THINKERS

A book review on a book that every teacher should own

Deepening Mathematical Thinking and the Skill of Generalisation

A Book Review :

Thinkers: Chris Bills; Liz Bills; Anne Watson & John Mason
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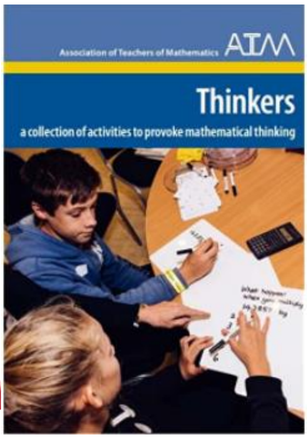


Ten years before the 2014 national curriculum for England was implemented, with its emphasis on problem-solving and reasoning , I came across an indispensable classroom and planning resource that has accompanied me on my professional journey ever since. I first found *'Thinkers'* when browsing through an ATM book stall at one of my first ever ATM annual conferences. It attracted me because it was A4 size (I'm not a fan of pocket guides) , it had a very simple red cover that appealed, and last, but by no means least, it was quite thin. As with most busy teachers, the thought of a large, many-paged refence book is off putting. *'Thinkers'* gets quickly and efficiently to the point, is easy to navigate, and is instantly applicable to a broad range of mathematics lessons across most ages and stages.

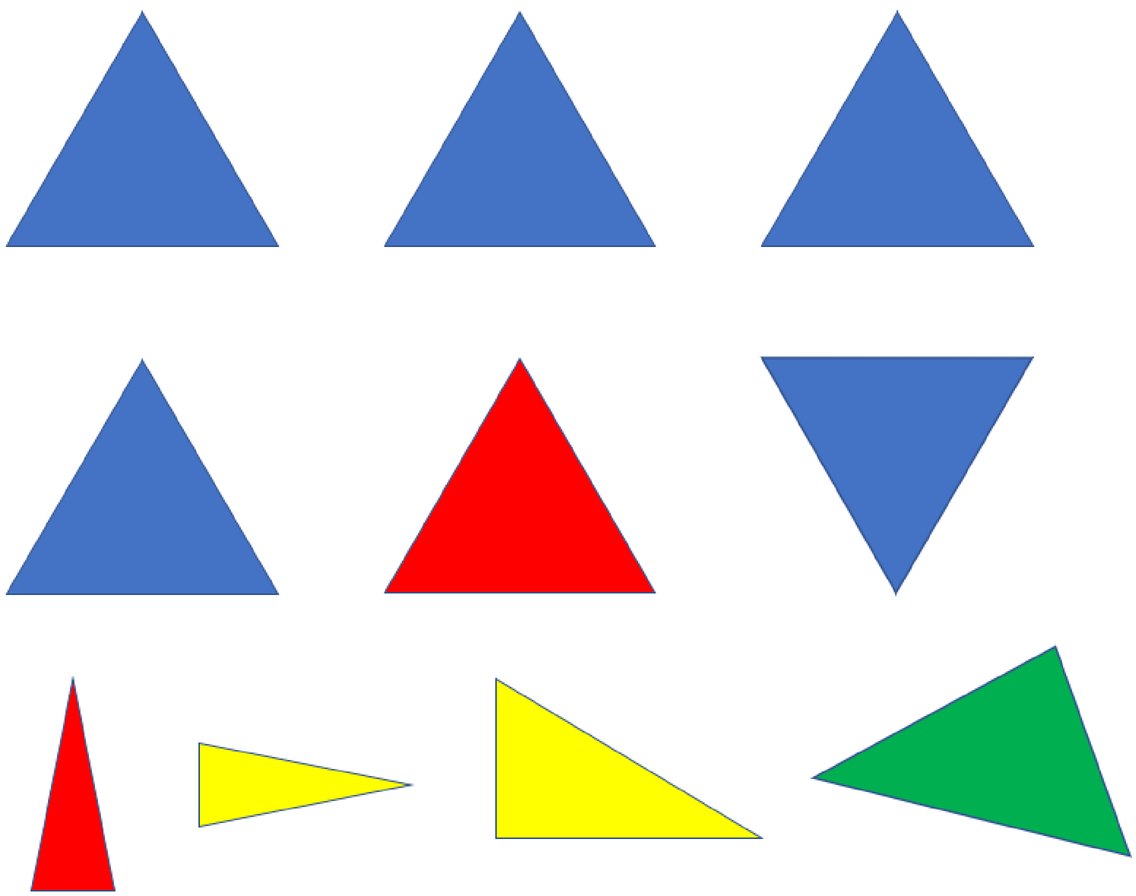
The books is a collection of thought provoking ideas for questions. Even if you are already using different questioning styles in your teaching, you will find new and alternative approaches here. The book contains sixteen different contexts for exemplifying and generalising the processes at the heart of doing mathematics that can be integrated into any mathematics lesson, together with examples of the different strategies and techniques across a range of mathematical topics.

