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# Problem Solving and Reasoning

## Mathematics Department Meeting

HIAS Maths team
September 2018
Final Version



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# Overview

## In this document

Some suggested slides for use in a mathematics department meeting to explore aspects of problem solving and reasoning, with links to the Y6 curriculum.

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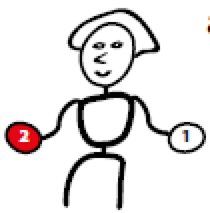
# The National Curriculum

## Aims:

- Become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to **recall and apply knowledge** rapidly and accurately.
- **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

# A bit of maths to get started….






**a** Sally has a red disc and a white disc.
The white disc has 1 on one side and 3 on the other. (1, 3)
The red disc has 2 on one side and 4 on the other. (2, 4)
The two discs are placed on a table and the scores added together e.g. 3 + 2 = 5.
What possible numbers could Sally get by adding the scores she can see on the discs?

**b** George has two discs.
The possible totals are 2, 4, 9, 11.
What could numbers could be on George's discs?
Is there only one possibility?

**c** Can you design some discs that give the totals 7, 8, 9, 10?
What about 13, 14, 15, 16?
Can you make any run of 4? Is your solution unique?

Sally has a red disc and a white disc.

The white disc has 1 on one side and 3 on the other. (1, 3)

The red disc has 2 on one side and 4 on the other. (2, 4)

The two discs are placed on a table and the scores added together e.g. 3 + 2 = 5.

What possible numbers could Sally get by adding the scores she can see on the discs?

George has two discs.

The possible totals are 2, 4, 9, 11.

What could numbers could be on George's discs?

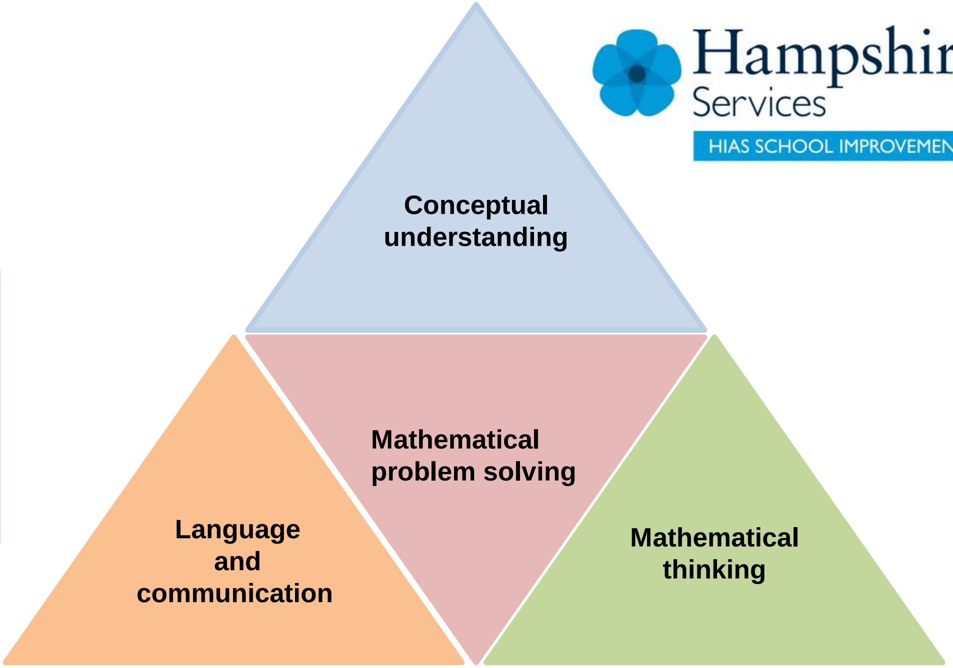
Is there only one possibility?

Can you design some discs that give the totals 7, 8, 9, 10?

What about 13, 14, 15, 16?

