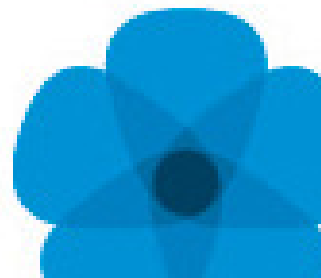


Bar Modelling for Key Stages 3 and 4

Equations

Hampshire Maths Team
January 2019
Final version

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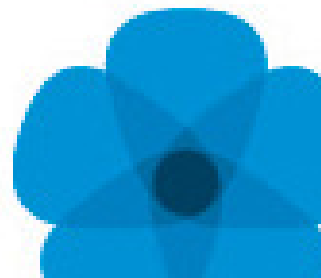


Overview

In this document are some ideas to support the development of bar modelling in KS3 and 4 to solve equations.

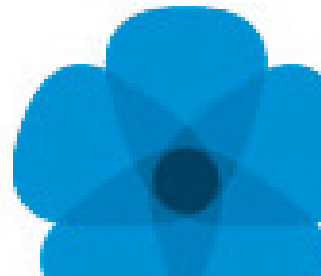
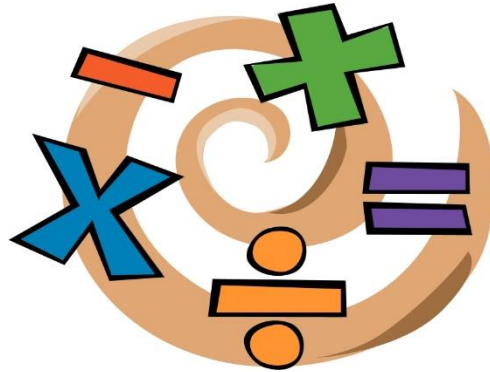
This can be used for staff training.

With thanks to the Hampshire Leading Maths Teachers.



Bar Modelling

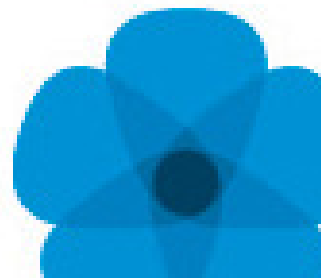
KS3 and 4



Why Bar Modelling ?

Core Competencies for Mastery:

- Visualisation
- Metacognition
- Generalisation
- Number-Sense
- Communication



The Part-Whole Model

Can you....

Say it ?

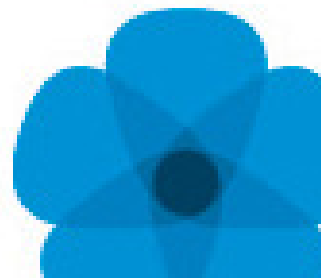
Make it ?

Draw it ?

Write it ?

Explain it?

- 1) Understand the problem
- 2) Model the problem situation
- 3) Draw the model
- 4) Solve the problem



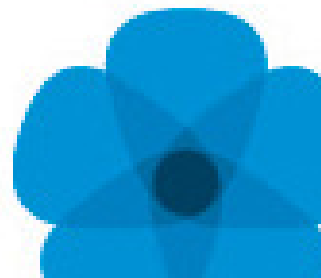
Starter:

Out of the strips supplied measure and cut three different coloured strips.....

One to a length of **3cm**

One to a length of **8cm**

One to a length of **11cm**



Creating and Rearranging Equations

Lesson Objectives:

To be able to use the Singapore bar method to model equations and then be able to use these models to rearrange simple and complex equations.

Skills for learning:

Modelling with the Singapore bar method.

Visualising equations.

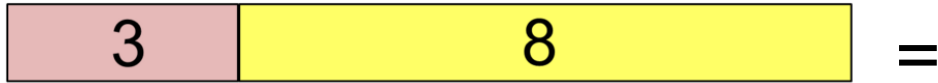
Rearranging equations.

