Teaching for Mastery

Department Meeting

HIAS Maths Team
August 2018
Final Version

Overview

In this document

You will:

- identify key aspects of Teaching for Mastery
- get a flavour of the changes at primary following the new curriculum
- have the opportunity to work on some Maths …looking at particular problems, and consider what can be generalised from them
- reflect on question types to develop reasoning
- explore bar models

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"Teaching for mastery begins with the premise that every student can succeed at mathematics"

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Essence of Maths Teaching for Mastery

- Maths teaching for mastery rejects the idea that a large proportion of people 'just can't do maths'.

- All students are encouraged by the belief that by working hard at maths they can succeed.

- Students engage through whole-class interactive teaching, where the focus is on **all** pupils working together on the same lesson content at the same time. This is a successful model in HPJs , such as Shanghai and Singapore. The aim is for all students to master concepts before moving to the next part of the curriculum sequence, allowing no student to be left behind.

Essence of Maths Teaching for Mastery

- High quality AfL identifies if a student is not yet grasping a concept or procedure. The key is to identify a potential problem early enough to either allow some pre-teaching or an intervention during the lesson. All students need to be ready to move forward with the whole class in the next lesson.

- Procedural fluency and conceptual understanding are developed in tandem because each supports the development of the other.

- It is recognised that practice is a vital part of learning, but the practice should be **intelligent practice** that both reinforces pupils' procedural fluency and develops their conceptual understanding. A few carefully chosen examples in greater depth, rather than a page of examples that are all similar.

Essence of Maths Teaching for Mastery

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- Significant time is spent developing deep knowledge of the key ideas that are needed to underpin future learning. The structure and connections within the mathematics are emphasised, so that students develop deep learning that can be sustained.

- Key facts such as multiplication tables and addition facts within 10 are learnt to automaticity to avoid cognitive overload in the working memory and enable students to focus on new concepts.

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National Centre for Excellence in the Teaching of Mathematics

The National Curriculum

Aims:

- Become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to **recall and apply knowledge** rapidly and accurately.
- **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

NCETM Teaching for Mastery…

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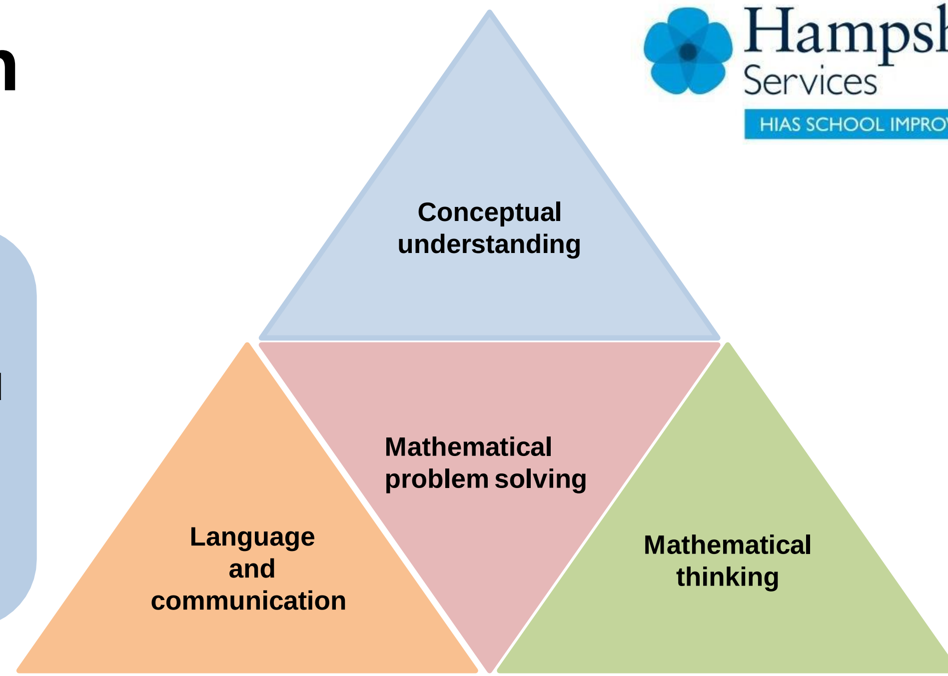
A possible checklist for what to look out for when assessing a pupils' understanding:

A pupil really understands a mathematical concept, idea or technique if they can:

- **describe it in their own words**;
- represent it in a variety of ways (CPA);
- explain it to someone else (using CPA perhaps);
- create examples and non-examples;
- see **connections with other facts and ideas;**
- recognise it in **new situations** and **contexts**;
- **make use** of it in **various ways**, including new situations.

From - Teaching for Mastery:
Questions, tasks and activities to support assessment
NCETM & Maths Hubs

Dimensions of depth



Conceptual understanding

Pupils deepen their understanding by representing concepts using objects and pictures, making connections between different representations and considering what different representations stress and ignore.

Language and communication

Pupils deepen their understanding by explaining, creating problems, justifying and proving using mathematical language. This use of language also acts as a scaffold for their thinking.

Mathematical thinking

Pupils deepen their understanding by asking and investigating great questions, by giving examples, by sorting and comparing, or by looking for patterns and rules in the mathematics they are exploring.

Drury. H (2014) Mastering Mathematics, Teaching to transform achievement, OUP ISBN: 978-0-19-835175-7

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Key ideas….

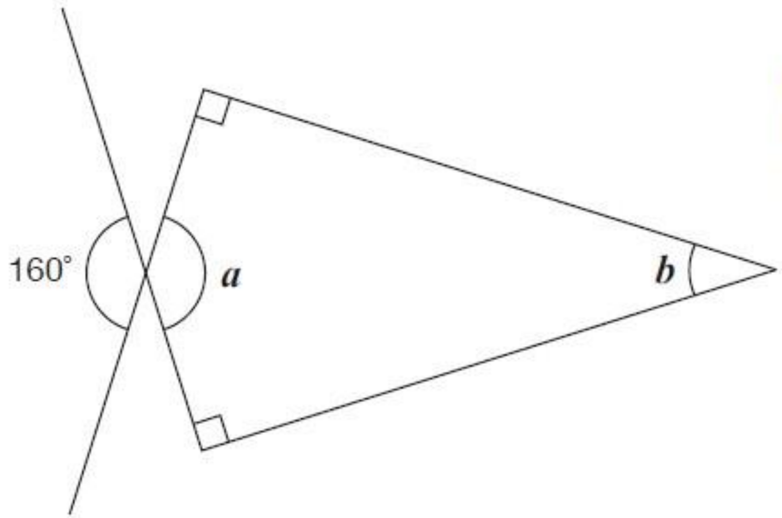
- CPA: concrete – pictorial - abstract
- Questioning
- Reasoning
- Problem solving
- Role of talk
- Linking aspects of maths
- Bar model

Teaching for Mastery

- Year 6 experience…..

