

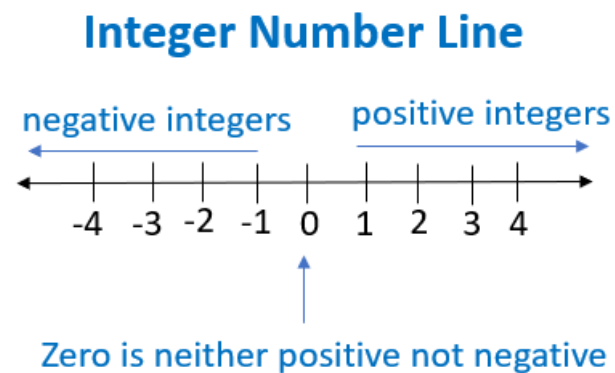
integer

-15 , 7 , 43
are all **integers**



Picture, model, or diagram

Non-Example



0.5 , -6.2 , 81.9
are not **integers**

divisor

$$18 \div 3 = 6$$

3 is the **divisor**.

Picture, model, or diagram

18		
6	6	6

18 divided between **three**
The number of groups is the divisor



$$18 \div 3 = 6$$

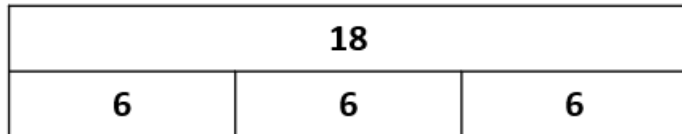
18 is the **dividend**

dividend

$$18 \div 3 = 6$$

18 is the **dividend**

Picture, model, or diagram



18 divided between three
The original amount to be divided is the dividend



$$18 \div 3 = 6$$

3 is the **divisor**.

quotient

$$18 \div 3 = 6$$

6 is the **quotient**

Picture, model, or diagram



Non-Example

18		
6	6	6

18 divided between three gives **six** in each group
The size of the group is the quotient
(The 'answer' to a division calculation)

$$18 \div 3 = 6$$

18 is the **dividend**

lowest common multiple (LCM)

The **LCM** of 24 and 36 is **72**

Picture, model, or diagram

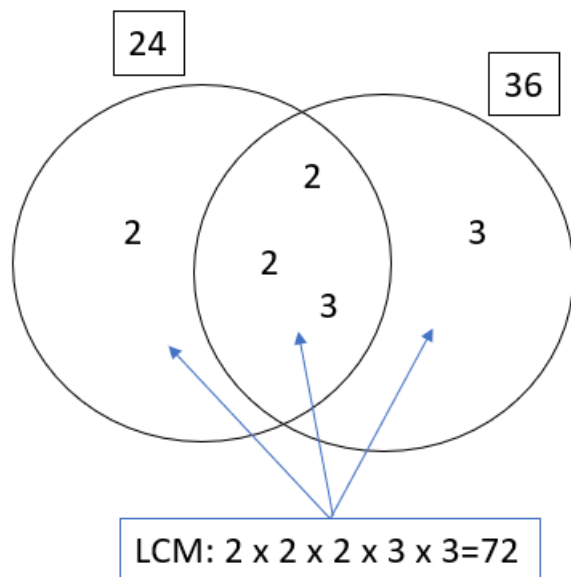


Non-Example

Prime Factor Form

$$24 = 2 \times 2 \times 2 \times 3$$

$$36 = 2 \times 2 \times 3 \times 3$$



The **HCF** of 24 and 36 is **12**
(highest common factor)

highest common factor (HCF)

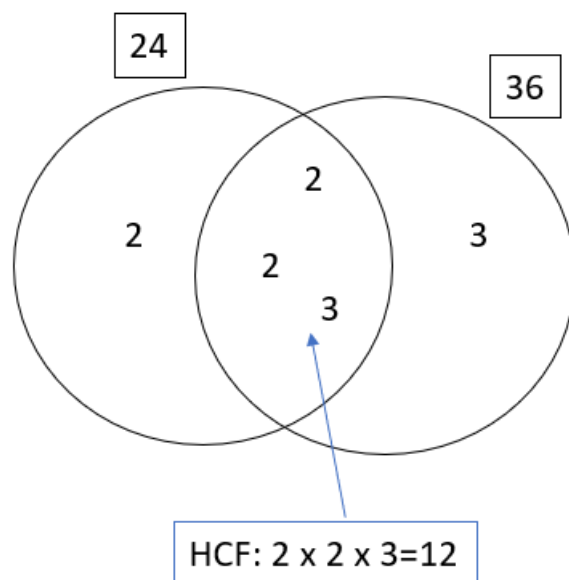
The **HCF** of 24 and 36 is **12**

Picture, model, or diagram



Non-Example

Prime Factor Form
 $24 = 2 \times 2 \times 2 \times 3$
 $36 = 2 \times 2 \times 3 \times 3$



The **LCM** of 24 and 36 is **72**
 (lowest common multiple)

square root

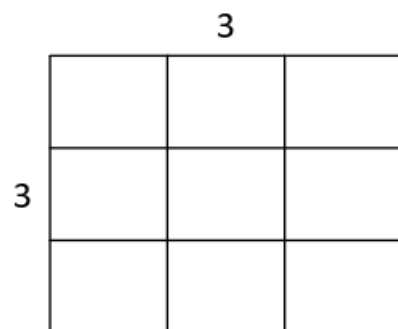
$$\sqrt{9} = + / - 3$$

$$\left\{ \begin{array}{l} 3 \times 3 = 9 \\ -3 \times -3 = 9 \end{array} \right\}$$

Picture, model, or diagram



Non-Example



$$3 \times 3 = 3^2$$

$$3 \times 3 = 9$$

$$\sqrt{3^2} = 3$$

$$3\sqrt{8} = 2$$

$$\{ 2 \times 2 \times 2 = 8 \}$$

cube root

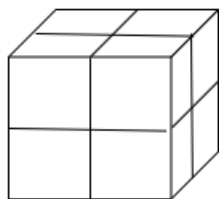
$${}^3\sqrt{8} = 2$$

$$\{2 \times 2 \times 2 = 8\}$$

Picture, model, or diagram



Non-Example



$$2 \times 2 \times 2 = 2^3$$

$$2 \times 2 \times 2 = 8$$

$${}^3\sqrt{8} = 2$$

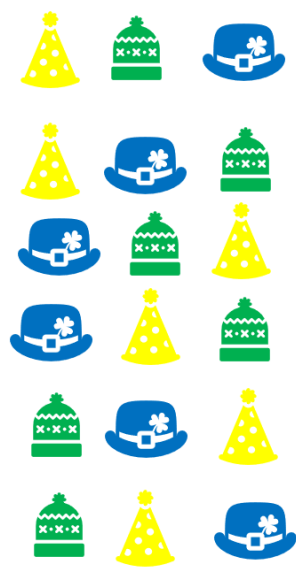
$$\sqrt{9} = + / - 3$$

$$\left\{ \begin{array}{l} 3 \times 3 = 9 \\ -3 \times -3 = 9 \end{array} \right\}$$

product rule

(for counting)

The number of different ways 3 hats can be distributed amongst 3 people is $3 \times 2 \times 1 = 6$



Person 1 has three choices (YGB)
Person 2 then has two choices (GB)
Person 2 than has one choice (B)

The probability of selecting yellow, then green, then blue is $\frac{1}{6}$

index (indices)

$$14^3 = 14 \times 14 \times 14 = 2744$$

14 is the base

3 is the index (or power)

Picture, model, or diagram



Non-Example

$$x^1 = 14$$

x^4	x^3	x^2	x^1	x^0
38416	2744	196	14	1

$$14 \times 3 = 42$$

surd

$$\sqrt{3}$$

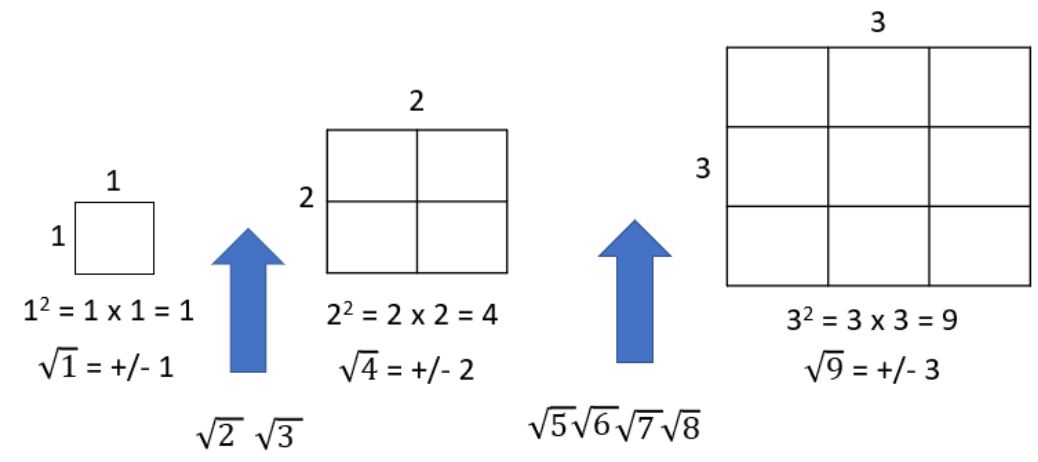
{3 is an 'imperfect square'}

{ A surd is not a whole number . It's decimal equivalent is a non-repeating, non-terminating decimal.
You cannot place it accurately on a number-line. }



