamiliar situation.

Websites such as NRIC (<https://nrich.maths.org/teacher-primary>; <https://nrich.maths.org/teacher-secondary>) include articles and activities to help develop problem-solving/reasoning skills.

Recommendation 3: Teach strategies for solving problems (extract from Education Endowment Foundation report)

! INEFFECTIVE

There is a lack of genuine problem solving tasks in teaching. Tasks tend to be routine and can be completed using a procedure that pupils know well.

Teachers lack knowledge and understanding of problem solving strategies. They do not feature in their teaching.

Teachers do not consciously vary the structure and context to problems.

Teachers rarely encourage pupils to use representations and manipulatives to represent problems mathematically.

continued...

🔄 IMPROVING

Some teachers select non-routine problems, but other teachers do not. Teachers sometimes do not feel confident enough to work on genuine, non-routine problem solving.

Teachers effectively model a range of problem-solving strategies. However, they do not effectively support pupils to self-regulate their use of strategies.

Teachers do pay attention to context and structure when setting problems, but this is not systematic and does not support improved understanding.

Some teachers encourage pupils to use representations and manipulatives to represent problems mathematically. However, this practice is not consistently adopted by teachers throughout the school.

continued...

✓ EXEMPLARY

Most teachers confidently select genuine, non-routine problem-solving tasks.

Teachers know a range of strategies, which they can model effectively for pupils. They teach pupils to carefully and consciously choose the most appropriate strategy for the problem at hand.

Teaching is organised so that problems with similar structures and different contexts are presented together, and, likewise, that problems with the same context but different structures are presented together. Pupils are taught to identify similar mathematics that underlies different situations, and identify and interrogate multiple relationships between variables in one situation.

Teachers encourage pupils to use representations and manipulatives to represent problems mathematically.

continued...

! INEFFECTIVE

...continued

Worked examples rarely feature in teaching.

Teachers rarely use the following approaches to improve pupils' use of strategies:

- Encouraging pupils to share and discuss strategies
- Encouraging pupils to interrogate and use their mathematical knowledge to solve problems
- Encouraging pupils to communicate their reasoning about their choice of strategies
- Requiring pupils to compare and evaluate multiple strategies

🔄 IMPROVING

...continued

Teachers deploy worked examples, but they are mainly used to consider steps in a procedure and are rarely used to examine problem-solving strategies.

Teachers are confident using only some of these approaches, or they could improve in some areas.

✓ EXEMPLARY

...continued

Teachers use worked examples to enable pupils to examine the use of different problem solving strategies.

Teachers are confident and capable when using all of these approaches to improving pupils' use of strategies.

BOX F: MODELLING METACOGNITION DURING PROBLEM SOLVING

While demonstrating the solving of a problem, a teacher could model how to plan, monitor, and evaluate their thinking by reflecting aloud on a series of questions. These could include:

- What is this problem asking?
- Have I ever seen a mathematical problem like this before? What approaches to solving it did I try and were they successful?
- Could I represent the problem with a diagram or graph?
- Does my answer make sense when I re-read the problem?
- Do I need help or more information to solve this problem? Where could I find this?

Developing metacognition is not straightforward and there are some important challenges to consider.

- Teachers need to ensure that pupils' metacognition does not detract from concentration on the mathematical task itself.³⁰ This might happen if pupils are expected to do too much, too early, without effective scaffolding from their teacher.
- Regardless of the strategy being taught, pupils need significant time to imitate, internalise, and independently apply strategies, with strategies used repeatedly across many maths lessons. It is likely that the time required to develop metacognition is much greater than for other skills and knowledge.
- Discussion and dialogue can be useful tools for developing metacognition, but pupils may need to be taught how to engage in discussion.³² Teachers should model effective discussion and 'what to do as a listener'.³³ Orchestrating productive discussions requires considerable skill and so may require targeted professional development.

Extracts from:

Improving mathematics in Key Stages Two and Three, guidance report and the self-assessment guidance materials that support it, Education Endowment Foundation.

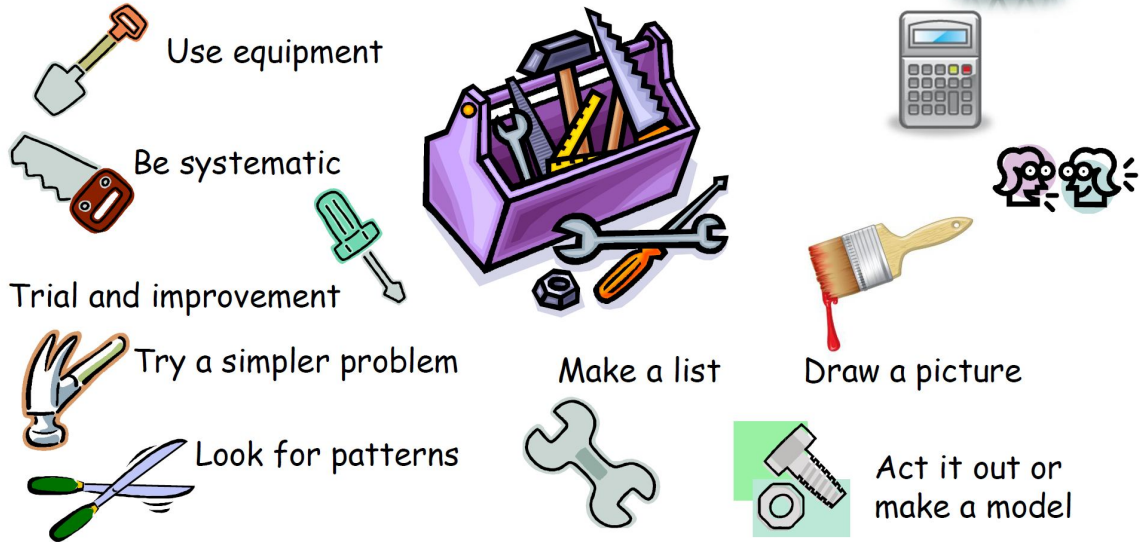
Teachers and pupils should also use heuristics as strategies for accessing a problem