Key Words:

Equal
Rearrange
Bar Method
Model
Visual

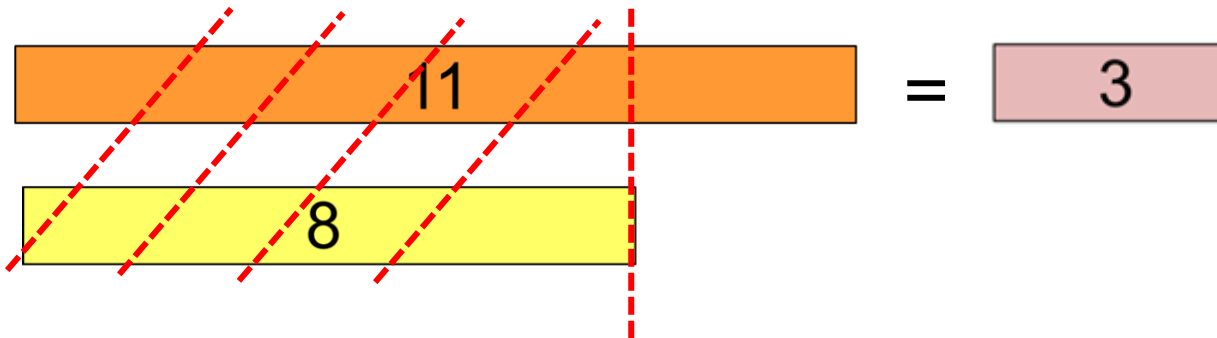


We can probably all agree on this fact...
(if we measured accurately)

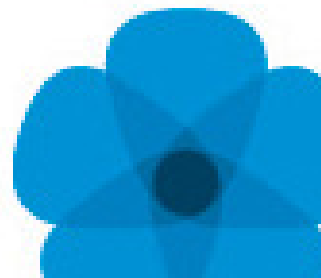


So $3 + 8 = 11$

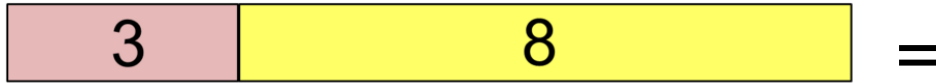
If we take the 11 bar, and chop off 8cm....



So $11 - 8 = 3$

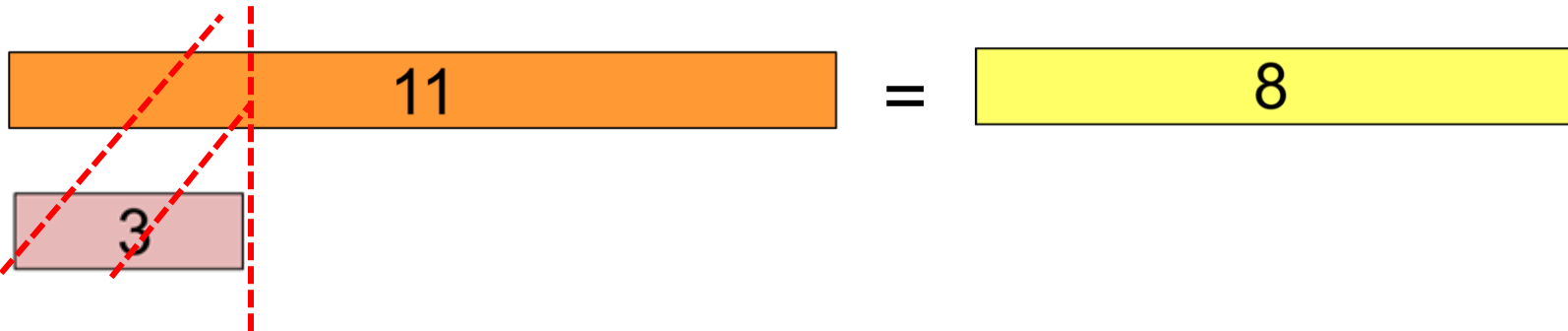


We can probably all agree on this fact...
(if we measured accurately)

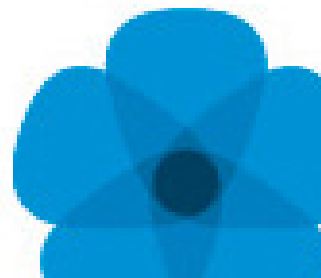


So $3 + 8 = 11$

If we take the 11 bar, and chop off 3cm....