Picture, model, or diagram

Non-Example



$$\sqrt{4}$$

{4 is an 'perfect square' and can be evaluated as +/- 2 }

rationalise (denominator)

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

3 is a rational number ~ you can place it on a number-line



Picture, model, or diagram

Non-Example

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

$$\frac{\sqrt{b}}{\sqrt{b}} = 1$$

$$\frac{2}{\sqrt{3}}$$

This fraction has an irrational (surd) denominator
A surd is not a whole number . It's decimal equivalent
is a non-repeating, non-terminating decimal. You cannot
place it accurately on a number-line

standard (index) form

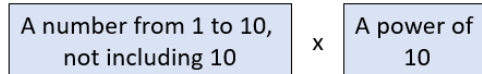
$$3.754 \times 10^5$$



Picture, model, or diagram

Non-Example

Numbers in standard form have two parts



Standard index form	Ordinary form
1×10^3	1000
1×10^2	100
1×10^1	10
1×10^0	1
1×10^{-1}	0.1
1×10^{-2}	0.01
1×10^{-3}	0.001

375 400

upper bound

The length of a book is 27.3 cm to one decimal place
What is the **upper bound** of the book length?

The upper bound is +0.05cm

$27.25 \leq \text{book length} < 27.35$

The **upper bound** is 27.35cm



Picture, model, or diagram

Non-Example

The length of a book is 27.3 cm to one decimal place
What is the longest length the book could be ?

27.25 27.26 27.27 27.28 27.29 27.3 27.31 27.32 27.33 27.34 27.35



Any number above
(and including) this value
rounds to 27.3 to one decimal place

LOWER BOUND

27.25

Any number below
(and **not** including) this value
rounds to 27.3 to one decimal place

UPPER BOUND

27.35

The lower bound is - 0.05cm

$27.25 \leq \text{book length} < 27.35$

The lower bound is 27.25cm

lower bound

The length of a book is 27.3 cm to one decimal place
What is the **lower bound** of the book length?

The lower bound is - 0.05cm

$27.25 \leq \text{book length} < 27.35$

The **lower bound** is 27.25cm



Picture, model, or diagram

Non-Example

The length of a book is 27.3 cm to one decimal place
What is the longest length the book could be ?

27.25 27.26 27.27 27.28 27.29 27.3 27.31 27.32 27.33 27.34 27.35



Any number above
(and including) this value
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LOWER BOUND

27.25

Any number below
(and **not** including) this value
rounds to 27.3 to one decimal place

UPPER BOUND

27.35

The upper bound is +0.05cm

$27.25 \leq \text{book length} < 27.35$

The upper bound is 27.35cm

substitute

Evaluate $7x - 10$ when $x = 12$

substitute $x = 12$ into the expression

$$7(12) - 10 = 74$$



x	x	x	x	x	x	x	-10
12	12	12	12	12	12	12	-10

74

Find x when $7x - 10 = 74$

Solve for x

$$7x = 84 \quad (\div 7)$$

$$x = 12$$

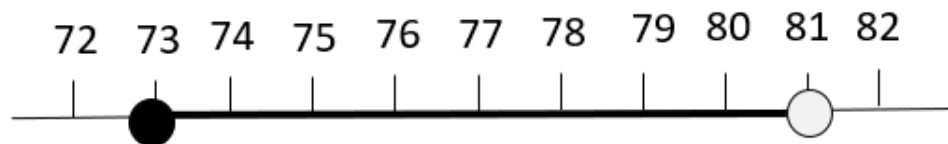
inequality

$$73 \leq t < 81$$

Picture, model, or diagram



Non-Example



$$t = 80$$

expand (brackets)

$$(x + 3)(x - 5) = x^2 - 2x - 15$$



Picture, model, or diagram

Non-Example

F = first pair
O = outer pair
I = inner pair
L = last pair

F $(x+3)(x-5) = x^2$

O $(x+3)(x-5) = x^2 - 5x$

I $(x+3)(x-5) = x^2 - 5x + 3x$

L $(x+3)(x-5) = x^2 - 5x + 3x - 15$

	x	+3
x	x^2	$+3x$
-5	$-5x$	-15

$$3 \times 4 + 7 \times 4 = 10 \times 4$$

factorise

$$x^2 - 2x - 15 = (x + 3)(x - 5)$$



Picture, model, or diagram

Non-Example

Factor pairs to -15

$$-15 = +3x - 5$$

$$-15 = -3x + 5$$

$$-15 = -1x + 15$$

$$-15 = +1x - 15$$

	x	+3
x	x^2	$+3x$
-5	$-5x$	-15

Which pair also sum to -2
For the x-term?

$$-2 = +3 - 5$$

$$10 \times 4 = 3 \times 4 + 7 \times 4$$

quadratic (function)

$$y = 3x^2 + 5x - 6$$

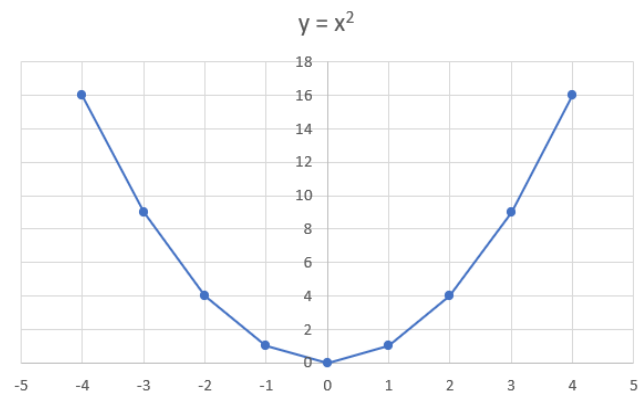
In general , **quadratic functions** are of the form

$$y = ax^2 + bx - c$$



Picture, model, or diagram

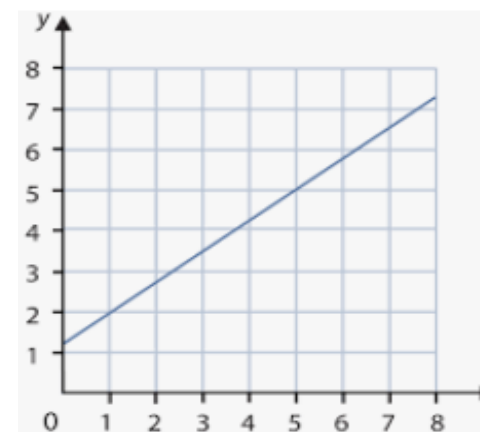
Non-Example



$$y = x^2$$

$$y = ax^2 + bx - c$$

(a = 1; b = 0; c = 0)



$$y = x + 1$$

n^{th} term

15, 18, 21, 24, 27 ...

The n^{th} term of this sequence is
 $3n+12$

$$U_n = 3n+12$$

$$n=1, U_1 = 3(1) + 12 = 15$$

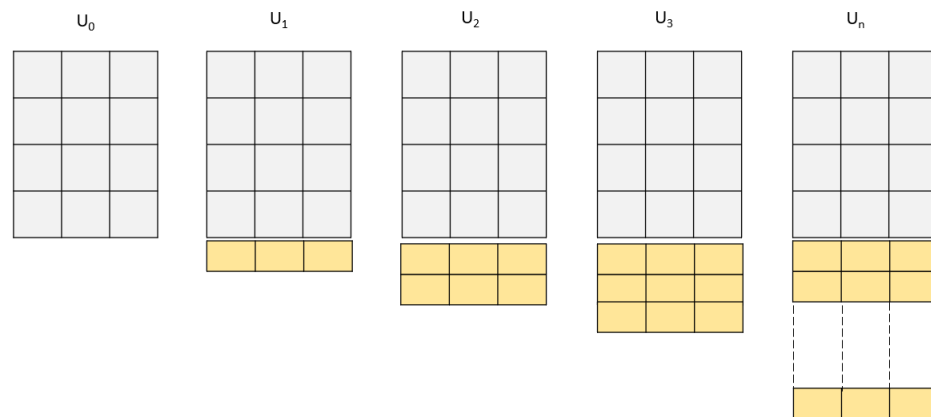
$$n=2, U_2 = 3(2) + 12 = 18$$

$$n=3, U_3 = 3(3) + 12 = 21$$



Picture, model, or diagram

Non-Example



$$U_n = 3n+12$$

15, 18, 21, 24, 27 ...