Can you make any run of 4? Is your solution unique?

# Dimensions of depth




## Conceptual understanding

Pupils deepen their understanding by representing concepts using objects and pictures, making connections between different representations and considering what different representations stress and ignore.

## Language and communication

Pupils deepen their understanding by explaining, creating problems, justifying and proving using mathematical language. This use of language also acts as a scaffold for their thinking.

## Mathematical thinking

Pupils deepen their understanding by asking and investigating great questions, by giving examples, by sorting and comparing, or by looking for patterns and rules in the mathematics they are exploring.

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Drury. H (2014) Mastering Mathematics, Teaching to transform achievement, OUP
ISBN: 978-0-19-835175-7

# Key ideas to consider….

- CPA: concrete – pictorial - abstract
- Questioning
- Reasoning
- Problem solving
- Role of talk
- Linking aspects of maths

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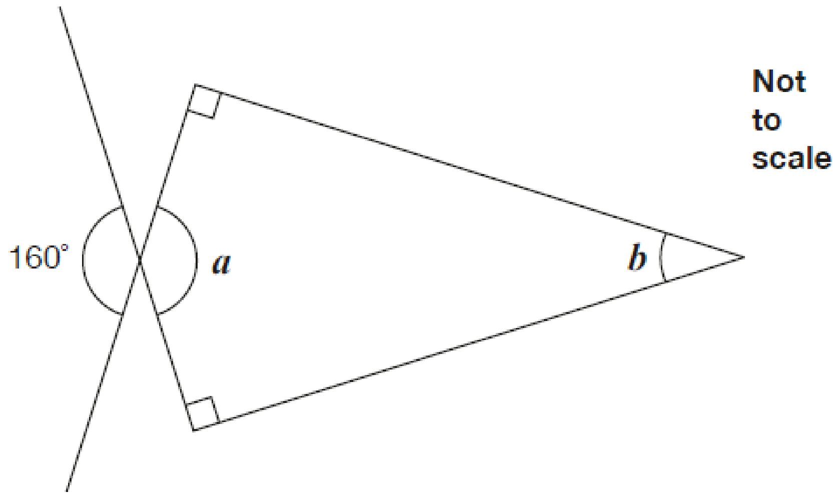
The next few slides give examples of problem solving at KS2 including SATs questions.

Consider how teaching in KS3 builds on this…

# Mathematics Paper 2: Reasoning

Calculate the size of angles $a$ and $b$ in this diagram.




$a =$ ______ °

1 mark

$b =$ ______ °

1 mark

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2016 national curriculum tests
Key stage 2
Mathematics
Paper 2: reasoning

**7** Write the two missing values to make these equivalent fractions correct.

$$\frac{\square}{3} = \frac{8}{12} = \frac{4}{\square}$$

1 mark

1 mark

Year 6 objective

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2018 national curriculum tests

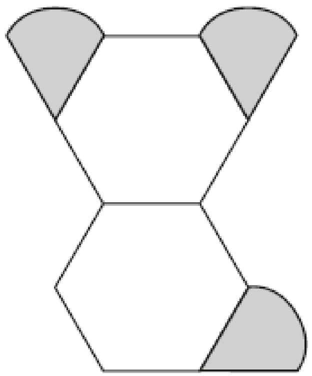
# Key stage 2

## Mathematics

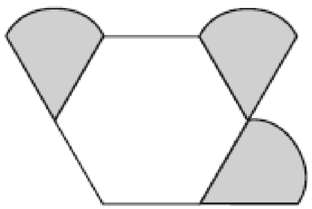
**Paper 2:** reasoning

**21** Amina is making designs with two different shapes.

She gives each shape a value.

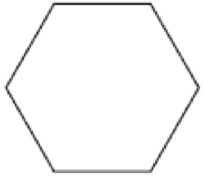


Total value is 147



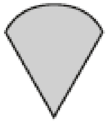
Total value is 111

Calculate the value of each shape.