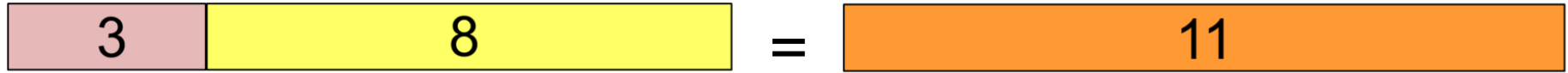


So $11 - 3 = 8$



We can probably all agree on this fact...

(if we measured accurately)

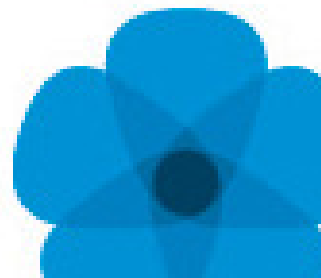


From these three slips of paper we can now create three equations....

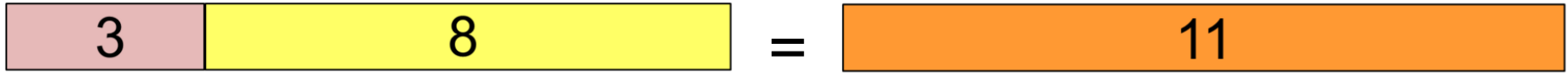
$$3 + 8 = 11 \quad (\text{and also } 8 + 3 = 11)$$

$$11 - 8 = 3$$

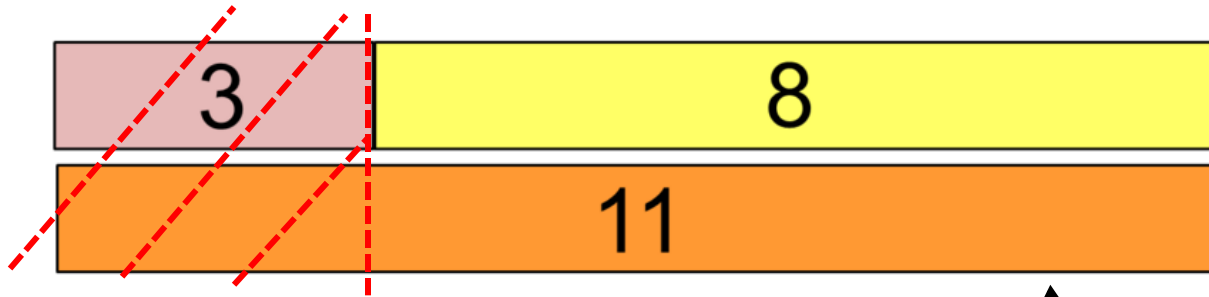
$$11 - 3 = 8$$



We can probably all agree on this fact...
(if we measured accurately)



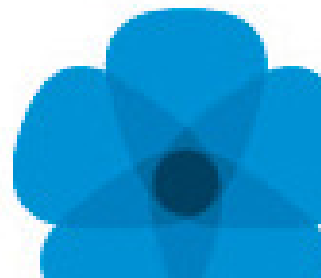
If we line the bars up like this.....



The three equations are easy to see!