Mathematics Paper 2: Reasoning

Calculate the size of angles a and b in this diagram.



Not to scale

$a =$ ______ °

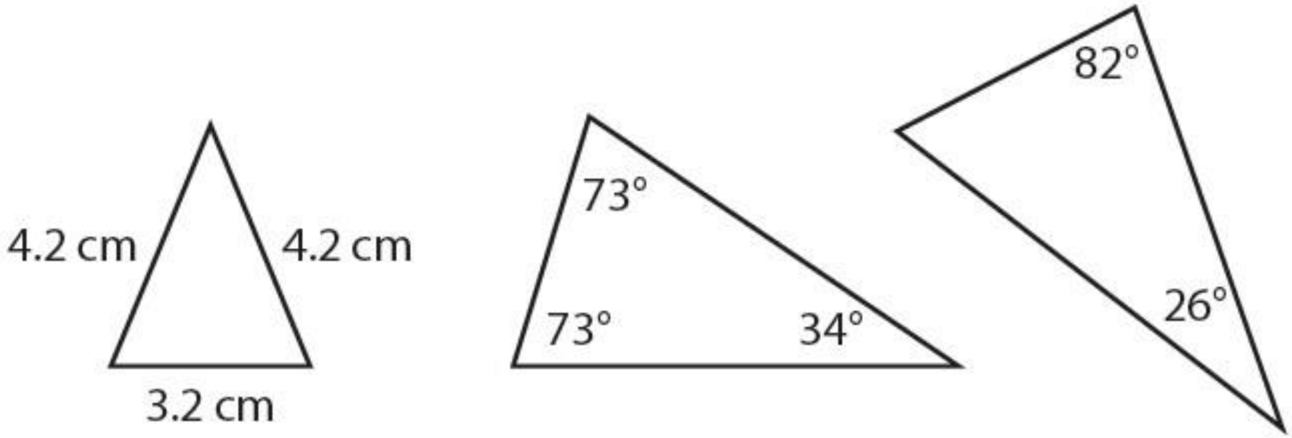
1 mark

$b =$ ______ °

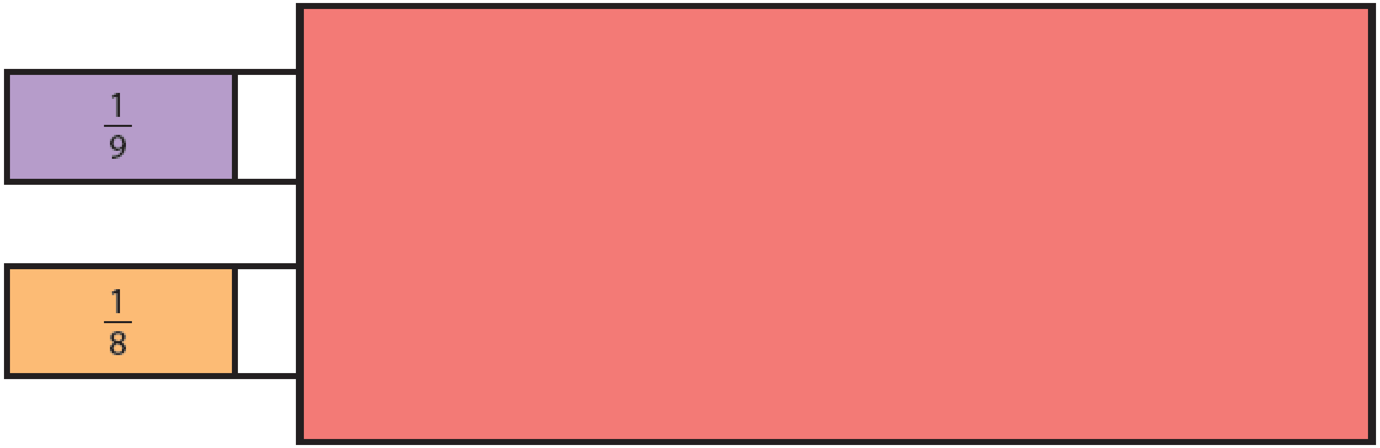
1 mark

Which of these triangles are isosceles?

Explain your decisions.



Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer.



Explain your reasoning.

Sam added two fractions together and got $\frac{7}{8}$ as the answer.

Write down two fractions that Sam could have added.

……and another, and another, and another

……can you think of an easy one, hard one, one that no one else will think of?

How do you know your solutions are right?

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2016 national curriculum tests
Key stage 2
Mathematics
Paper 2: reasoning

7 Write the two missing values to make these equivalent fractions correct.

$$\frac{\square}{3} = \frac{8}{12} = \frac{4}{\square}$$

1 mark

1 mark

Year 6 objective

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On New Year's Eve, Polly has £3·50 in her money box. On 1 January she puts 30p into her money box. On 2 January she puts another 30p into her money box. She continues putting in 30p every day.

How much money is in the box on 10 January?

How much money is in the box on 10 February?

Write a sequence-generating rule for working out the amount of money in the money box on any day in January.

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The diameter of a golf ball is 4 cm. I want to make a box which will hold six golf balls.

What size could my box be?

Is there more than one answer?

How do you know you have all of the possibilities?

If you had 12 golf balls, how many different boxes could you design?

If you had to produce a box, which would you choose? Why?

Key Stage 3

- A focus on consolidation of key skills from KS2 and exploring topics in readiness for more formal teaching at KS4.

- Developing problem solving techniques and resilience are seen as key areas for secondary students.

Key Stage 3 continued….

The challenges in key stage 3 are:

- All students expected to be introduced to traditionally "Higher" topics such as quadratics, non-linear graphs and trigonometry during key stage 3
- A focus on multiplicative reasoning
- Increased opportunities for the use of generalisations, reasoning and making conjectures with opportunities to develop more formal proof
- A focus on modelling mathematical ideas including, ready access to ICT to enable lower attaining students to explore more complex relationships.

KS3 scheme of work: Things to consider

- How effective will it be for preparing students for GCSE?
- Does it ensure that students are taught for understanding?
- Does it encourage students to make links?
- Are there opportunities for reasoning and problem solving woven through it?