= [ ]

1 mark



= [ ]

1 mark

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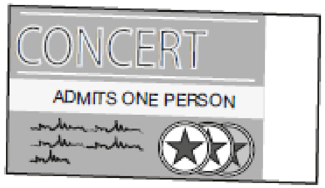
**18** Last year, Jacob went to four concerts.

Three of his tickets cost £5 each.


The other ticket cost £7




What was the **mean** cost of the tickets?

Show your method

£

2 marks

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2018 national curriculum tests

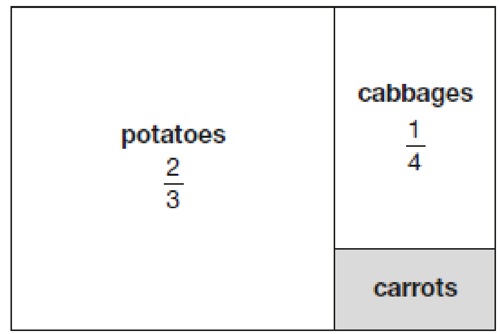
# Key stage 2

## Mathematics

**Paper 3:** reasoning

**18** This is a diagram of a vegetable garden.

It shows the fractions of the garden planted with potatoes and cabbages.




**Not to scale**

The remaining area is planted with carrots.

What **fraction** of the garden is planted with carrots?

Show your method

2 marks

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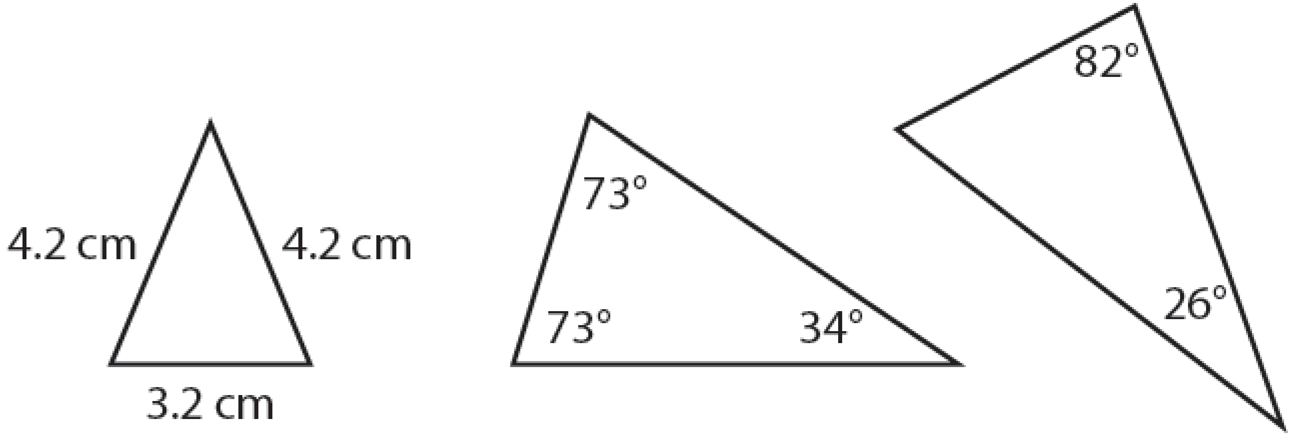
NCETM - Year 6: Teaching for Mastery

Which of these triangles are isosceles?

Explain your decisions.



# NCETM - Year 6: Teaching for Mastery

Sam added two fractions together and got $\frac{7}{8}$ as the answer.

Write down two fractions that Sam could have added.

……and another, and another, and another

……can you think of an easy one, hard one, one that no one else will think of?

How do you know your solutions are right?