The term-to-term rule for this sequence is 'add 3'

algebraic fraction

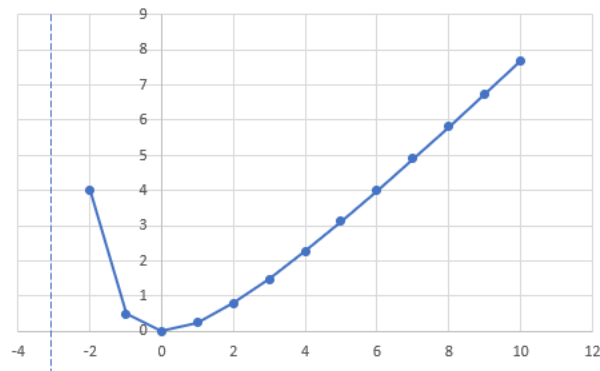
$$\frac{x^2}{x+3}$$

Picture, model, or diagram



Non-Example

$$y = \frac{x^2}{x+3} \text{ for } x > -3$$



This graph has an asymptote at $x=-3$

$$2x(x+4)$$

identity (\equiv)

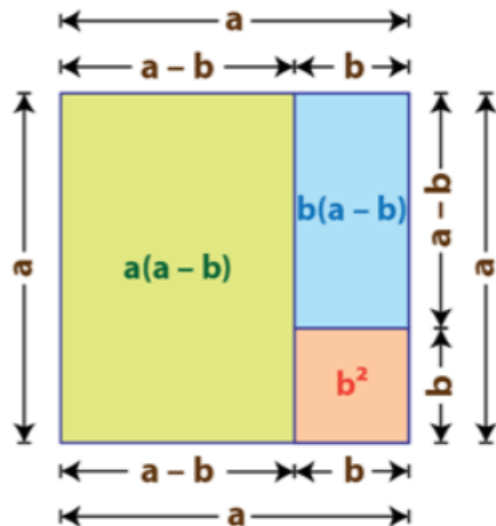
$$a^2 - b^2 \equiv (a + b)(a - b)$$

$$(a - b)(a + b) = a^2 + ab - ab - b^2$$



Picture, model, or diagram

Non-Example



$$ab \approx 17$$

function

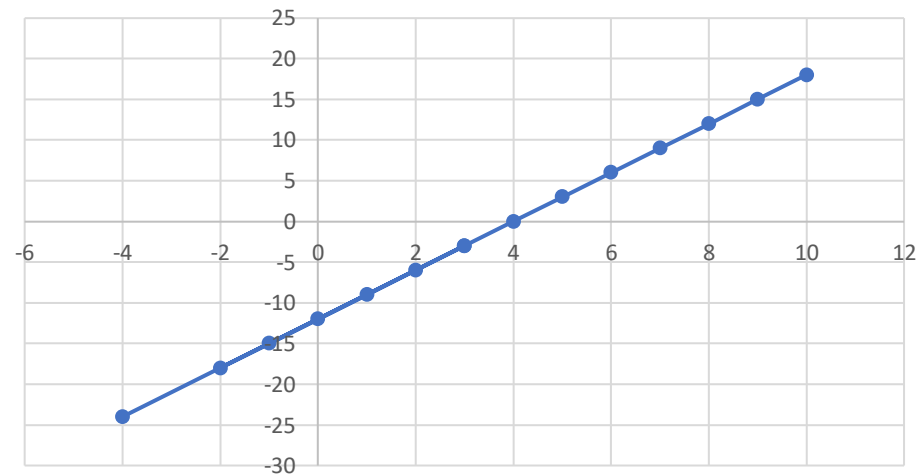
$$f(x) = 3x - 12$$



Picture, model, or diagram

Non-Example

$$f(x) = 3x - 12$$



$$3x - 12$$

inverse function

$$f^{-1}(x)$$

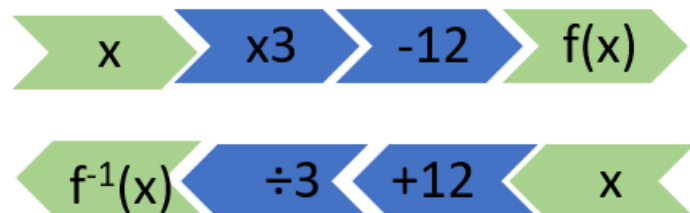
$$f(x) = 3x - 12$$

$$f^{-1}(x) = \frac{x + 12}{3}$$



Picture, model, or diagram

Non-Example



$$3x - 12$$

composite function

$gf(x)$

$$f(x) = x + 4$$

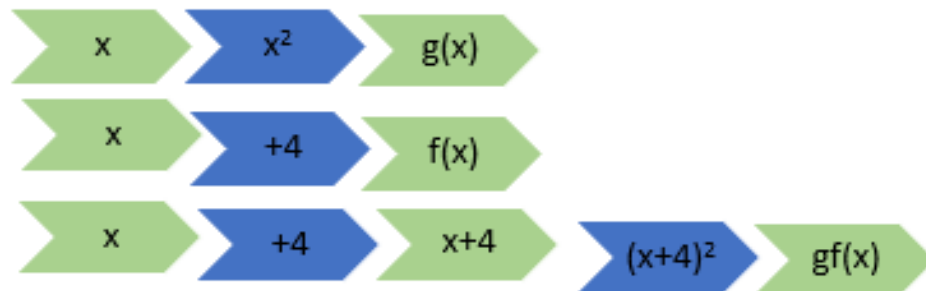
$$g(x) = x^2$$

$$gf(x) = (x + 4)^2$$

Picture, model, or diagram



Non-Example



$$g(x) = x^2$$

turning point

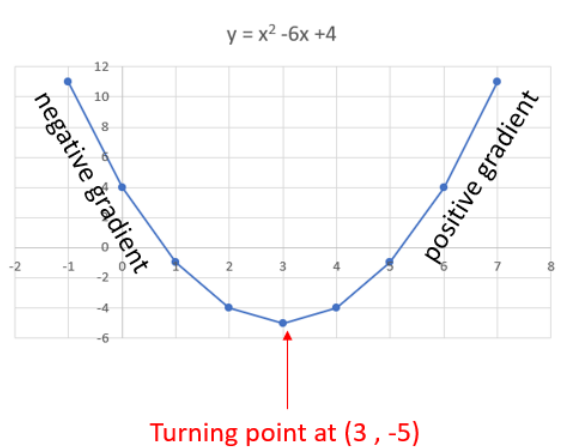
$$y = x^2 - 6x + 4$$

Complete the square

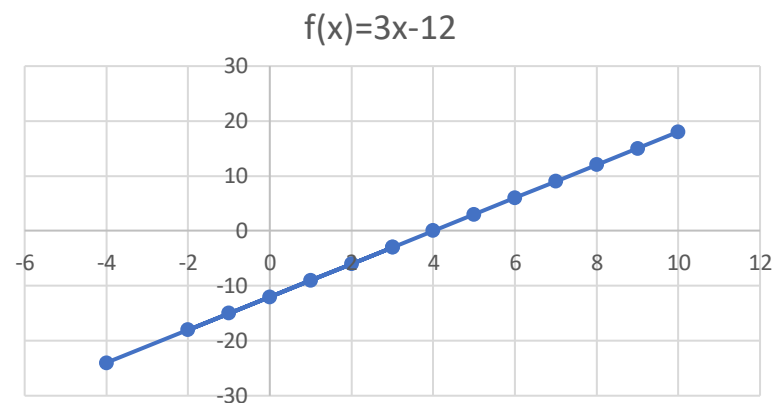
$$y = (x-3)^2 - 5$$

turning point is at (3,-5)

Picture, model, or diagram



Non-Example



This linear graph has no turning point

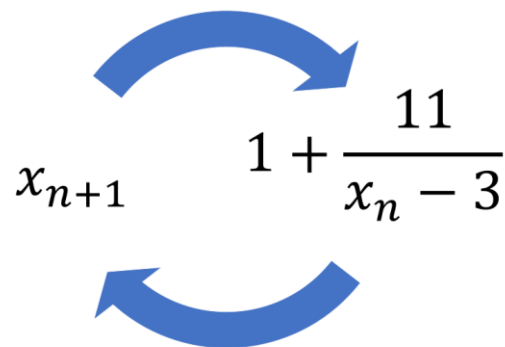
iteration

Use the **iterative formula** $x_{n+1} = 1 + \frac{11}{x_n - 3}$
and the starting value $x_1 = -2$ to find a value for x_4

$$x_2 = 1 + \frac{11}{-2-3} \quad x_2 = -1.2$$

$$x_3 = 1 + \frac{11}{-1.2-3} \quad x_3 = -1.619$$

$$x_4 = 1 + \frac{11}{-1.619-3} \quad x_4 = -1.381$$

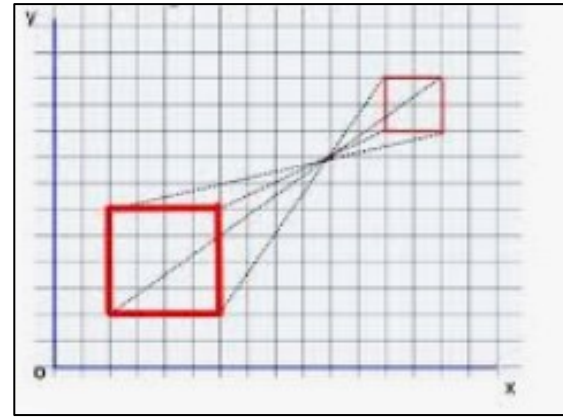


Substitute $x = -2$ into the equation
 $y = 1 + \frac{11}{x-3}$ to find the value of y