E.g. $11 - 3 = 8$

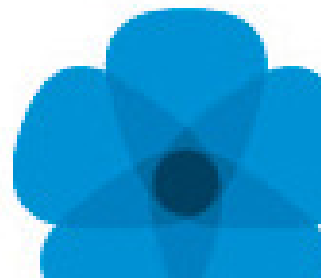
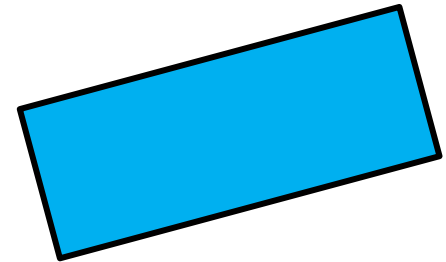
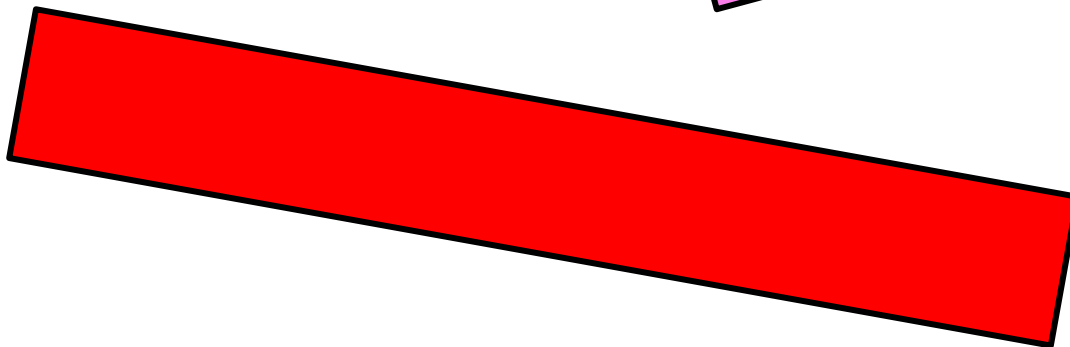
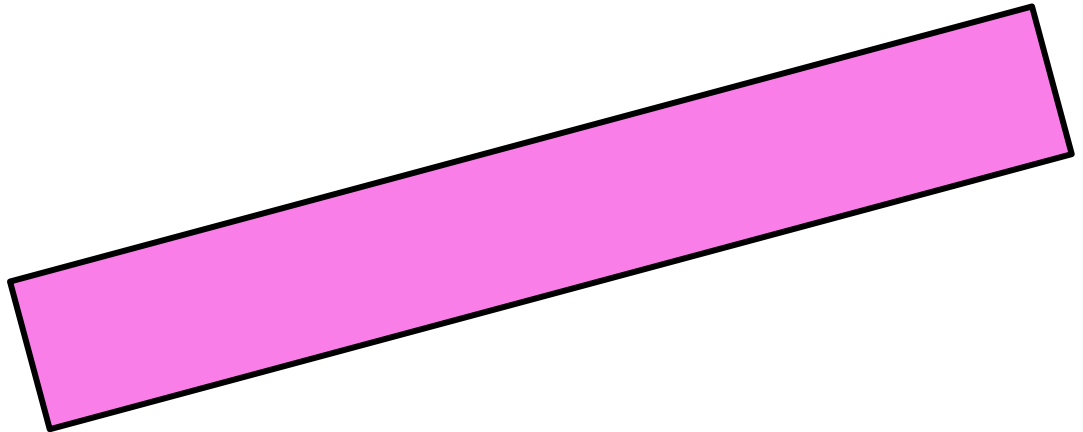
Glue it into your books like this!



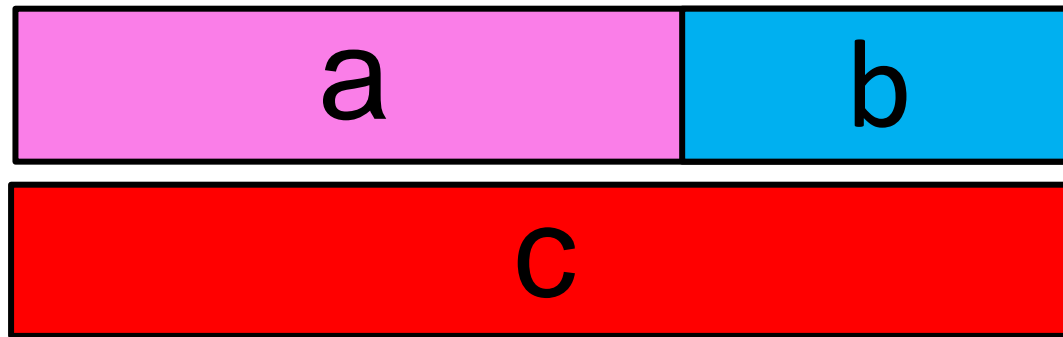
This method can be used for algebra too!