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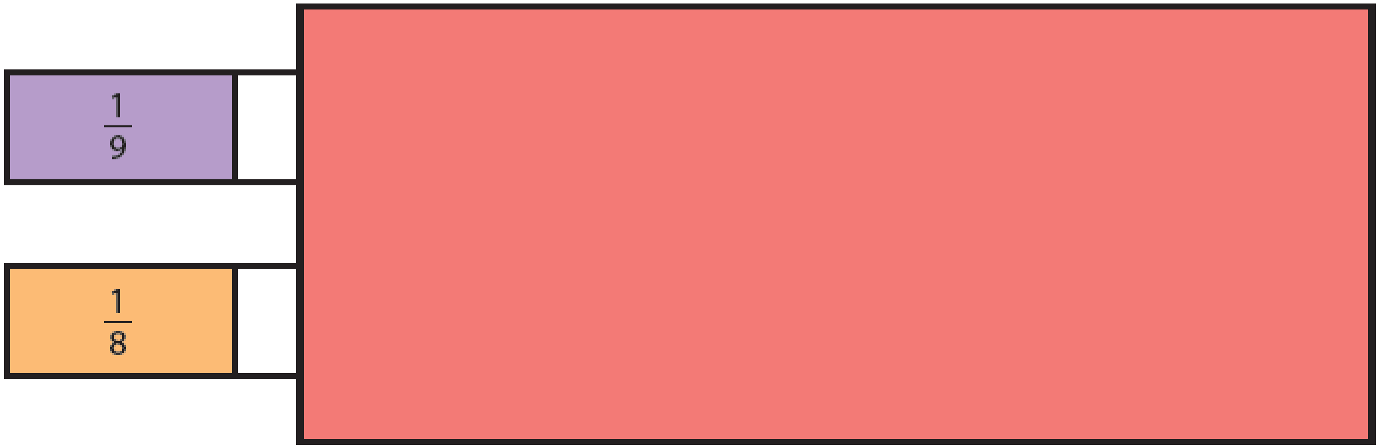
NCETM - Year 6: Teaching for Mastery

Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer.




Explain your reasoning.

# Key Stage 3

- A focus on consolidation of key skills from KS2 and exploring topics in readiness for more formal teaching at KS4.

- Developing problem solving techniques and resilience are seen as key areas for secondary students.

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National Centre for Excellence in the Teaching of Mathematics

MathsHUBS

# Teaching for Mastery

## Questions, tasks and activities to support assessment in KS3

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## NUMBER

Selected National Curriculum Programme of Study statements

Pupils should be taught to:

- Use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, both positive and negative
- Use conventional notation for the order of operations, including brackets, powers, roots and reciprocals
- Recognise and use relationship between operations including inverse operations

### The Big Ideas

This group of statements focuses on arithmetic procedures.

Pupils need to develop understanding of, and hence fluency in using, operations. Acronyms, such as BIDMAS inhibit deep understanding and can cause misconceptions. Considering the order of operations, Watson (2016) writes in a blog (https://educationblog.oup.com/secondary/maths/order-and-disorder) about the ambiguities of BIDMAS. Watson continues to say that the 'new curriculum for primary Year 6 offers strong guidance that algebra should be introduced to express what children already know about number operations and relations; the new curriculum for Key Stage 3 includes a similar statement. If students know that ambiguities need to be sorted out, they are likely to be more willing to learn how mathematicians sort them out through precise notation. The notation follows from their mathematical needs, rather than their learning needs following from the notation.'

### Mastery

A teacher asked her class to calculate $\frac{2}{3} \times \frac{1}{3}$. The answers from her students included…

- $\frac{2}{3} \times \frac{1}{3} = \frac{2}{6}$
- $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$
- $\frac{2}{3} \times \frac{1}{3} = \frac{3}{6}$
- $\frac{2}{3} \times \frac{1}{3} = \frac{3}{9}$

Did she get a correct answer? Explain what mistakes have might have been made for any incorrect answers.

### Mastery with Greater Depth

Is it always, sometimes or never true that the use of brackets in an addition and multiplication calculation will change the value of the answer? Explain your thinking.