The journey for KS3

- Reasoning and problem solving….scaffold and model
- Language of maths and the role of talk
- Bar model
- Linking aspects of Maths

Some types of questions that are appearing in sample questions to assess the new GCSE specification

…..What are the implications for teaching in KS3?

Which of $\frac{2}{5}$ or $\frac{5}{8}$ is closer in value to $\frac{1}{2}$?

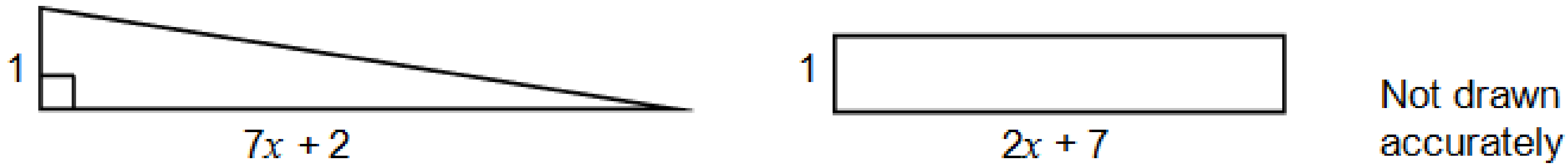
You **must** show your working.

Alternative method 1		
$\frac{16}{40}$ or $\frac{25}{40}$ or $\frac{20}{40}$	M1	
Valid comparison eg $\frac{16}{40}$ and $\frac{25}{40}$ and $\frac{20}{40}$ or $\frac{4}{40}$ and $\frac{5}{40}$	M1	oe
$\frac{2}{5}$	A1	Must see working
Alternative method 2		
0.4 or 0.625	M1	40(%) or 62.5(%)
0.4 and 0.625 or 0.1 and 0.125	M1	40(%) and 62.5(%) or 10(%) and 12.5(%)
$\frac{2}{5}$	A1	Must see working

Work out the area of the shapes

The triangle and the rectangle have the same area.

All lengths are in cm.



What is 0.9 as a percentage?

Circle your answer.

0.009%	0.09%	9%	90%

Circle the value of 2^4

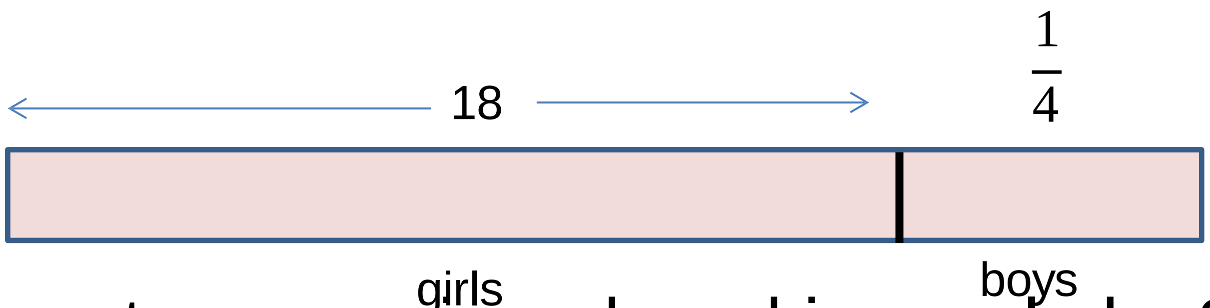
6 8 16 24

Bar Model - NCETM

- The bar model supports the **transformation of real life problems into a mathematical form** and can bridge the gap between concrete mathematical experiences and abstract representations.
- It is not a method for solving problems, but a way of **revealing the mathematical structure** within a problem and gaining insight and clarity as to how to solve it.

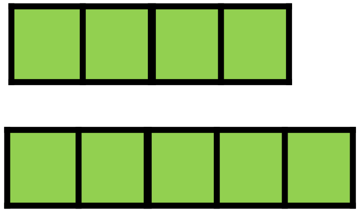
Bar model

- In a class, 18 of the children are girls
  A quarter of the children are boys
  Altogether, how many children are there in the class?

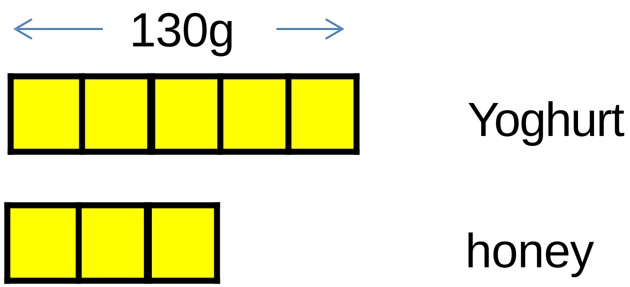


- A computer game is reduced in a sale by 30%. Its reduced price is £77. How much was the original price?

- Two numbers are in the ratio 4:5. They both sum to 135. Identify both numbers.



- A herbal skin treatment uses yoghurt and honey in the ratio 5 : 3. How much honey is needed to mix with 130 g of yoghurt?



Reasoning

- Reasoning is integral to the development of conceptual understanding and problem-solving skills.
- Of the three National Curriculum aims, it is the least well developed currently.
- Not all classrooms have a positive ethos that encourages learning from mistakes.
- Tasks are not used well enough to develop reasoning.
- Talk often focuses on the 'how' rather than the 'why', 'why not', and 'what if' in:
  - teachers' explanations and questions
  - pupils' responses.

Ofsted

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Reasoning….

Can you show me an example of….
and another, and another…?

What is the same and different?

True or false, why?

Which is the odd one out?

Have you found all of the solutions?

Developing Reasoning Skills…

- Use of teacher modelling, verbalise mathematical thinking
- Discuss the mathematical language
- Use of reasoning sentence starters to scaffold process for students
- Consider how to get ‘inside the student’s heads’
- Providing examples that fit the rule or non-examples
- How can you convince me? Prove it to me

Reasoning Sentence Starters…

- It is always / sometimes / never true because…
- I think this because…
- If this is true then…
- I know that the next one is…because…
- This can't work because…
- When I tried……I noticed that…
- The pattern looks like…
- All the numbers begin with…
- This won't work because…

Kate said……

A three digit number is always bigger than a two digit number

Do you agree?

Explain your answer

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Can you give me an example of a pair of rectangles whose areas differ by 2 …..and another….and another….

Can you generalise?

Can you give me an example of a pair of rectangles whose perimeters differ by 2 …..and another….and another….

