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Thinking About Maps and Graphs

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www.hants.gov.uk

Overview

In this document

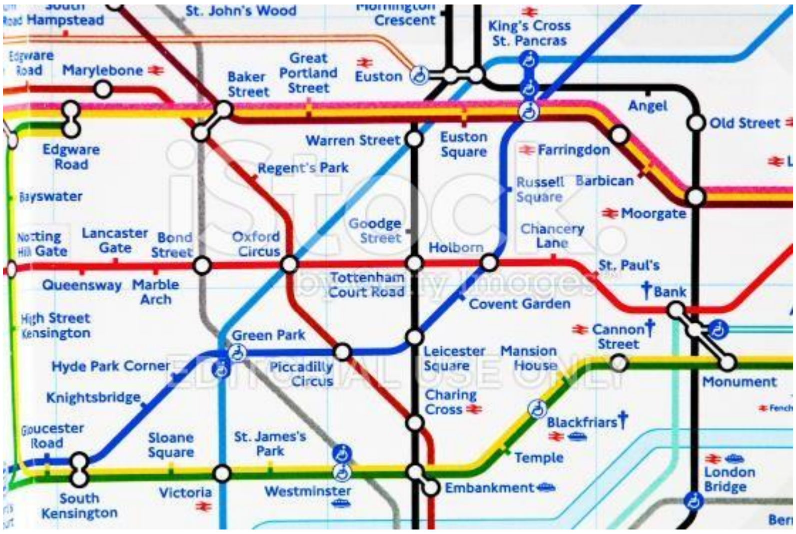
This article is designed to inform and promote thinking around graph theory in a secondary context

Points to consider when using this resource

Teachers should look for other examples of graphs and link this to work on polygons, vectors.

Maps and Graphs

Every now and again, I like to visit the capital city. I travel from Winchester train station to Waterloo and then set off underground. Working out your route on the London Underground requires you to be able to read a very special and very well-known map.



www.istockphoto.com *(London Underground Map, Close up on Zone 1 royalty-free stock photo)*

The Tube map is a schematic transit map of the lines, stations and services of London's public transit systems. The London Underground is the base of the map, but these days the map also includes the Docklands Light Railway and the London Overground network. In newer versions, the Emirates Air Line cable car, TfL Rail and the London Tramlink are also included.

As a schematic diagram, it does not show the geographic locations but rather the relative positions of the stations, lines, the stations' connective relations, and fare zones. The basic design concepts have been widely adopted for other network maps around the world, and for maps of other sorts of transport networks. The precursor for the current map was first designed by Harry Beck in 1931.

Once you have decided on the line you need to take, the map becomes less complex since you only need to see the map of one particular line
Clearly these maps are not geographically accurate and are simple representations of the journey - a schematic.
The London Underground is just over 150 years old but the problem of efficient travel maps and how best to represent them is an old mathematical problem.

The **Seven Bridges of Königsberg** is a historically notable problem in mathematics. The city of Konigsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River and included two large islands which were connected to each other and the mainland by seven bridges. The problem was to devise a walk through the city that would cross each bridge once and only once, with the provisos that: the islands could only be reached by the bridges and every bridge once accessed must be crossed to its other end. The starting and ending points of the walk need not be the same.

In 1736, a famous mathematician called Leonhard Euler proved that the problem has no solution. He did this by reducing the city map to a schematic. This was the foundation of a branch of mathematics called **graph theory**, which then developed the idea of **topology** (sometimes called ‘rubber band mathematics’).

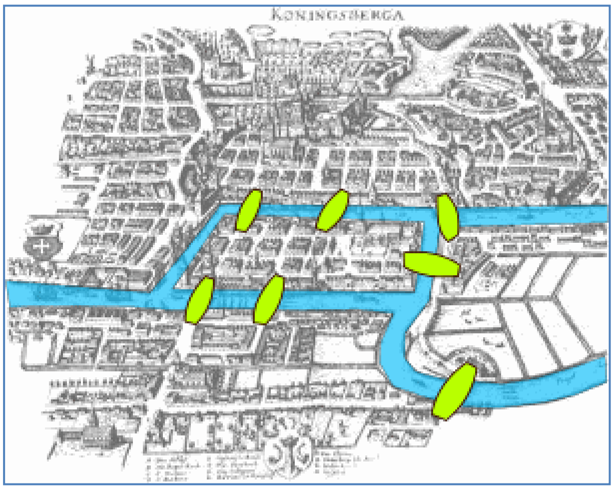


*Image : Bogdan Giuşcă : **GNU Free Documentation License***

This is a map of the seven bridges in Euler’s time. First, Euler pointed out that the choice of route inside each land mass is irrelevant. The only important feature of a route is the sequence of bridges crossed. This allowed him to reformulate the problem in abstract terms (laying the foundations of graph theory), eliminating all features except the list of land masses and the bridges connecting them. In modern terms, one replaces each land mass with an abstract "vertex" or node, and each bridge with an abstract connection, an "edge", which only serves to record which pair of vertices (land masses) is connected by that bridge. The resulting mathematical structure is called a graph.

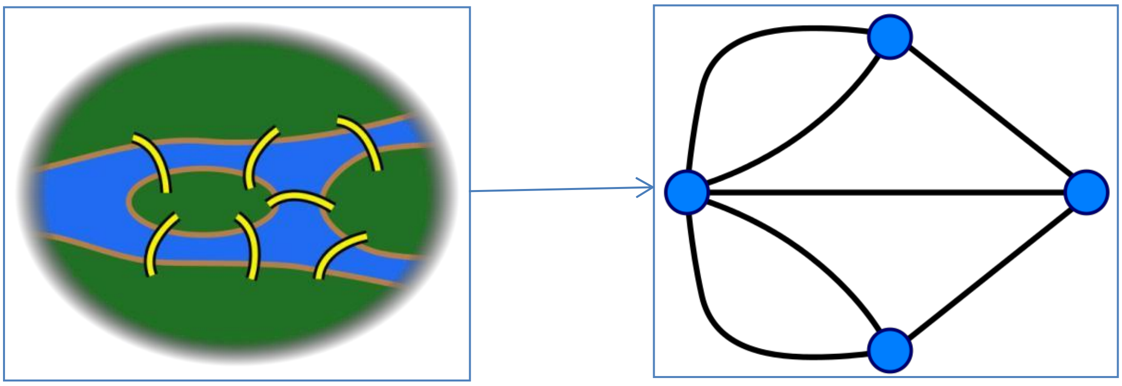


*Image : Bogdan Giuşcă : **GNU Free Documentation License***

Euler now had a schematic version that ignored distance and geographical features and focussed only on how things are connected.

This idea was also developed and explored by William Rowan Hamilton. He looked at polyhedra such as the dodecahedron and tried to create a map (or graph) that travelled along each edge of the dodecahedron visiting each vertex (or node) exactly once. This is called a Hamiltonian path and the dodecahedron problem is known as *Hamilton's puzzle* or the **icosian game.**

You might like to explore routes around different polyhedra. Which ones are 'Hamiltonian' (meaning that you can travel around the edges, visiting each vertex exactly once)? You could start by working with polygons (2 dimensional shapes) first.

Ideas and information to start you off can be found at:
https://en.wikipedia.org/wiki/Hamiltonian_path and http://wild.maths.org/bridges-k%C3%B6nigsberg

You might like to further explore schematic maps, such as the one for the London Underground, thinking about the work of Euler and Hamilton.

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