Secondary Puzzle Page – Solutions

Same Surface, Different Depth Problems

These linked problems are taken from Craig Barton's excellent website:
https://ssddproblems.com/the-cubic-equation/

The Cubic Equation with solutions

Show that the equation $x^3 - 7x + 5 = 0$ has a solution between $x = 2$ and $x = 3$

$f(2) = -1$
$f(3) = 11$
Change in sign and function is continuous, therefore, root must be in interval [2,3]

$f(x) = x^3 - 7x + 5$
$g(x) = x - 1$

Find $fg(x)$

$fg(x) = x^3 - 3x^2 - 4x + 11$

Let $x_{n+1} = x_n^3 - 7x_n + 5$
Given that $x_0 = 2$, find x_3 to 3 significant figures

$x_0 = 2$
$x_1 = -1$
$x_2 = 11$
$x_3 = 1259$
$x_3 = 1260$ (to 3 sig figs)

Find the remainder when $x^3 - 7x + 5$ is divided by $(x - 5)$
[Further Maths GCSE]

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