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Always, sometimes, never true

$x^2 > x$

Division always makes smaller

A quadrilateral can have four acute angles

Thinkers: ATM

Confounding Expectations

(and then another, and another….)

Give, find, construct an example of….

- A symmetrical shape which is not regular
- An equation of a line that does not go through the first quadrant
- A pair of fractions whose product is greater than 1

What is the same and what is different?

- What is the same about a square and a rhombus? What is different?
- What is the same about the graph of $y = 2x + 3$ and $y = 5x + 3$? What is different?

Which is the odd one out?

$x^2 + 6x + 5$	$x^2 + 7x + 12$
$x^2 - 5x + 6$	$x^2 + 7x + 13$
$x^2 + 6x – 5$	$x^2 + 7x + 14$
$x^2 + 7x + 6$	$x^2 + 7x + 15$
$x^2 - 7x + 6$	$x^2 + 7x + 16$

Thinkers: ATM

Reasoning….

Can you show me an example of….
and another, and another…?

What is the same and different?

True or false, why?

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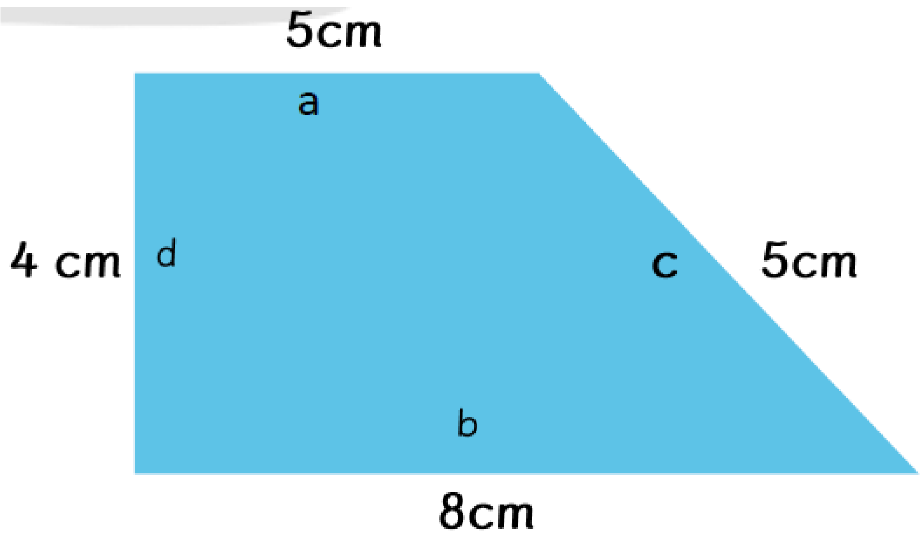
Have you found all of the solutions?

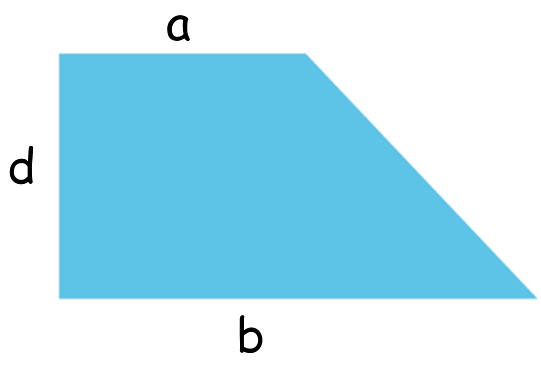
The Trapezium

Areas of Shapes

What is this shape called?
Can I find the area of this shape?

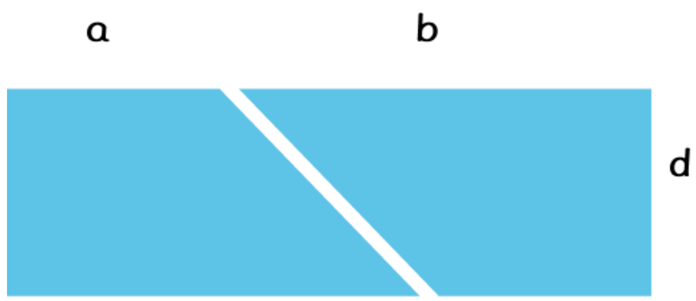


How can I find the area of this shape?
How many ways can you find?