Using another three strips, can you model this equation?

$$a + b = c$$

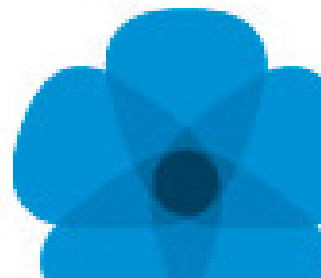


This method can be used for algebra too!



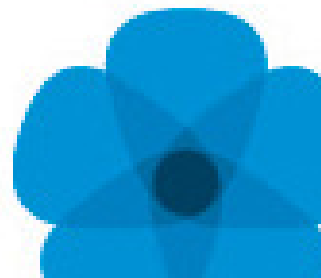
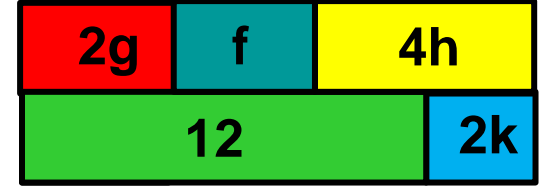
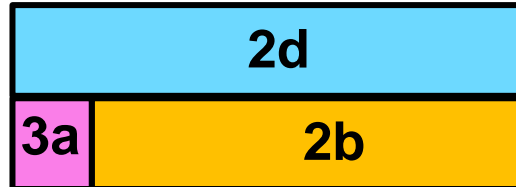
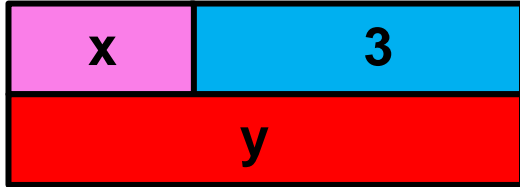
Glue it
into your
books like
this!

What are the four equations we can make from this model?



Using the Bar Method:

Write as many equations as you can from each.



Using the Bar Method:

Model these equations and write as many equations as you can from each (you can draw them or cut and stick).

$$x = y + 5$$

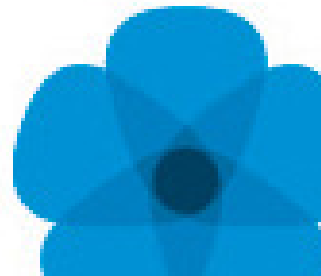
$$12 = z + w$$

$$2g + h = b$$

$$d - 3 = k$$

$$2j - 4 = p$$

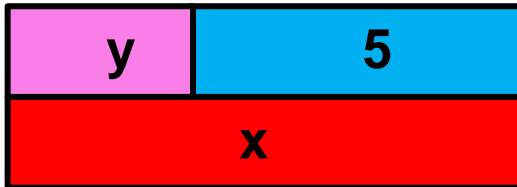
$$3h = 2q - v$$



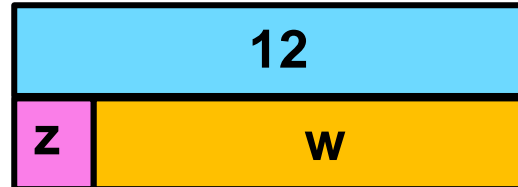
Using the Bar Method Solutions:

Remember, your bars don't need to be the same size as mine, as the quantities and proportions are unknown!