$81 \times 36 = 2916$.

Explain how you can use this fact to devise calculations with answers 29.16, 2.916, 0.2916.

# Which is bigger; 2n or $n^2$ ?

Mastery

Which is bigger?

| | | |
|---|---|---|
| $-3x+6$ | $9+x$ | $2x-7$ |
| $X^2$ | $\frac{X^2}{2}$ | $\frac{10}{x}$ |

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# Mastery with Greater Depth

Pete is solving a linear equation. He draws this bar model to help.

- What equation is Pete solving?
- What is the value of t?

| | | | |
|---|---|---|---|
| t | t | t | 7 |
| t | t | 10 | |

Explain how you know.

A piece of wood was 40cm long. It was cut into 3 pieces. The lengths in cm are: $2x - 5$, $x + 7$, $x + 6$.

What is the length of the longest piece? Explain your answer.

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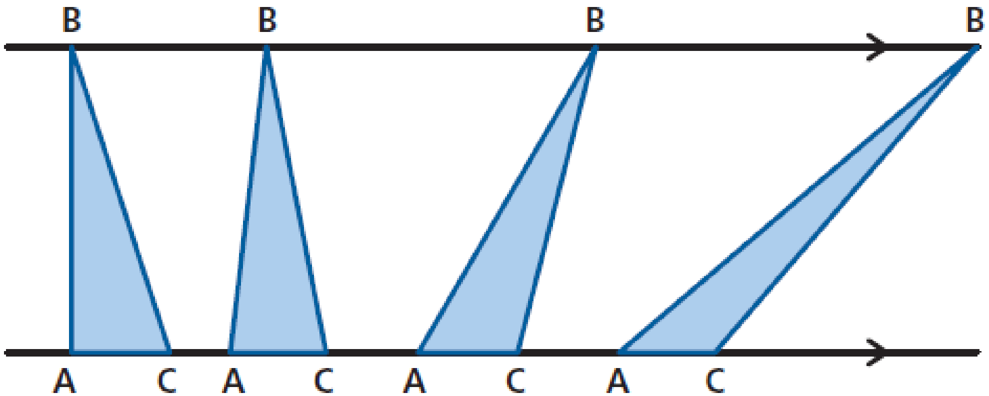
# Mastery with Greater Depth

A piece of elastic is fixed at two points A and C and point B slides along the line.

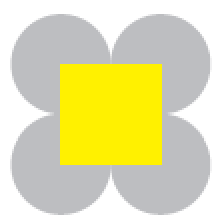



Some of the triangles that can be made are shown here.

Which triangle has the greatest area? How do you know?

Using π as 3.14 gives the grey area as 150.72cm$^2$

What's the perimeter of the yellow section?



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# Bloom's Revised Taxonomy

| | Question | Level | Some key words |
|---|---|---|---|
| HIGHER ORDER | Can the pupil create a new point of view or product? | ***Creating:*** | assemble, construct, create, design ,modify, write |
| | Can the pupil justify a stand or decision? | ***Evaluating:*** | appraise, conclude, defend, interpret, evaluate |
| | Can the pupil distinguish between different parts? | ***Analysing:*** | analyse, breaks down, compare, contrast, criticise, discriminates, distinguish |
| LOWER ORDER | Can the pupil use information in a new way? | ***Applying:*** | demonstrate, illustrate, solve, choose, dramatise, interpret |
| | Can the pupil explain ideas or concepts? | ***Understanding:*** | classify, describe, explain, discuss, identify, report |
| | Can the pupil recall or remember the information? | ***Remembering:*** | define, list, memorise, duplicate, recall |

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**Bloom's Taxonomy** presents us with a progression in the complexity of tasks.


