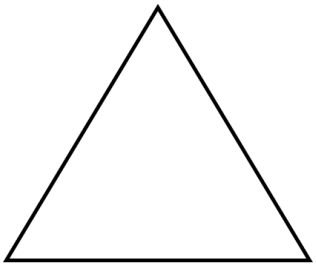
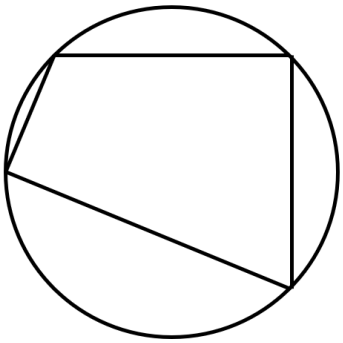
Two-piece jigsaw

Prove by geometry, not by chopping up paper, that two triangles with sides 8,7 and 5cm and 8,7 and 3cm, can be fitted together to form either an equilateral triangle or a cyclic quadrilateral.





Taken from 'Forty Harder Problems for the Classroom',
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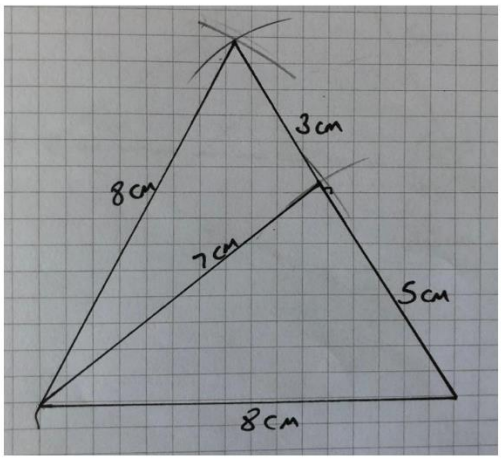
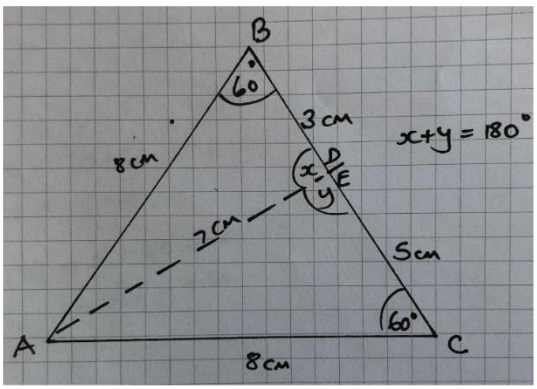


Solution (part one)

Start with a ruler and compasses construction of
a triangle with a base of 8 cm and two sides of 7cm and 5cm.

Now use the side of 7cm to construct a triangle with a base
of 7cm and two sides of 8am and 3cm as shown

This constructs an equilateral triangle of side lengths 8cm



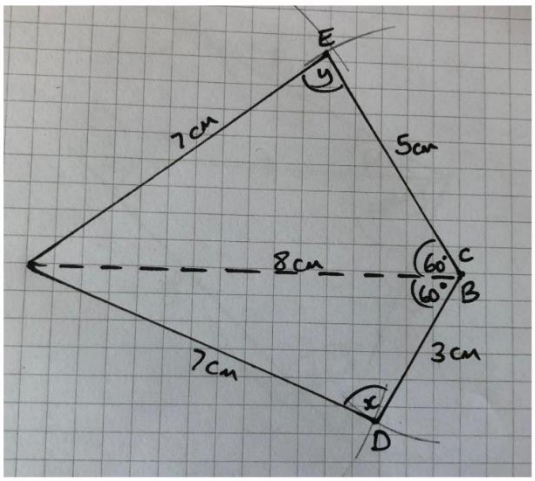
Solution (part two)

Given that triangle ABC is equilateral, since all sides are of length 8cm, label as many known angles as possible.

There are 60^o angles at vertices B and C, since the triangle is equilateral.

BC is a straight line of length 8cm, with BD = 3cm and EC = 5cm

Since BC = BD + EC, then the angles at D and E must sum to 180^o
So $x + y = 180^o$



Solution (part three)

To show a cyclic quadrilateral, we can use the angle knowledge gained from the equilateral triangle, using the same labels

We need to re-position the two triangles as shown,
so that both have a base of 8cm.

We know that $x + y = 180^o$

This satisfies the condition for a cyclic quadrilateral since pairs of opposite angles sum to 180^o in a cyclic quad.

It follows that the other pair of opposite angles must also sum to 180^o since the angle sum of a quadrilateral is 360^o