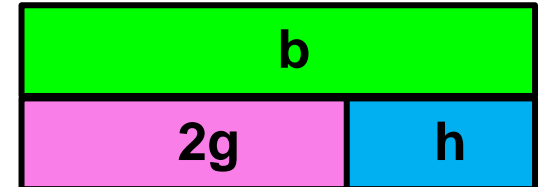
$$x = y + 5$$



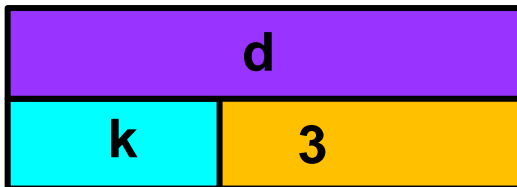
$$12 = z + w$$



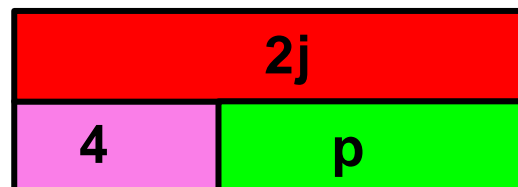
$$2g + h = b$$



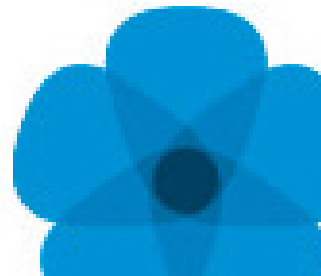
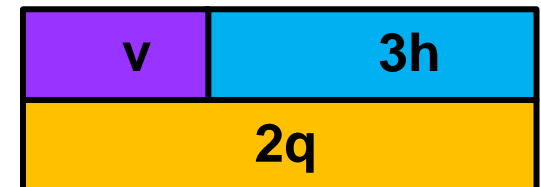
$$d - 3 = k$$



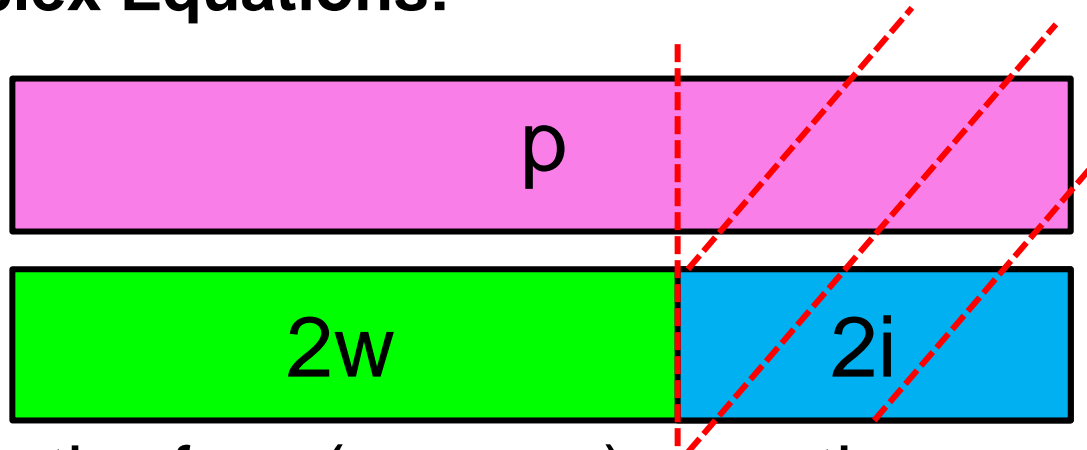
$$2j - 4 = p$$



$$3h = 2q - v$$



More Complex Equations:



What are the four (or more) equations we can make from this model?

$$p = 2w + 2i$$

$$2w + 2i = p$$

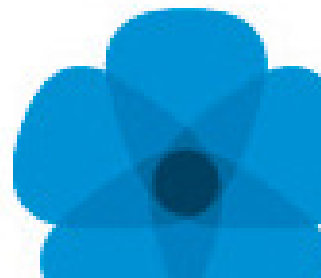
$$p - 2w = 2i$$

$$p - 2i = 2w$$

What if I want to make **w** the subject of the formula?