It is important to note that there are no lesson plans or 'activities' in this book. At the very heart of *'Thinkers'* is the belief that regular use of high quality, insightful questions will enhance the teaching and learning of mathematics for new and experienced teachers, and for learners from 8 to 18 (and beyond). The ideas in the book are suitable for learners from KS2 to A 'level and are designed to stimulate mathematical thinking. The prompts or questions ask learners to construct or deal with some mathematical ideas which exemplify a concept or technique. The strategy of asking learners to generate examples for themselves is fundamental to the premise of this book. It is suggested that the process of generating our own examples 'can provide the foundations for recognition, articulation and appreciation of a generality. Teacher-generated examples always illustrate the teachers' understanding , and sometimes seem, to the learner, as if they are chosen at random'

What is mathematical generalisation and why is it important ?

'Being able to generalise a situation involves identifying its underlying mathematical structure. Having transferred our thinking from one example to another, to another, to another ..., the emerging similarities and differences offer us insight into what will always be true in that situation, which in turn can be explained by its underlying mathematical structure.' (Nrich) [Mastering Mathematics: the Challenge of Generalising and Proof (maths.org)]

Let us think about how we approach the teaching of the properties of shape. If young learners have only ever seen specific triangles, equilateral ones, red plastic ones in the shape box, and so on, they are unlikely to be able to generalise about different shapes going forward. This is often evident when children do not recognise the same triangle in different orientations, or are unhappy when it is presented 'upside down'. We provide our young learners with lots of different triangles, colour, thickness, shapes, orientations, to help them discriminate mathematically and broaden their 'example spaces'.



Mathematical generalisation is the process of expanding our example space to include things that are different mathematically, but still share a ‘label’, such as ‘quadrilateral’ or ‘polygon’. Furthermore, if the learner has lots of examples under a particular label to internalise and categorise , with some of them looking a bit like the previous example but not quite, we have the opportunity to deepen the understanding through sorting out the small differences and similarities, as well as the larger, more obvious ones.

‘Thinkers’ offers questions, tasks or prompts under sixteen different headings in no particular order. This means that teachers can select a task sequence or question to suit the needs of a particular group of learners or curriculum schedule without feeling that they have missed out on previous bits. The book focusses on developing the learner’s ability to create and question in mathematics by generalising from examples.

Sixteen types of questions and prompts

1. Give an example of….. and another….and another
2. Pointing toward generality (particular, peculiar, general)
3. Hard and easy
4. Additional conditions
5. Comparing / contrasting three
6. Confounding expectations
7. Impossible constructions
8. One the spot generalisation
9. Open and closed questions / exhaustive lists
10. Always, sometimes, never true
11. Odd one out
12. Sorting
13. Ordering
14. Equivalent statements
15. With and across the grain
16. Burying the bone.

In the sample page below, we see how the book exemplifies how best to use the prompt and then follows this with a list of possible concepts or contexts.
(with thanks to the ATM website for this image)

2 Pointing Toward Generality (Particular, Peculiar, General)

Learners can also be led to generalisation by asking for a particular then a peculiar example. Once they develop fluency in locating peculiar examples, they can also be encouraged to try to say in what way an example is generic, and to express the form of a general example. For instance:

Give me an example of a fraction that is equivalent to $\frac{2}{3}$

Give me a really peculiar example.

Give me a general example.

Thinking about the peculiar can help with thinking about the general. To create more and more peculiar examples we need to double any number for the numerator and treble it for the denominator. For example 2 billion over 3 billion or 2×3.22 over 3×3.22 or 2π over 3π. In the classroom learners can compete to make the examples more and more peculiar – numerator and denominator can be integers, decimals, fractions, algebraic expressions, ...

From here it is a short step to an expression of the general:
'2 times something over 3 times something is a fraction equivalent to $\frac{2}{3}$
or '$\frac{2x}{3x}$ is equivalent to $\frac{2}{3}$ for any value of x other than zero'.

Here are some more starters for the PPG treatment:

Give me an example, a peculiar example, a general example, of

1. an even number.
2. a number with exactly three factors.
3. a number which leaves remainder 1 when divided by 3.
4. a parallelogram.
5. a fraction equivalent to 0.2.
6. a fraction bigger than 3.
7. a shape with rotational symmetry of order 2.
8. an algebraic fraction which is equivalent to $\frac{5}{6}$.
9. a quadratic with a root of 2.
10. a straight-line graph.
11. an angle whose cosine is 0.5.
12. an odd function.

… any of the 'Give an example …another and another' tasks could be set in the PPG mode.

I recommend this book to all teachers of mathematics, at any stage in their career, irrespective of the age and stage in which they teach. A constant companion to me , it has provided me with insights, ideas and understanding in a compact, precise and accessible way.

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