- **Remember:** recall facts and basic concepts (define, duplicate, list, memorise, repeat, state)
- **Understand:** explain ideas or concepts (classify, describe, discuss, explain, identify, locate, recognise, report, select, translate).
- **Apply:** use information in new situations (execute, implement, solve, use, demonstrate, interpret, operate, schedule, sketch)
- **Analyse:** draw connections between ideas or organise information (differentiate, organise, relate, compare, distinguish, examine, experiment, question, test).
- **Evaluate:** justify o decision (appraise, argue, defend, judge, select, support, value, critique, weigh).
- **Create:** produce a new or original work (design, assemble, construct, conjecture, develop, formulate, author, investigate).

# REASONING

**NRICH offers a five-step progression in reasoning:**

- A spectrum that shows us whether children are moving on in their reasoning from novice to expert.
- Children are **unlikely to move fluidly from one step to the other,** rather flow up and down the spectrum settling on a particular step that best describes their reasoning skills at any one time.

# Nrich Progression


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- **Describing:** saying what you did you solve a problem.
- **Explaining:** saying why you did what you did, even if it is wrong.
- **Convincing:** becoming confident that there is a right way to do something, leading to inductive reasoning.
- **Justifying:** offering a correct logical argument, that creates a chain of explanation.
- **Proving:** a mathematically sound generalisation that leads to underlying patterns and proof.

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**The following slides are a selection of Nrich problems and articles for KS3.**

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# Peaches Today, Peaches Tomorrow...

A little monkey had 75 peaches.

Each day, he kept a fraction of his peaches, gave the rest away, and then ate one.

These are the fractions he decided to keep:

1/2 1/4 3/4 3/5 5/6 11/15

In which order did he use the fractions so that he was left with just one peach at the end?

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# Peaches Today, Peaches Tomorrow...

A little monkey had 60 peaches.

On the first day he decided to keep 3/4 of his peaches.
He gave the rest away. Then he ate one.

On the second day he decided to keep 7/11 of his peaches.
He gave the rest away. Then he ate one.

On the third day he decided to keep 5/9 of his peaches.
He gave the rest away. Then he ate one.

On the fourth day he decided to keep 2/7 of his peaches.
He gave the rest away. Then he ate one.

On the fifth day he decided to keep 2/3 of his peaches.
He gave the rest away. Then he ate one.

**How many did he have left at the end?**

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# Peach Rationing

The monkey always keeps a fraction of his peaches each day, gives the rest away, and then eats one.

**If he started with fewer than 100 peaches,**
**could he make his peaches last for more than a week?**

Each fraction must be in its simplest form and must be less than 1.

The denominator is never the same as the number of peaches left (e.g. if there are 8 peaches left, he can't keep 7/8 of them)

**What is the longest that you can make them last?**

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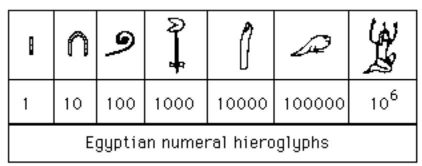
# History of Fractions

**Stage: 2 and 3**
Article by Liz Pumfrey
Published November 2004,December 2004,February 2011.

Did you know that fractions as we use them today didn't exist in Europe until the 17th century? In fact, at first, fractions weren't even thought of as numbers in their own right at all, just a way of comparing whole numbers with each other. Who first used fractions? Were they always written in the same way? How did fractions reach us here? These are the sorts of questions which we are going to answer for you. Read on ...

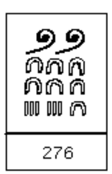
The word fraction actually comes from the Latin "fractio" which means to break. To understand how fractions have developed into the form we recognise, we'll have to step back even further in time to discover what the first number systems were like.

From as early as 1800 BC, the Egyptians were writing fractions. Their number system was a base 10 idea (a little bit like ours now) so they had separate symbols for 1, 10, 100, 1000, 10000, 100000 and 1000000. The ancient Egyptian writing system was all in pictures which were called hieroglyphs and in the same way, they had pictures for the numbers:



Egyptian numeral hieroglyphs

Here is an example of how the numbers were made up:




Could you write down 3581 in hieroglyphics?