- Draw a diagram.
- Be systematic.
- Use a formula.
- Make a table.
- Use a model.
- Write an equation.
- Use guess and check – trial and improvement.
- Work backwards.
- Look for a pattern.
- Ask “*Have I seen anything like this before?*”.

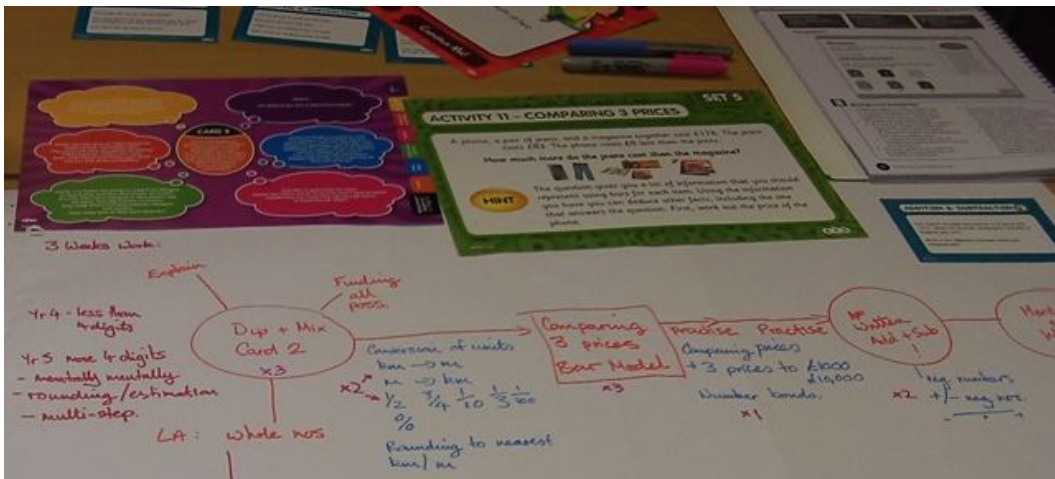
What is in **your** mathematical toolbox ?

Problem-solving skills (heuristics)

Facts, relationships and links to generate and check



When planning, build in opportunities for problem-solving



Decide the order of the key tasks and how long pupils might need to really *get it* – think about the practice that will be needed, eg converting units, key number facts.

Using the digits 2, 3 and 4 in the multiplication below. How many solutions are there?

$$\square \square \times \square = ?$$

How close can you get to 100?

Plan for rich mathematical tasks that require pupils to think and reason.

Encourage talk for learning. Here we see KS2 pupils' collaborative work and pupils at KS3 exploring algebra together.

Develop a community of inquiry in your classroom

Some ideas and thoughts from Jennie Pennant:

Developing a classroom culture that supports a problem-solving approach to mathematics (2013).

Aspects to consider	More information on each aspect
1. Who does most of the talking in whole-class parts of the lesson?	Generally, in a strong problem-solving environment the teacher needs to be doing around 30% of the talking and the students 70%. What do you notice about the balance in your classroom? What type of things are you saying when you are talking? Explaining how to do something? Asking questions?
2. What questions do I ask?	Do you ask closed questions such as, 'can you see how the system works?' or open questions such as, 'what system can you see emerging in this problem?'
3. Who answers the questions?	Is it the mostly the same students? Is it the more articulate ones? Is it more often boys or girls?
4. How well do I listen to the students' answers and seek to understand what they are saying?	Do I respond by telling the whole class what I think a particular student said without checking with them? Do I slightly adjust what they said to make better sense or fit a 'better/right answer'? Do I ask the student a 'clarification' question, such as 'can I just check what I think you said was ...'?
5. What do I do with the students' answers?	Do I praise them for a fabulous answer? Do I simply evaluate their answers with comments such as 'Good', 'Well done', 'Right', 'OK', 'No', 'Think again'? Do I carry on with the next thing I was going to say? Do I ask other students to comment on what was said? Do I ask another follow-up question such as 'are you sure?' or 'how do you know that?'?
6. How do I facilitate the learning?	Do I explain how it needs to be done and make sure they understand it as fully as possible before working on their own? Do I give them key pointers/hints/clues to help them? Do I pull out the learning from the students' thinking and use that to develop the journey of the lesson?
7. How confident are the students to take a risk, to try out ideas, to make mistakes?	What evidence is there of the students taking a risk in what they offer to the discussion or ideas that they try out? What evidence is there that the students are trying out their ideas rather than replicating mine? When is it helpful for them to replicate mine? What do I do when a student makes a mistake or follows a 'dead end' line of thought?
8. What does my body language communicate?	Do I communicate interest/acceptance/frustration/disapproval ...? How does my body language change through the lesson?

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