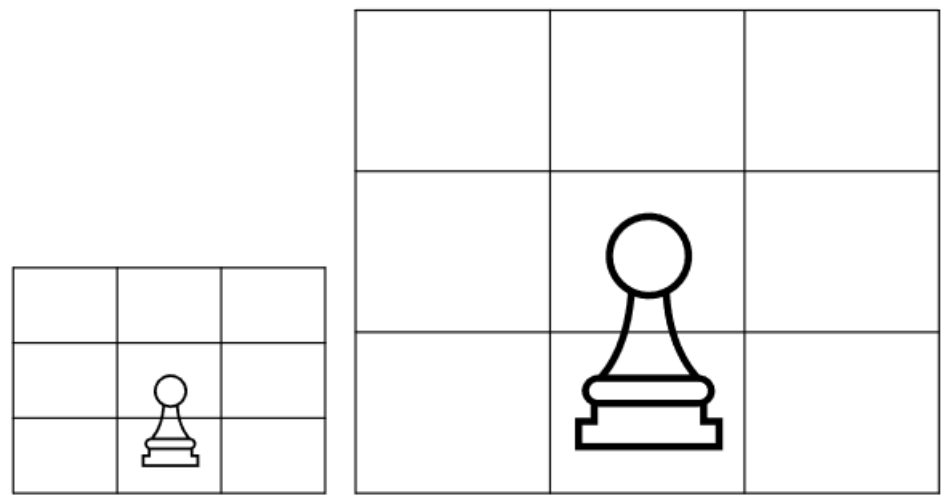
$$y = 1 + \frac{11}{-2-3}$$


$$y = -1.2$$

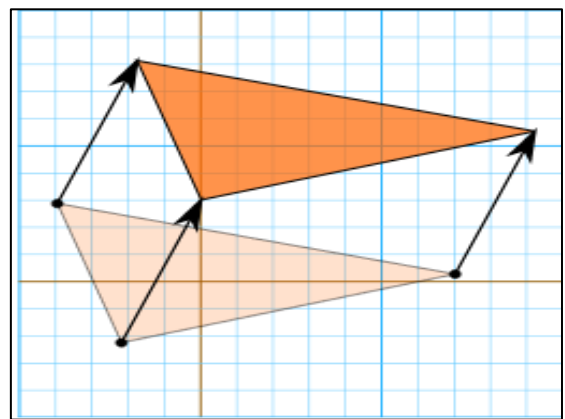
scale factor



This is an enlargement with a **scale factor of -2**

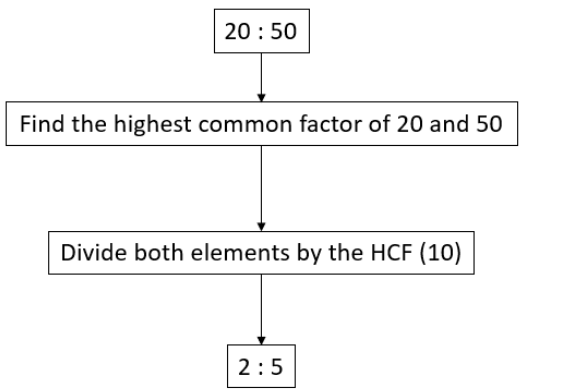
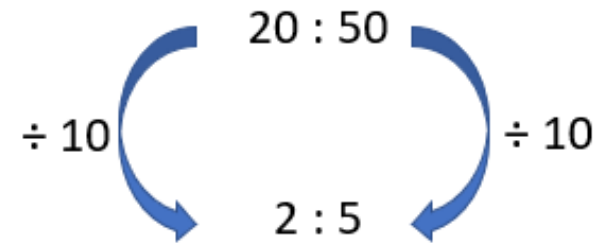


Scale factor 2  corresponding edges are twice as long

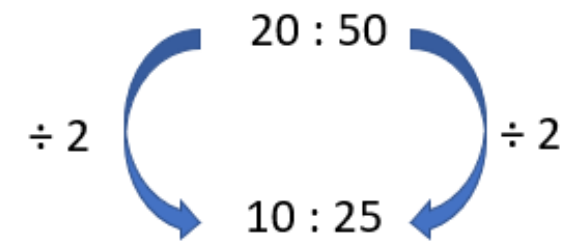


This is a translation.

reduce (to simplest form)



10	10	10	10	10	10	10
2		5				



percentage (decrease)

A watch is bought at a car boot sale for £50. It is later sold in a shop for £25. What is the percentage loss?

The watch has decreased by £25

The **percentage decrease** is

$(\text{difference} / \text{original}) \times 100 = 25/50 \times 100 = 50\%$

A 50% loss has been made.



original cost	
sold price	loss (difference)

The **percentage decrease** is $(\text{difference} \div \text{original}) \times 100$

A watch is bought at a car boot sale for £40. It is later sold in a shop for £50. What is the percentage profit?

The watch has increased by £10

The **percentage increase** is

$(\text{difference} / \text{original}) \times 100 = 10/40 \times 100 = 25\%$

25% profit has been made.

percentage (increase)

A watch is bought at a car boot sale for £40. It is later sold in a shop for £50. What is the percentage profit?

The watch has increased by £10

The **percentage increase** is

$(\text{difference} / \text{original}) \times 100 = 10/40 \times 100 = 25\%$

25% profit has been made.



sold price	
original cost	difference

The **percentage increase** is $(\text{difference} \div \text{original}) \times 100$

A watch is bought at a car boot sale for £50. It is later sold in a shop for £25. What is the percentage loss?

The watch has decreased by £25

The **percentage decrease** is

$(\text{difference} / \text{original}) \times 100 = 25/50 \times 100 = 50\%$

A 50% loss has been made.

compound (units)

Calculate the density of aluminium if 20 cm³ has a mass of 54 g.

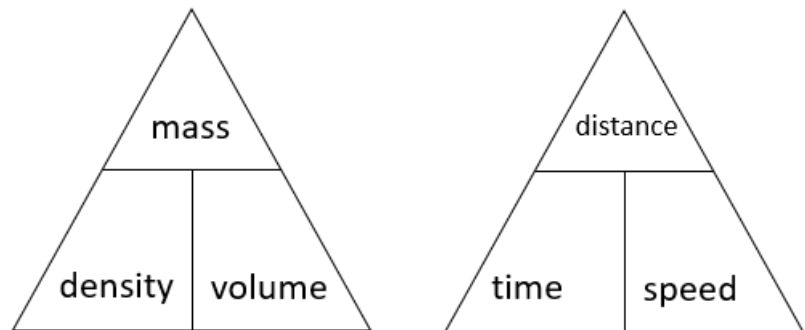
$$\begin{aligned} \text{Density} &= \text{mass} \div \text{volume} \\ &= 54 \div 20 \end{aligned}$$