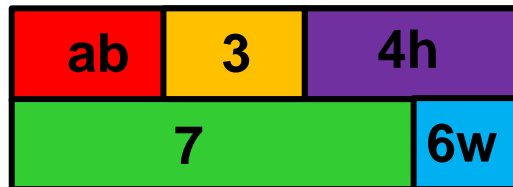
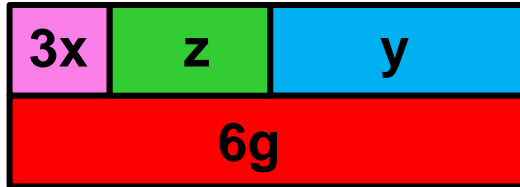
We want **w** on its own.....so we have to divide both sides by 2!

$$\begin{array}{l}
 p - 2i = 2w \\
 \div 2 \qquad \div 2 \\
 \hline
 \frac{p - 2i}{2} = w
 \end{array}$$



Using the Bar Method:

Write as many equations as you can from each.



Extension:

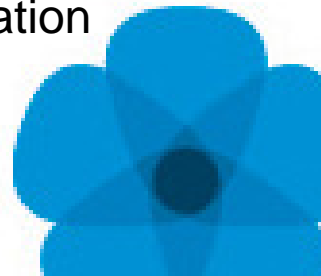
Make x the subject of the equation

Extension:

Make w the subject of the equation

Extension:

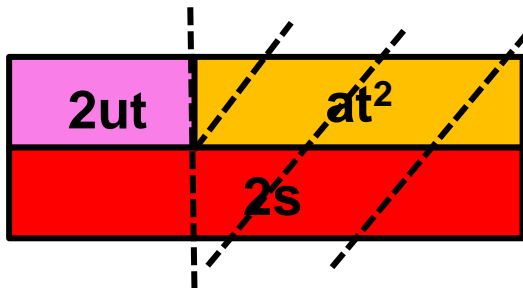
Make u the subject of the equation



Bar vs Balance

Make **u** the subject of the equation

$$2s = 2ut + at^2$$



When we draw a bar model we see...

$$2s - at^2 = 2ut$$

$$\div 2t \quad \div 2t$$

$$\frac{2s - at^2}{2t} = u$$

Remember $2ut$ means $2 \times u \times t$ so to get u alone we divide by $2t$

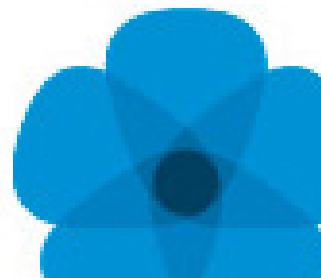
$$2s = 2ut + at^2$$

$$-at^2 \quad -at^2$$

$$2s - at^2 = 2ut$$

$$\div 2t \quad \div 2t$$

$$\frac{2s - at^2}{2t} = u$$



$$3x = 2y + z + e$$

Main: Make **z** the subject of the equation

Extension: Make **y** the subject of the equation

$$4b + 2d = a + c + e$$

Main: Make **c** the subject of the equation

Extension: Make **d** the subject of the equation

$$a - 2pz = 3f + s$$

Main: Make **f** the subject of the equation

Extension: Make **p** the subject of the equation

$$g = a + 3b - zfq$$

Main: Make **b** the subject of the equation

Extension: Make **f** the subject of the equation

$$3a^2 + 2b - 6 = tr$$

Main: Make **b** the subject of the equation

Extension: Make **a** the subject of the equation

$$a^2 + 4b^2 = 6ts - 5$$

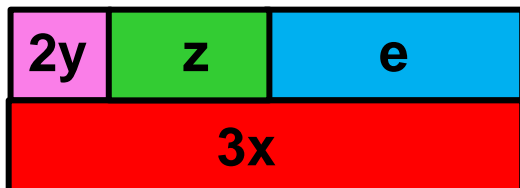
Main: Make **t** the subject of the equation

Extension: Make **b** the subject of the equation



Rearranging equations: Use the method that suits you...