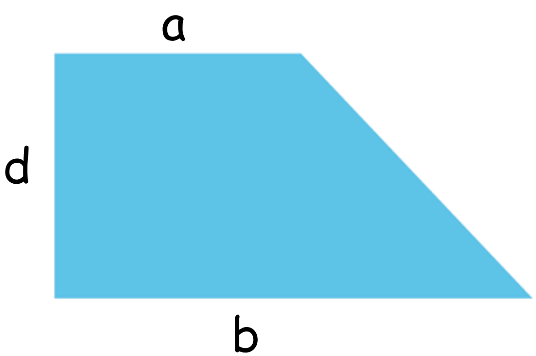
Deriving the formula

Area of shapes

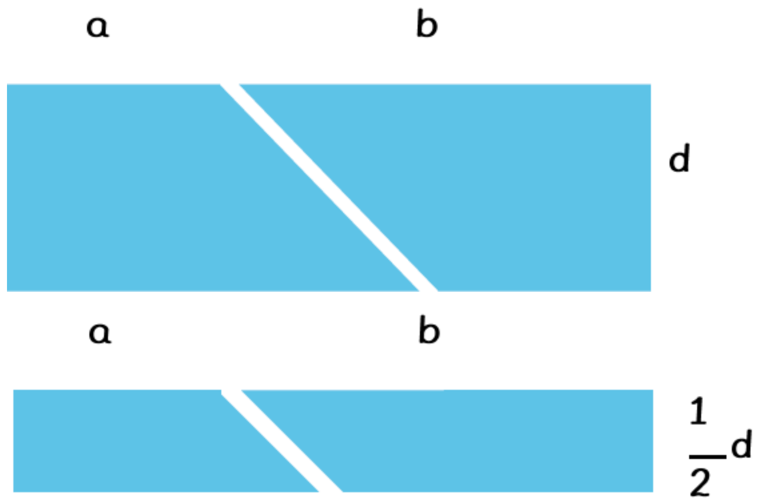


$$= \frac{d(a+b)}{2}$$

Deriving the formula



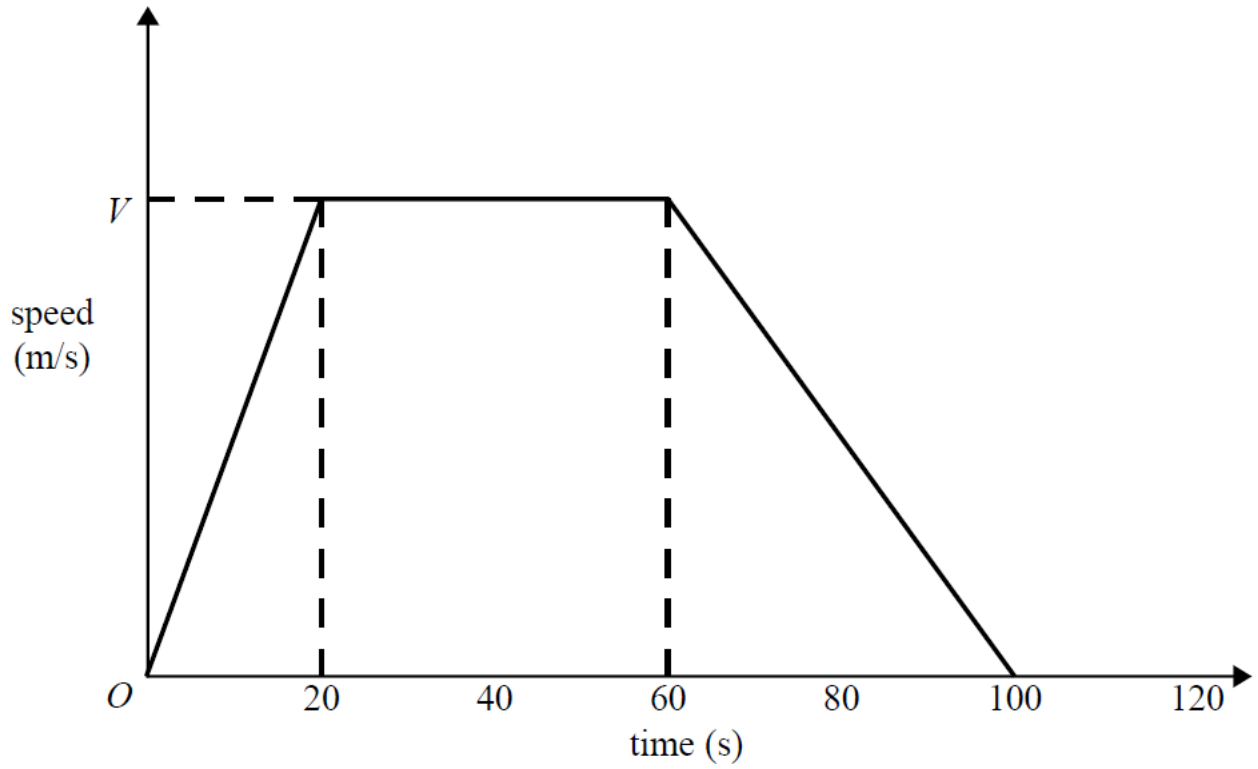
Area of shapes



$$= \frac{d(a+b)}{2}$$

$$= \frac{1}{2}d(a+b)$$

Here is a speed-time graph for a car journey.
The journey took 100 seconds.



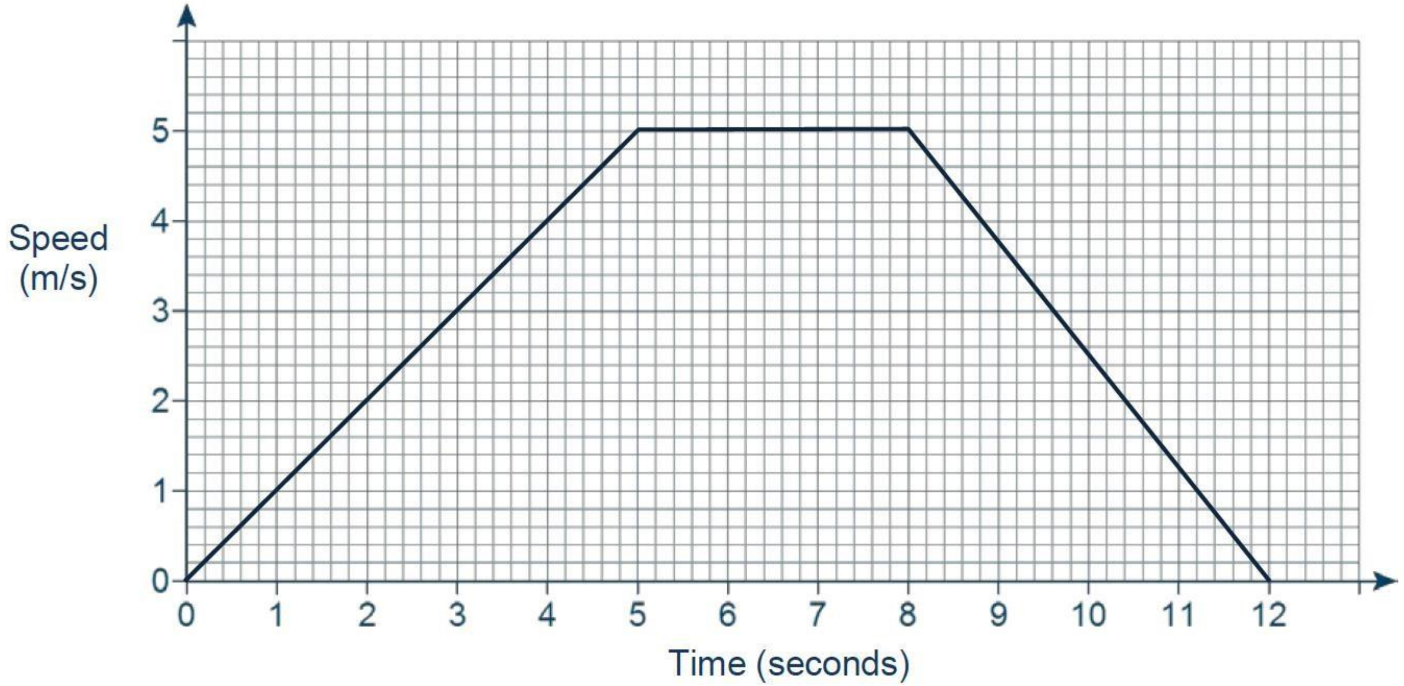
The car travelled 1.75km in the 100 seconds.

(a) Work out the value of V.

Higher tier

Meera runs for 12 seconds.

Her speed, in metres per second, is shown on the graph.



For how many seconds does she run at a constant speed?

Work out the total distance she runs.