$$= 2.7 \text{ g / cm}^3 \text{ (grams per cubic centimetre)}$$

Density is measured using **compound units**



Picture, model, or diagram



Non-Example

The mass of one apple is 10 grams

Calculate the mass of 5 apples

$$\underline{10\text{g} \times 5 = 50\text{g}}$$

directly proportional

y is directly proportional to x

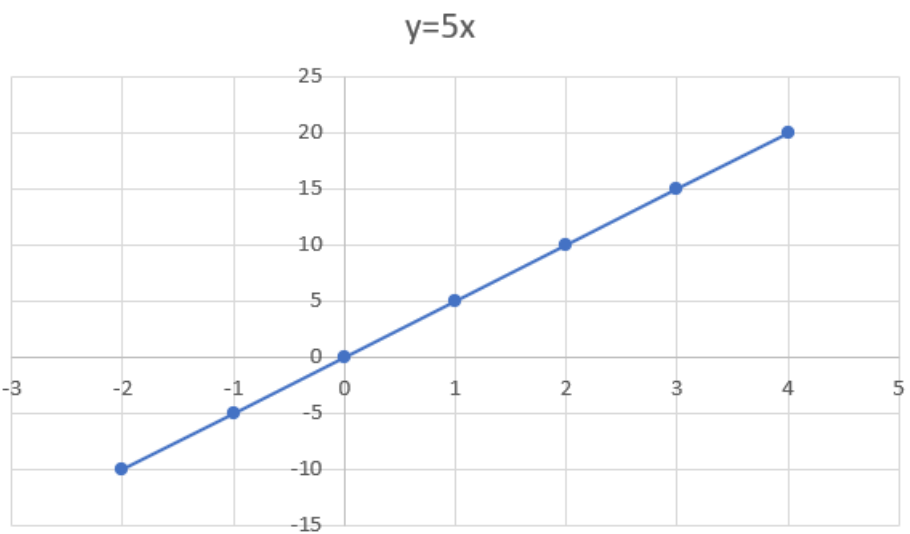
when $x = 3$, $y = 15$

write an equation for y in terms of x

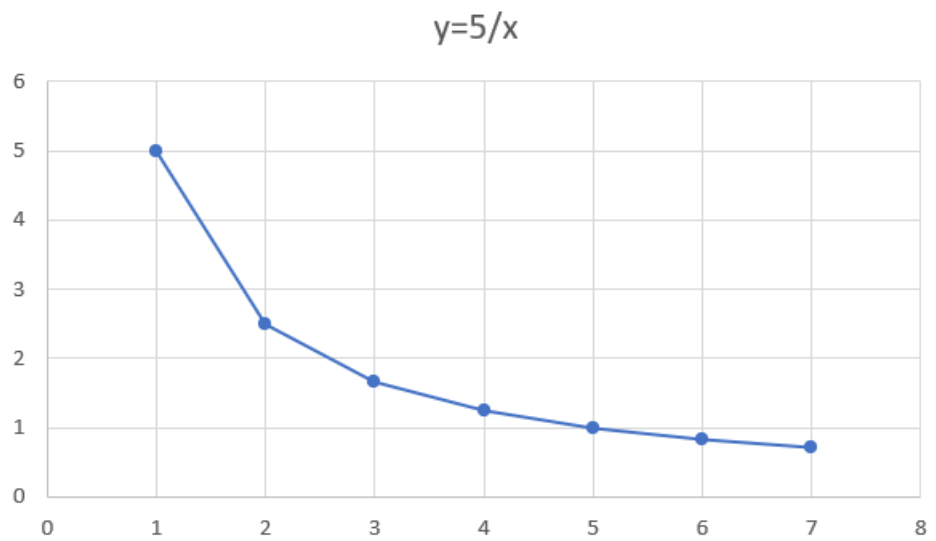


Picture, model, or diagram

Non-Example



This graph shows direct proportion



This graph shows inverse proportion

inversely proportional

y is **inversely proportional** to x

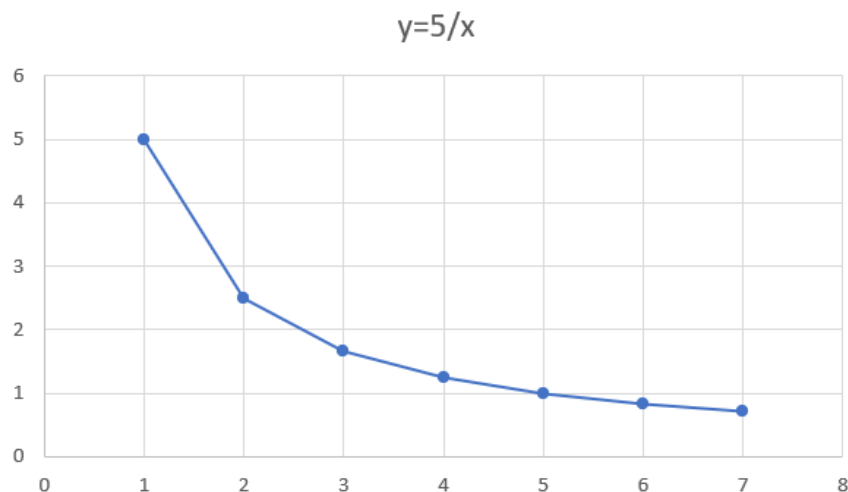
when $x = 5$, $y = 1$

write an equation for y in terms of x

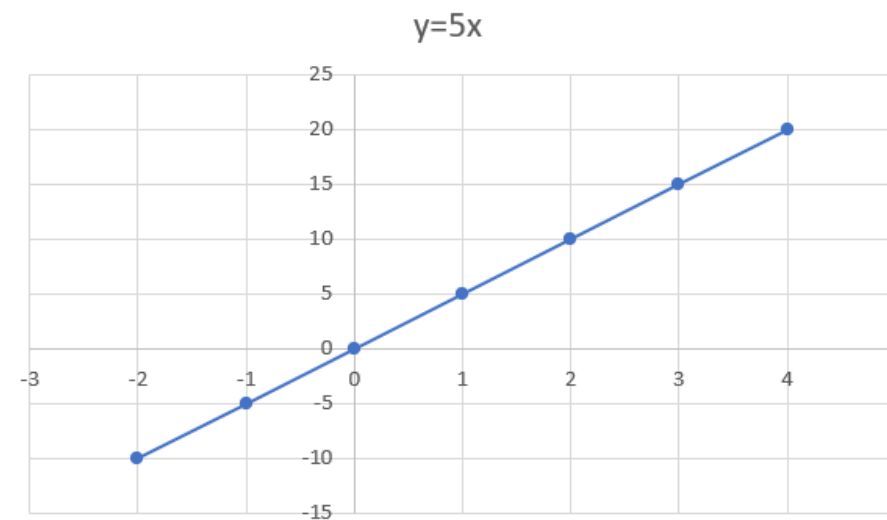
$$\begin{array}{|l}
 y \propto \frac{1}{x} \\
 \\
 y = \frac{k}{x}
 \end{array}
 \longrightarrow
 \begin{array}{|l}
 1 = \frac{k}{5} \\
 \\
 k = 5 \\
 \\
 y = \frac{5}{x}
 \end{array}$$

Picture, model, or diagram

Non-Example



This graph shows inverse proportion



This graph shows direct proportion

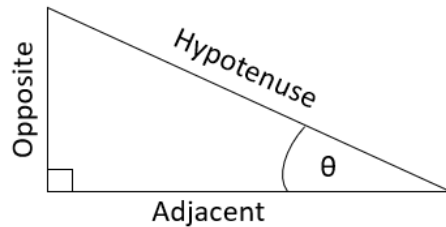
trigonometric ratio (trigonometry)

$$\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

Picture, model, or diagram

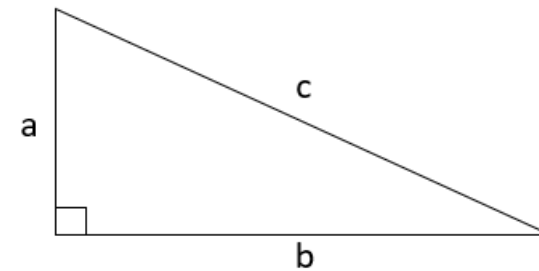


Non-Example



SoH CaH ToA

$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$



$$a^2 + b^2 = c^2$$

exponential growth

A population of rabbits doubles every month.

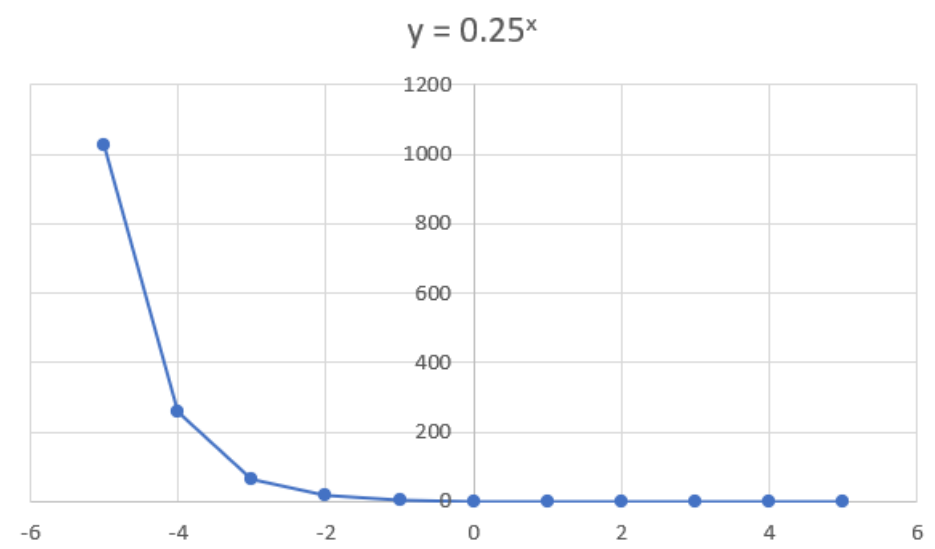
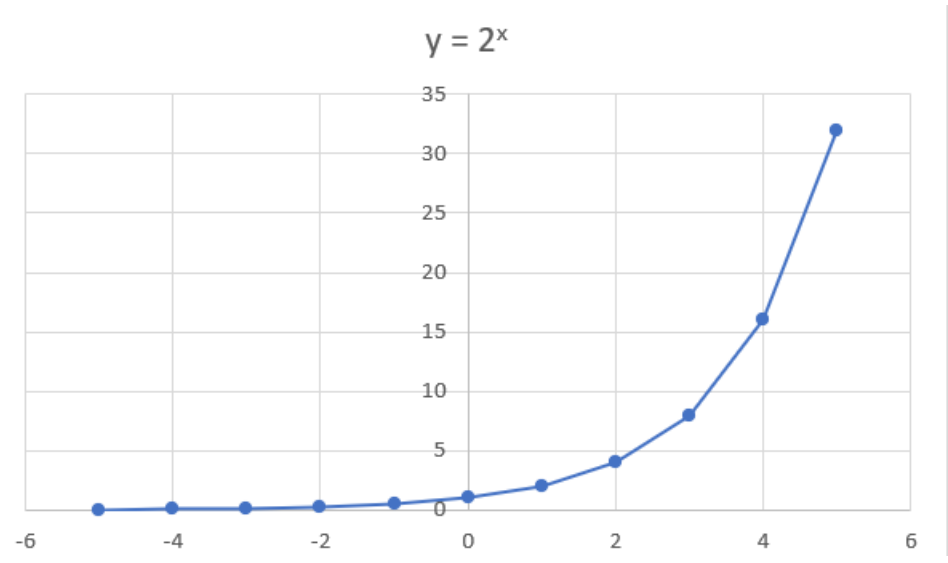
The population is **growing exponentially**.