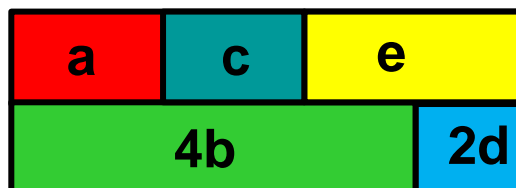
$$3x = 2y + z + e$$



$$z = 3x - 2y - e$$

$$y = \frac{3x - z - e}{2}$$

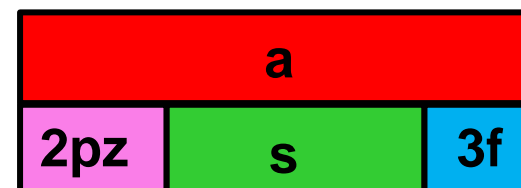
$$4b + 2d = a + c + e$$



$$c = 4b + 2d - a - e$$

$$d = \frac{a + c + e - 4b}{2}$$

$$a - 2pz = 3f + s$$



$$f = \frac{a - 2pz - s}{3}$$

$$p = \frac{a - s - 3f}{2z}$$

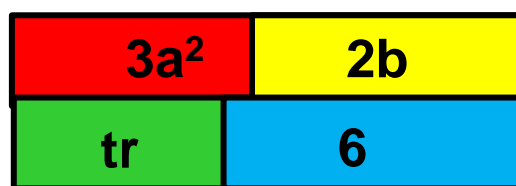
$$g = a + 3b - zfq$$



$$b = \frac{g + zq - a}{3}$$

$$f = \frac{a + 3b - g}{zq}$$

$$3a^2 + 2b - 6 = tr$$



$$b = \frac{tr + 6 - 3a^2}{2}$$

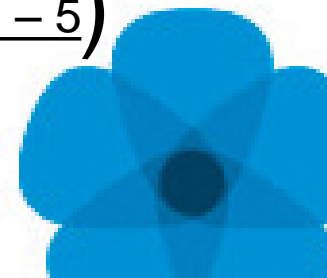
$$a = \sqrt{\frac{tr + 6 - 2b}{3}}$$

$$a^2 + 4b^2 = 6ts - 5$$



$$t = \frac{a^2 + 4b^2 + 5}{6s}$$

$$b = \sqrt{\frac{6s}{4(6ts - a^2 - 5)}}$$

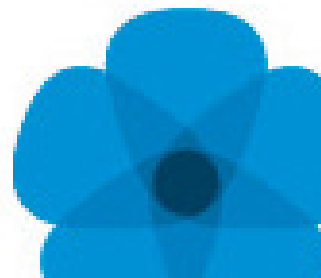


Plenary Check:

What can you tell me about this?

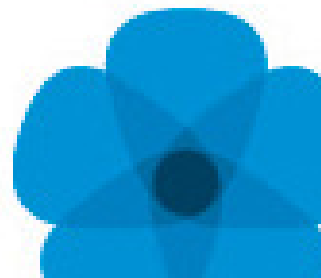


Extension: Think of other formulas we know that could be modelled and rearranged using this method...



Stretch B – Draw a bar picture and write the equation for each question.

- 1) The total cost of six apples and one banana is £5.
- 2) A cheese sandwich and two bags of crisps costs £4.50.
- 3) Joe has £20. He buys a CD and a chocolate bar. He has £8 left.
- 4) Sam has £50. He buys three books, a t-shirt and a pair of socks. He has £X left.



HIAS Maths Team

Jo Lees – Area Inspector – Mathematics

Email: jo.lees@hants.gov.uk

Tel: 02380 816139

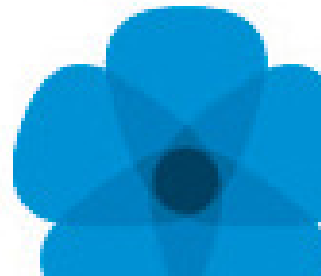
Jacqui Clift – Area Inspector – Mathematics

Email: jacqui.clift@hants.gov.uk

Tel: 02380 816139

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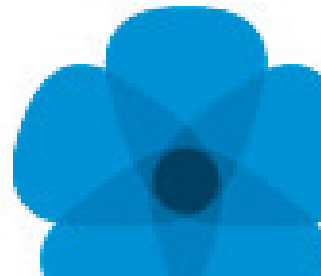
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