Area / perimeter…..

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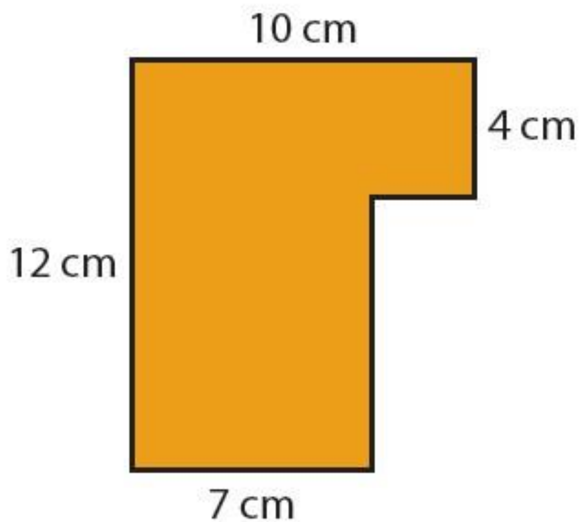
Mastery with Greater Depth

Sami worked out the area of the orange shape as $10 \times 4 + 8 \times 7 = 96\,cm^2$.

Razina worked out the area as $12 \times 7 + 3 \times 4 = 96\,cm^2$.

Lukas worked out the area as $10 \times 10 - 2 \times 2 = 96\,cm^2$.



Some Maths to get us started…

Are you convinced by Sami, Razina or Lukas's reasoning?

Explain your answer.

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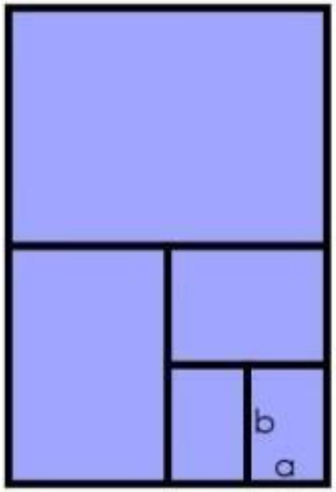
Perimeter Expressions

Stage: 3 ★



Charlie took a sheet of paper and cut it in half.

Then he cut one of those pieces in half, and repeated until he had five pieces altogether.

He labelled the sides of the smallest rectangle, *a* for the shorter side and *b* for the longer side.

Here is a shape that Charlie made by combining the largest and smallest rectangles:

How can you convince someone else that this is the perimeter?

Check you agree that the perimeter is $10a+4b$.

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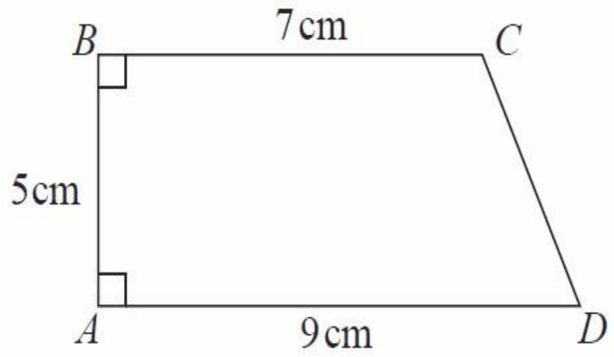
http://nrich.maths.org/7283

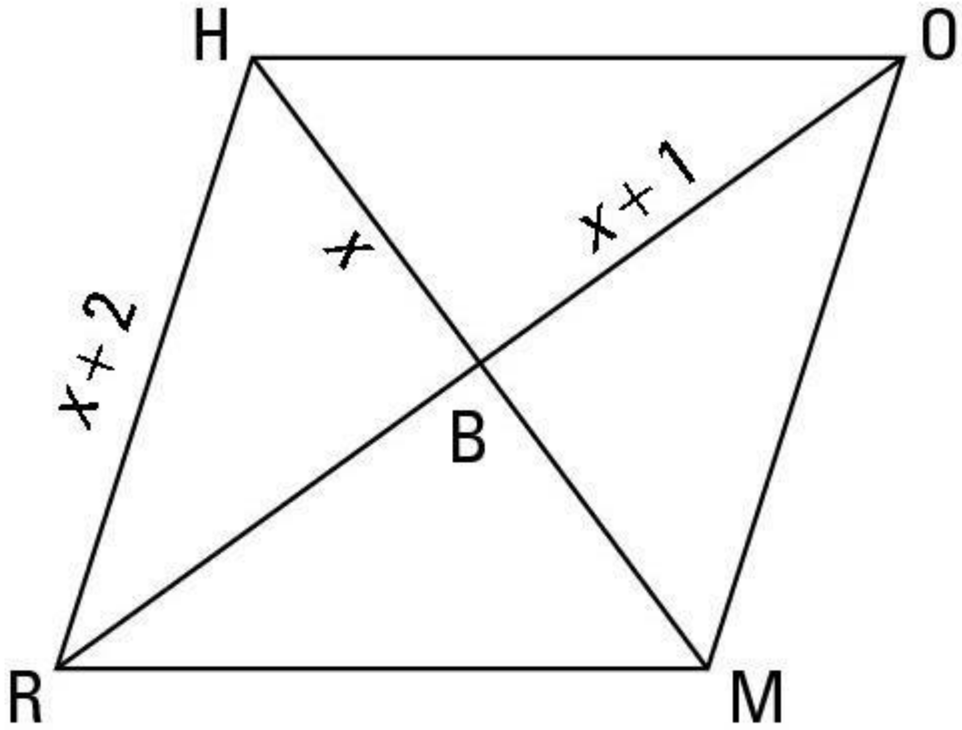
ABCD is a trapezium.



A square has the same perimeter as this trapezium.

Work out the area of the square.
Give your answer correct to 3 significant figures.

Find the perimeter of rhombus RHOM



Maths task….

- Right angled or not?
- How do you decide?
- How does a pupil decide?

Pythagorean triples

http://www.mathsisfun.com/pythagorean_triples.html

List of the First Few

Here is a list of the first few Pythagorean Triples (**not** including "scaled up" versions mentioned below):

(3,4,5)	(5,12,13)	(7,24,25)	(8,15,17)	(9,40,41)
(11,60,61)	(12,35,37)	(13,84,85)	(15,112,113)	(16,63,65)
(17,144,145)	(19,180,181)	(20,21,29)	(20,99,101)	(21,220,221)
(23,264,265)	(24,143,145)	(25,312,313)	(27,364,365)	(28,45,53)
(28,195,197)	(29,420,421)	(31,480,481)	(32,255,257)	(33,56,65)
(33,544,545)	(35,612,613)	(36,77,85)	(36,323,325)	(37,684,685)
... infinitely many more ...				