Month	Number of rabbits
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$
4	$16 = 2^4$
5	$32 = 2^5$
6	$64 = 2^6$
n	2^n



Picture, model, or diagram

Non-Example



exponential decay

Year	Grams of salt
1	2500000
2	1250000
3	625000
4	312500
5	156250
6	78125
7	39063
8	19531
9	9766
10	4883
11	2441
12	1221
13	610
14	305
15	153

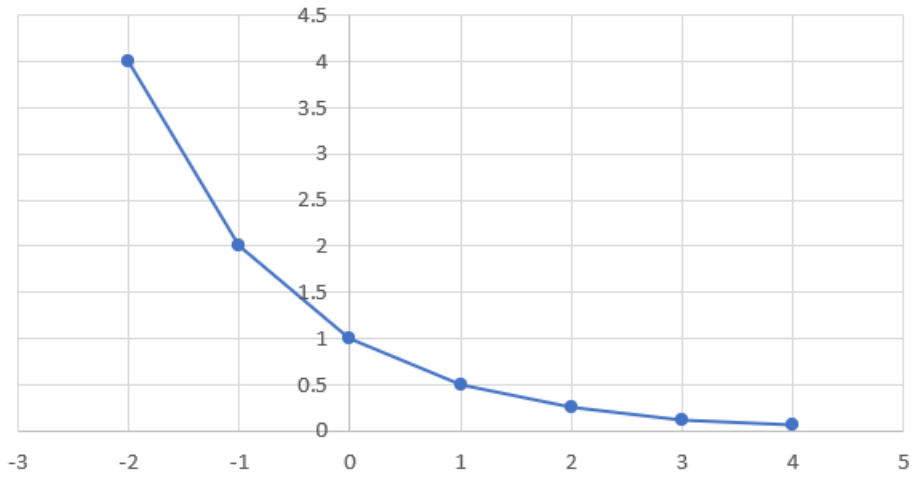
A restaurant decreases its salt use by 50% per year. In year 0, the restaurant uses 5,000,000 grams of salt. Salt usage is **decaying exponentially**.



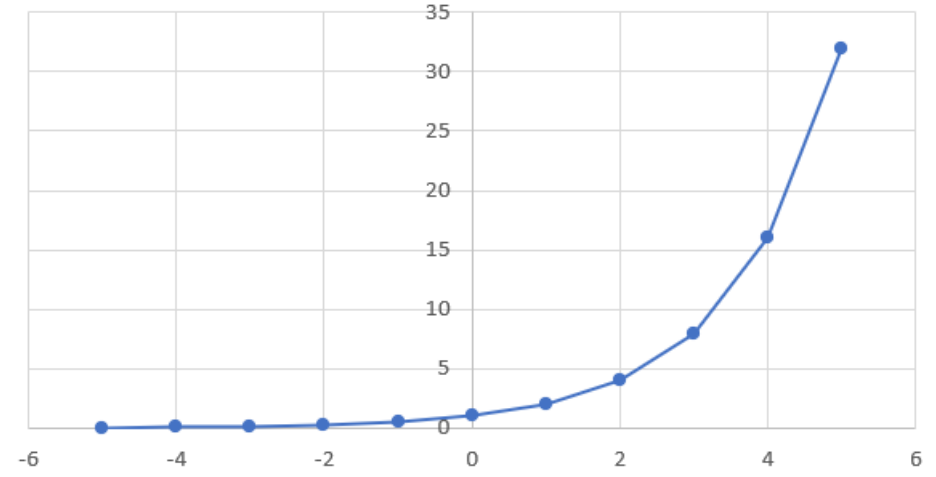
Picture, model, or diagram

Non-Example

$y=0.5^x$



$y = 2^x$



Word

Example

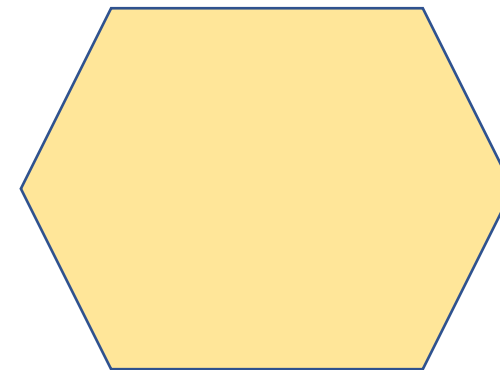
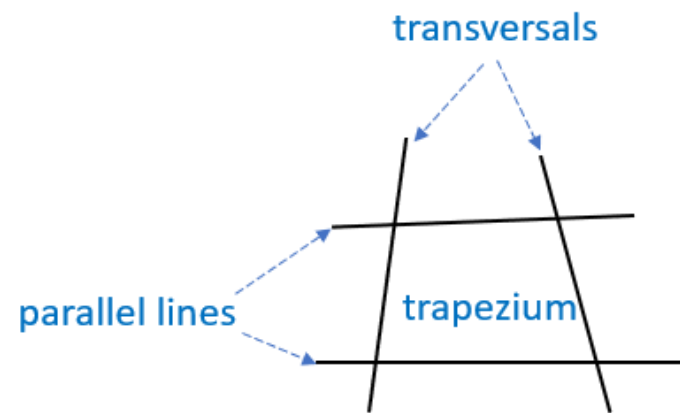
trapezium (-a)