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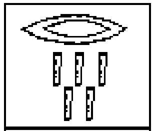
The Egyptians wrote all their fractions using what we call unit fractions. A unit fraction has 1 as its numerator (top number). They put a mouth picture (which meant part) above a number to make it into a unit fraction. For example:

Here is one fifth.



1/5

Can you work out how to write one sixteenth?

They expressed other fractions as the sum of unit fractions, but they weren't allowed to repeat a unit fraction in this addition. For example this is fine:

$$\frac{3}{4}=\frac{1}{2}+\frac{1}{4}$$

But this is not:

$$\frac{2}{7}=\frac{1}{7}+\frac{1}{7}$$

The huge disadvantage of the Egyptian system for representing fractions is that it is very difficult to do any calculations. To try to overcome this, the Egyptians made lots of tables so they could look up answers to problems.

# Keep it Simple

**Stage: 3 ★**

**Unit fractions (fractions which have numerators of 1) can be written as the sum of two different unit fractions.**

For example

$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$

Charlie thought he'd spotted a rule and made up some more examples.

$\frac{1}{2} = \frac{1}{10} + \frac{1}{20}$

$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$

$\frac{1}{3} = \frac{1}{7} + \frac{1}{21}$

$\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$

Are all his examples correct?

What do you notice about the sums that are correct?

Find some other correct examples..

How would you explain to Charlie how to generate lots of correct examples?

**Alison started playing around with $\frac{1}{6}$ and was surprised to find that there wasn't just one way of doing this.**

She found:

$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$

$\frac{1}{6} = \frac{1}{8} + \frac{1}{24}$

$\frac{1}{6} = \frac{1}{9} + \frac{1}{18}$

$\frac{1}{6} = \frac{1}{10} + \frac{1}{15}$

$\frac{1}{6} = \frac{1}{12} + \frac{1}{12}$ (BUT she realised this one didn't count because they were not different.)

**Charlie tried to do the same with $\frac{1}{8}$. Can you finish Charlie's calculations to see which ones work?**

$\frac{1}{8} = \frac{1}{9} + ?$

$\frac{1}{8} = \frac{1}{10} + ?$

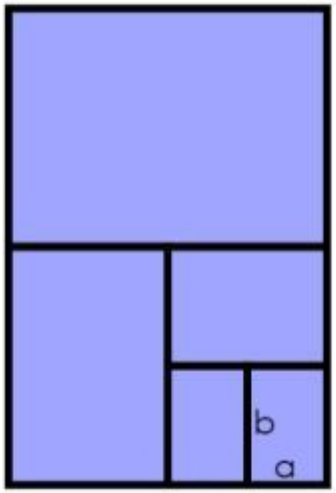
$\frac{1}{8} = \frac{1}{11} + ?$

..........

Can all unit fractions be made in more than one way like this?

Choose different unit fractions of your own to test out your theories.

# Perimeter Expressions **Stage: 3** ★



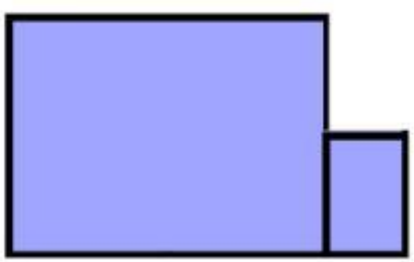

Charlie took a sheet of paper and cut it in half.

Then he cut one of those pieces in half, and repeated until he had five pieces altogether.

He labelled the sides of the smallest rectangle, *a* for the shorter side and *b* for the longer side.

How can you convince someone else that this is the perimeter?

Here is a shape that Charlie made by combining the largest and smallest rectangles:



Check you agree that the perimeter is $10a+4b$.