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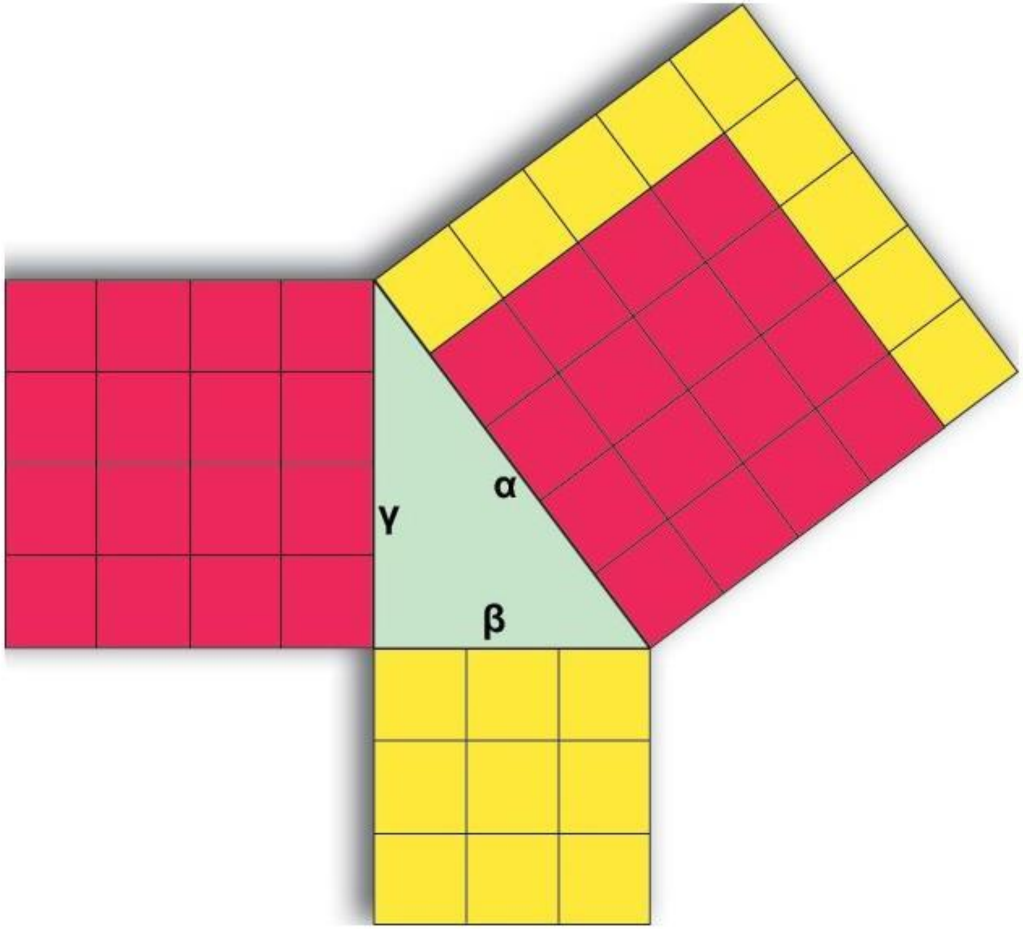
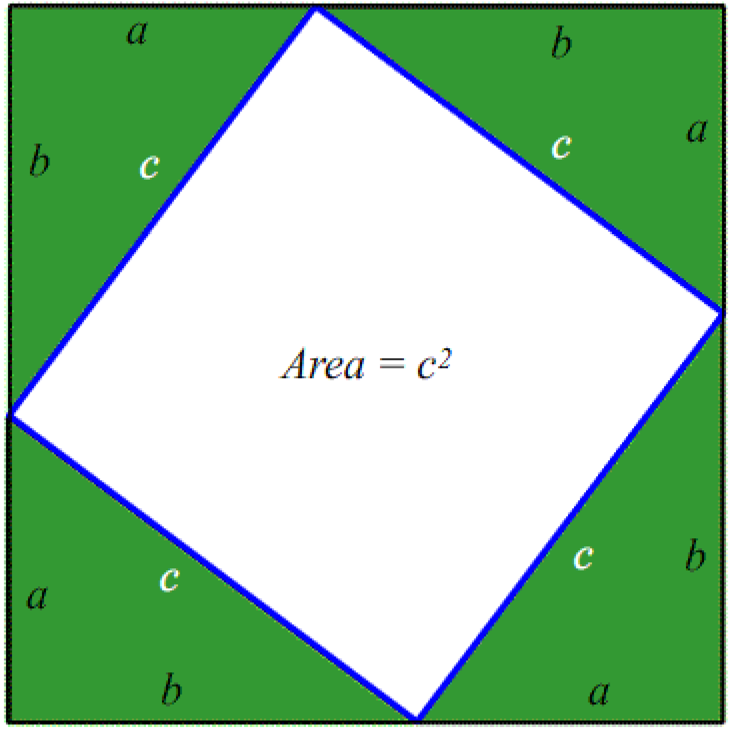
Teaching for understanding