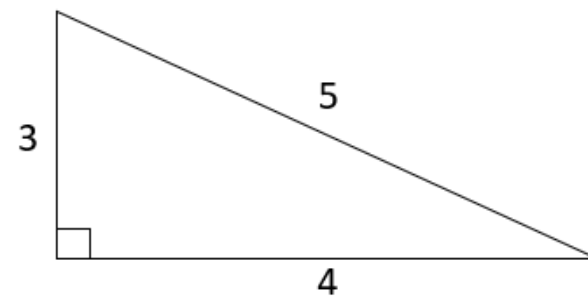
Picture, model, or diagram



Non-Example



Pythagoras' Theorem

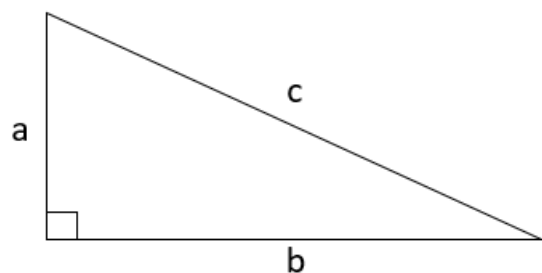


$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$

Picture, model, or diagram



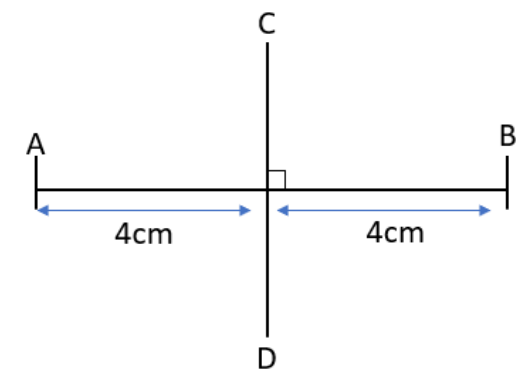
Non-Example



$$a^2 + b^2 = c^2$$

$$\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

perpendicular bisector



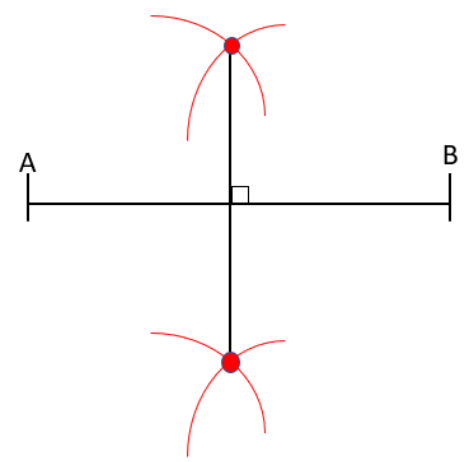
AB = 8cm

CD bisects AB at 90°

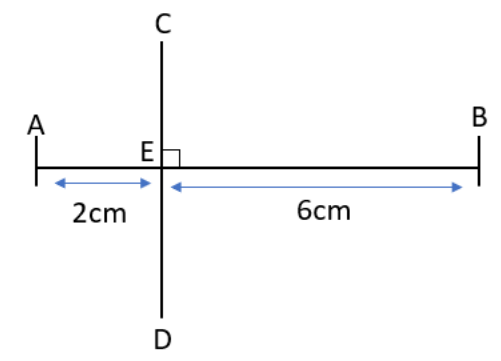
Picture, model, or diagram



Non-Example



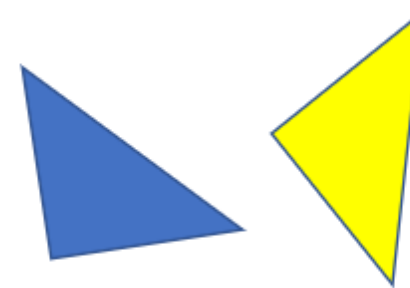
Use a pair of compasses.
 Place the point on A and open the compasses to over half way
 Make an arc above and below the line.
 Repeat from B
 Join the arcs



AB = 8cm

CD intersects AB at 90° at E

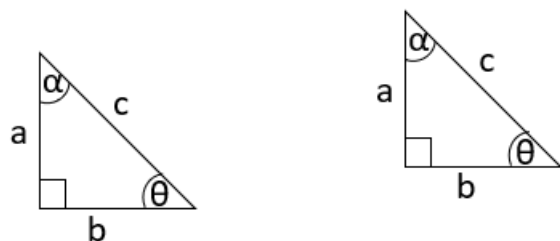
congruent



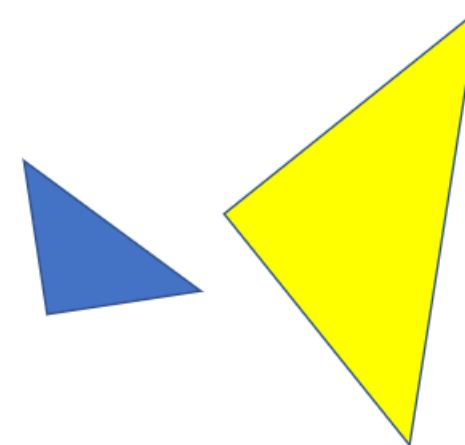
Picture, model, or diagram



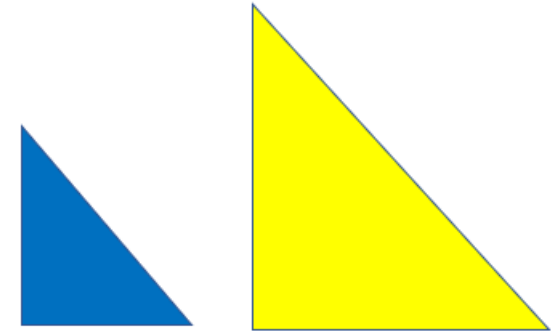
Non-Example



These triangles are congruent
All corresponding angles are the same size
All corresponding sides are the same length



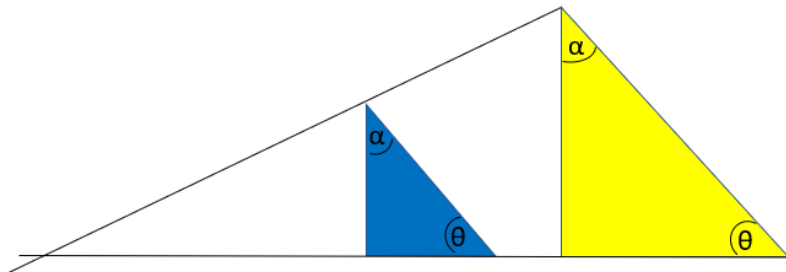
similar



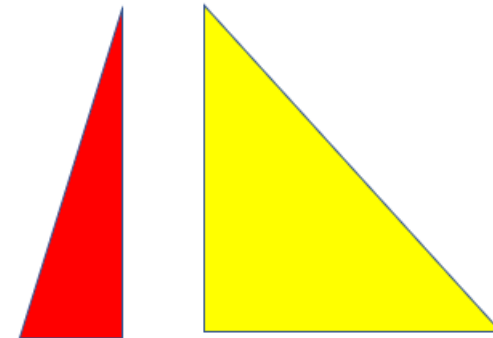
Picture, model, or diagram



Non-Example



These triangles are similar
All corresponding angles are the same
All corresponding sides are proportional



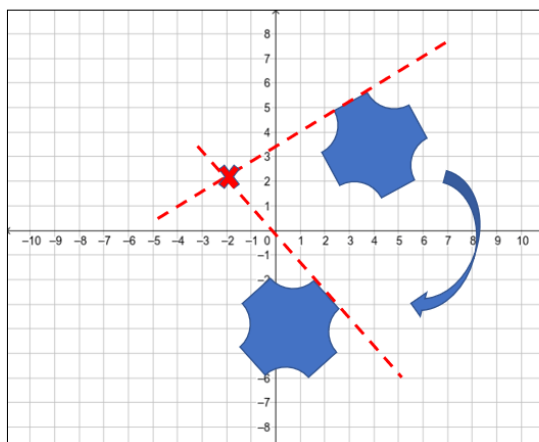
rotation



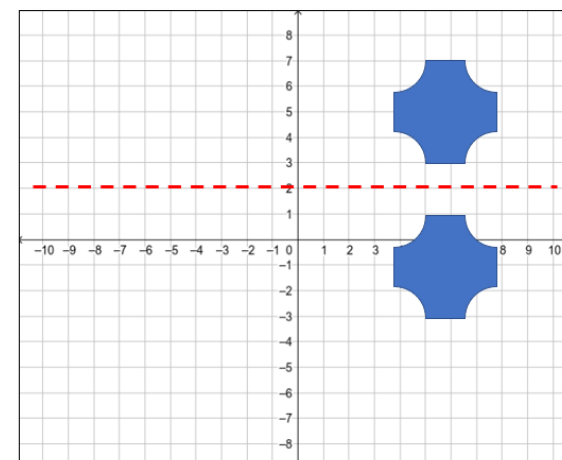
Picture, model, or diagram



Non-Example



This is a rotation of 90° clockwise about the point $(-2, 2)$

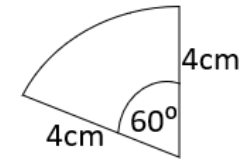


Word

sector

Example

Calculate the area of this **sector** to 1 decimal place

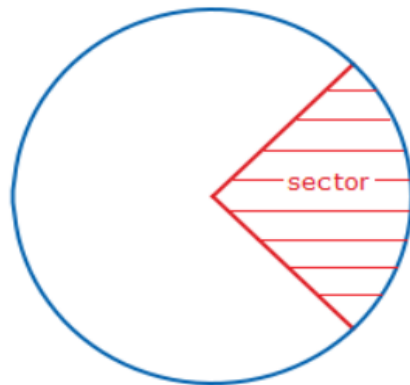


Sector area is $\frac{60}{360}$ of the circle area

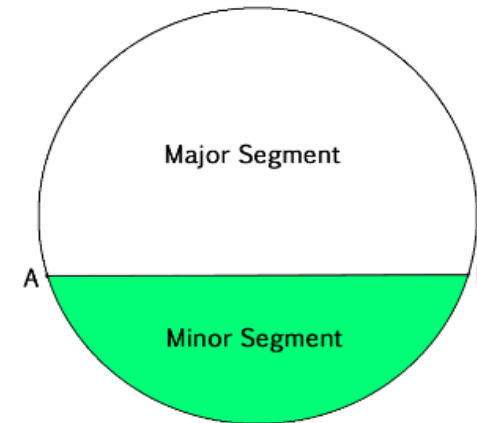
$$\text{Sector area} = \frac{1}{6} \times \pi \times 4 \times 4$$

$$\text{Sector area} \approx 8.4 \text{ cm}^2$$

Picture, model, or diagram



Non-Example



segment

A circle has a radius of length 3cm
The area of a major **segment** is twice the area of the minor **segment**.
What is the area of the minor **segment**?