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# A journey toward problem solving and reasoning at GCSE …

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AQA

GCSE
**Mathematics**
**Specification (8300/1F)**
Paper 1 Foundation tier

Which of $\frac{2}{5}$ or $\frac{5}{8}$ is closer in value to $\frac{1}{2}$?

You **must** show your working.

| | | |
|---|---|---|
| **Alternative method 1** | | |
| $\frac{16}{40}$ or $\frac{25}{40}$ or $\frac{20}{40}$ | M1 | |
| Valid comparison<br>eg $\frac{16}{40}$ and $\frac{25}{40}$ and $\frac{20}{40}$<br>or $\frac{4}{40}$ and $\frac{5}{40}$ | M1 | oe |
| $\frac{2}{5}$ | A1 | Must see working |
| **Alternative method 2** | | |
| 0.4 or 0.625 | M1 | 40(%) or 62.5(%) |
| 0.4 and 0.625<br>or 0.1 and 0.125 | M1 | 40(%) and 62.5(%)<br>or 10(%) and 12.5(%) |
| $\frac{2}{5}$ | A1 | Must see working |

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# Work out the area of the shapes

The triangle and the rectangle have the same area.

All lengths are in cm.


Not drawn accurately

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## Regular opportunities to reason will help conceptual understanding:

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# Always, sometimes, never true

$x^2 > x$

Division always makes smaller

A quadrilateral can have four acute angles

Thinkers: ATM

# What is the same and what is different?

- What is the same about a square and a rhombus? What is different?
- What is the same about the graph of $y = 2x + 3$ and $y = 5x + 3$? What is different?

Thinkers: ATM

# Which is the odd one out?

| | |
|---|---|
| $x^2 + 6x + 5$ | $x^2 + 7x + 12$ |
| $x^2 - 5x + 6$ | $x^2 + 7x + 13$ |
| $x^2 + 6x – 5$ | $x^2 + 7x + 14$ |
| $x^2 + 7x + 6$ | $x^2 + 7x + 15$ |
| $x^2 - 7x + 6$ | $x^2 + 7x + 16$ |

Thinkers: ATM

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# Reasoning….

Can you show me an example of….
and another, and another…?

What is the same and different?

True or false, why?

Which is the odd one out?

Have you found all of the solutions?

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**Are there enough opportunities across both KS3 and KS4 for pupils to develop reasoning and problem solving skills?**

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# HIAS Maths Team

Jo Lees – Area Inspector - Mathematics
Email: jo.lees@hants.gov.uk
Tel: 02380 816139

Jacqui Clifft – Area Inspector - Mathematics
Email: jacqui.clifft@hants.gov.uk
Tel: 02380 816139

Jenny Burn – Inspector/Adviser – Mathematics
Email: jenny.burn@hants.gov.uk
Tel: 01962 876207

Tessa Ingrey – Teaching & Learning Adviser – Mathematics (P/T)
Email: tessa.ingrey@hants.gov.uk
Tel: 01962 876207

Dave Parnell – Teaching & Learning Adviser – Mathematics
Email: dave.parnell@hants.gov.uk
Tel: 01962 876207

Rebecca Reynolds – Teaching & Learning Adviser – Mathematics
Email: rebecca.reynolds@hants.gov.uk
Tel: 01962 876207

Brenda Robertson – Inspector/Adviser – Mathematics
Email: brenda.robertson@hants.gov.uk
Tel: 01962 876207

Kate Spencer – Teaching & Learning Adviser – Mathematics
Email: kathryn.spencer@hants.gov.uk
Tel: 01962 876207

Natalie Ivey – Inspector/Adviser – Mathematics (P/T)
Email: natalie.ivey@hants.gov.uk
Tel: 01962 876207

For further details on the full range of services available please contact us using the following details:

Tel: 01962 874820 or email: **hias.enquiries@hants.gov.uk**

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- Searchable course catalogue linked to the Learning Zone.
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- In-house training opportunities.
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