Deriving the formula



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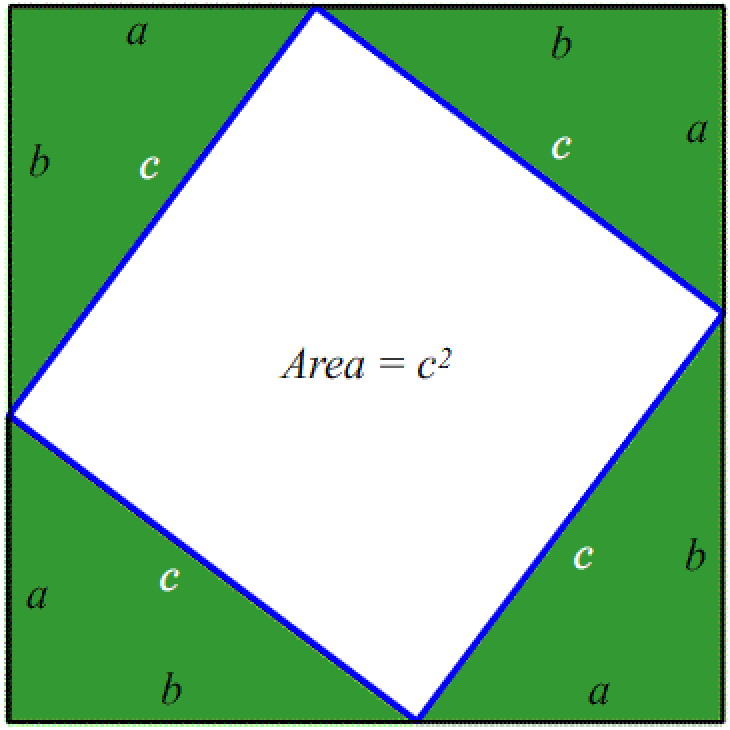
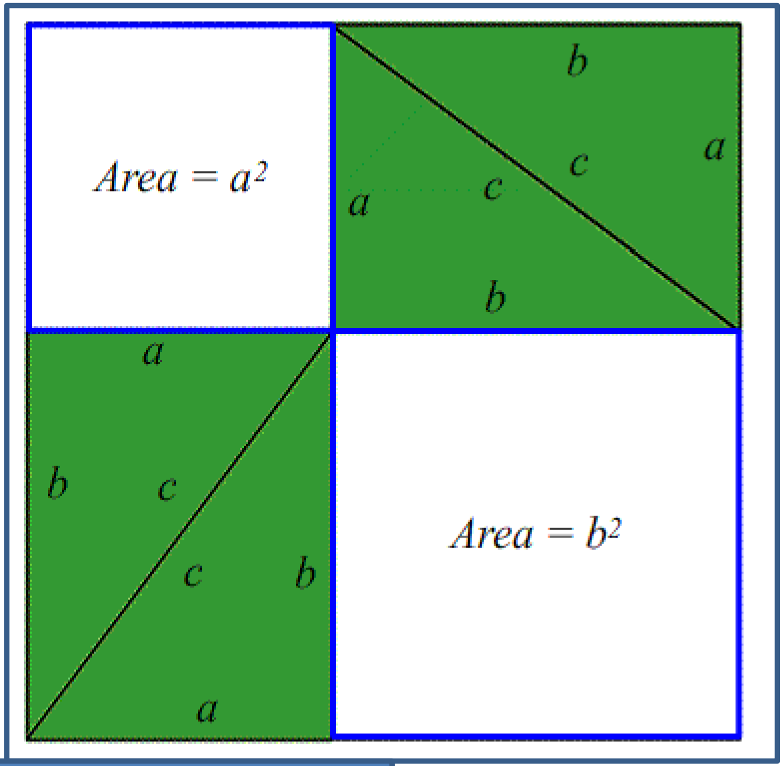
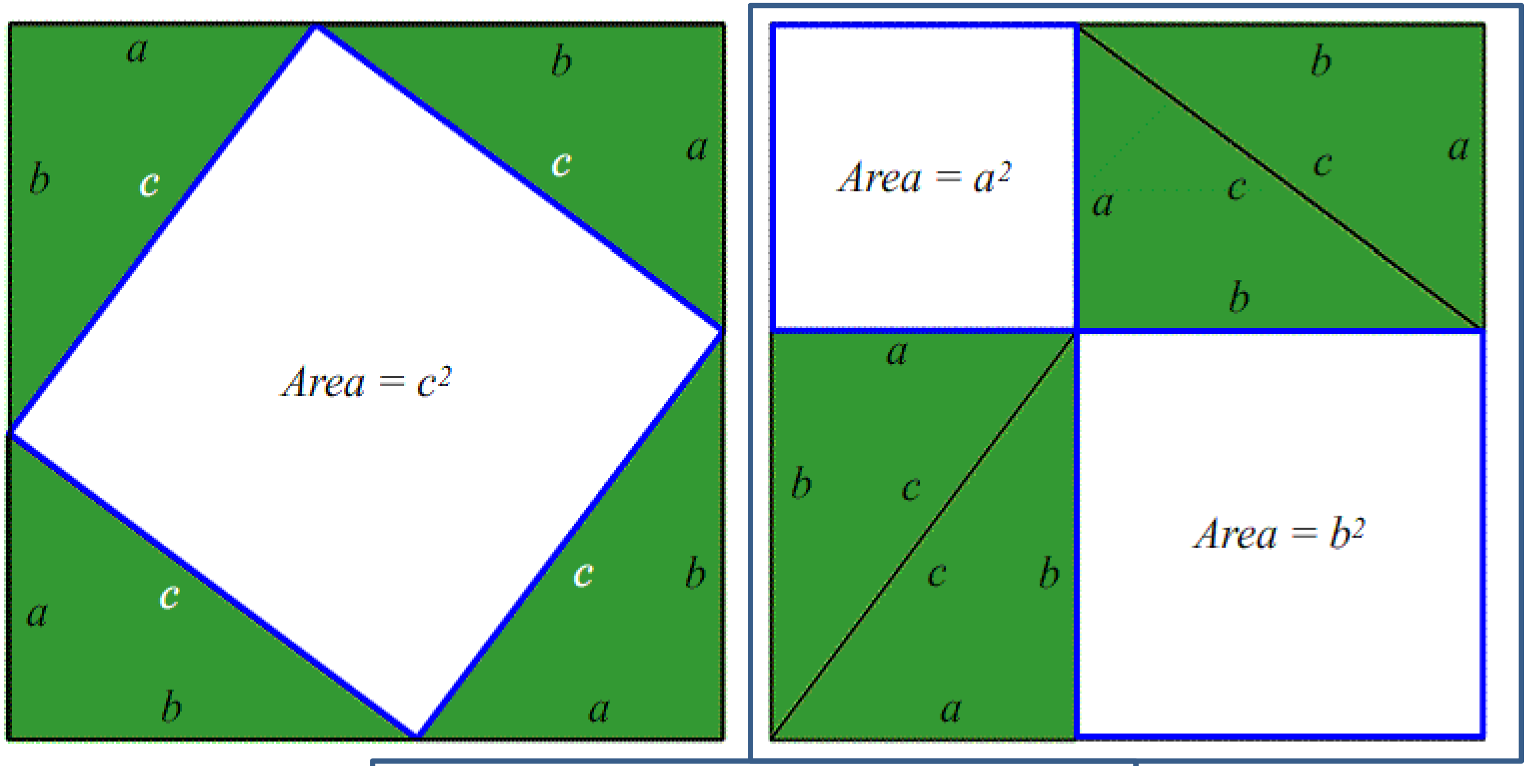




Deriving the formula



The white space has the same area in both diagrams because the triangles do not change size within the containing square. Therefore, $a^2 + b^2 = c^2$

What am I going to take away from today?

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