$$\text{Area of circle} = \pi r^2$$

$$\text{Area} = 9\pi \text{ cm}^2$$

$$\text{Area}_{\text{minor}} : \text{Area}_{\text{major}}$$

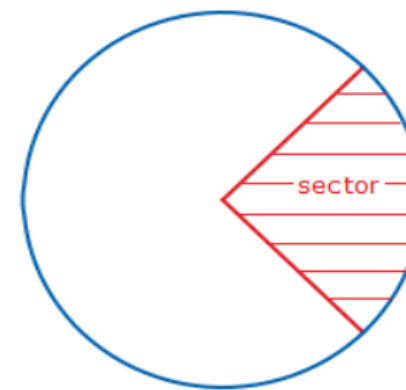
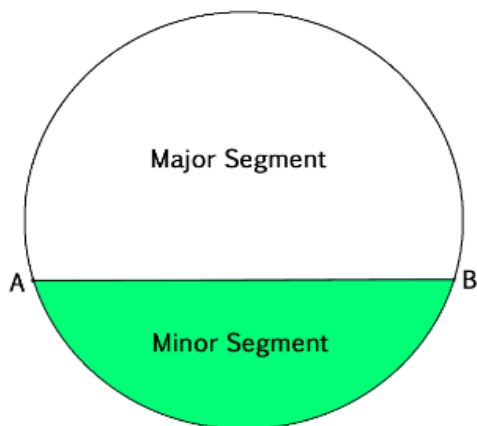
$$1 : 2$$

$$3 : 6$$

$$\text{Area of minor segment} = 3\pi \text{ cm}^2$$

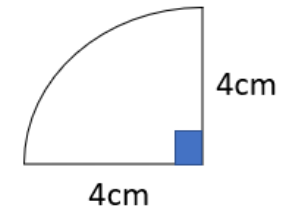
Picture, model, or diagram

Non-Example



arc

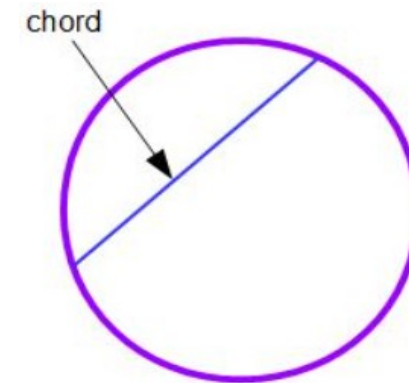
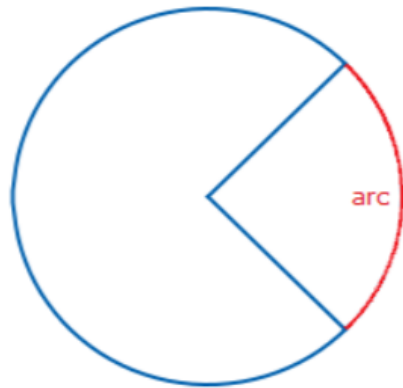
Calculate the **arc** length to 2 decimal places



Arc length is 0.25 of the circumference

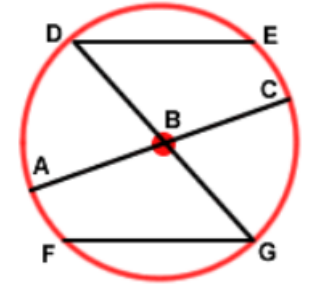
Arc length = $0.25 \times \pi \times 8$

Arc length $\approx 6.28\text{cm}$

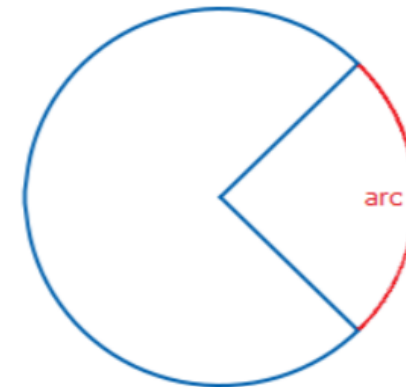
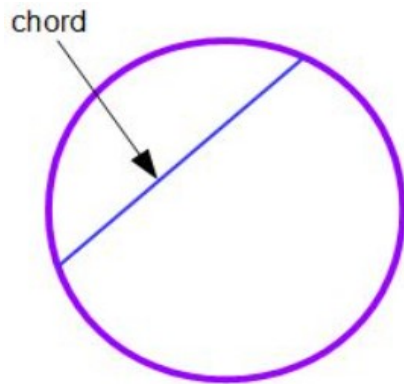


chord

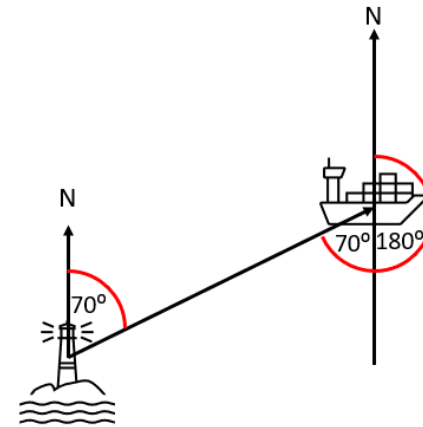
All these lines are chords
DG and AC are also diameters



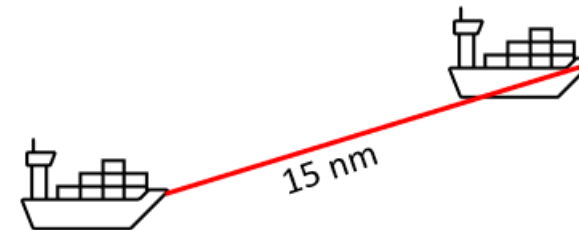
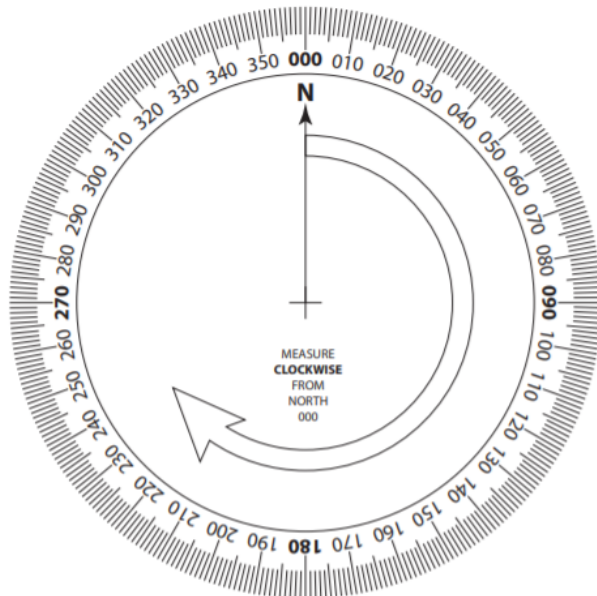
The diameter is the longest possible chord in a circle



bearing

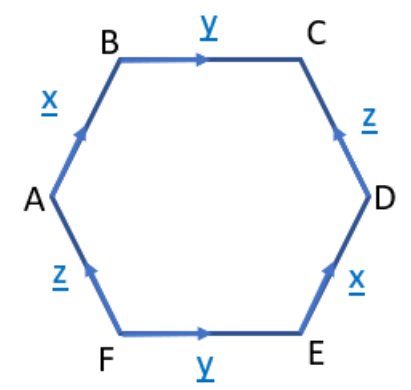


The bearing of the lighthouse from the ship is 250°



These ships are 15 nautical miles apart

vector



ABCDEF is a regular hexagon

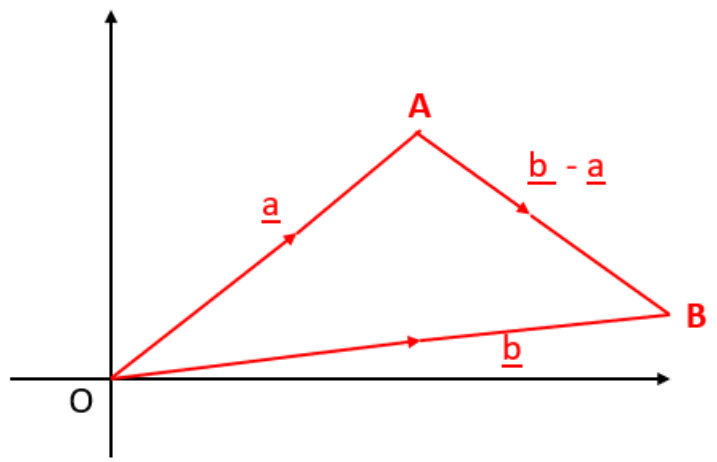
$$\overrightarrow{AB} = \overrightarrow{ED} = \underline{x}$$

$$\overrightarrow{BC} = \overrightarrow{FE} = \underline{y}$$

$$\overrightarrow{DC} = \overrightarrow{FA} = \underline{z}$$

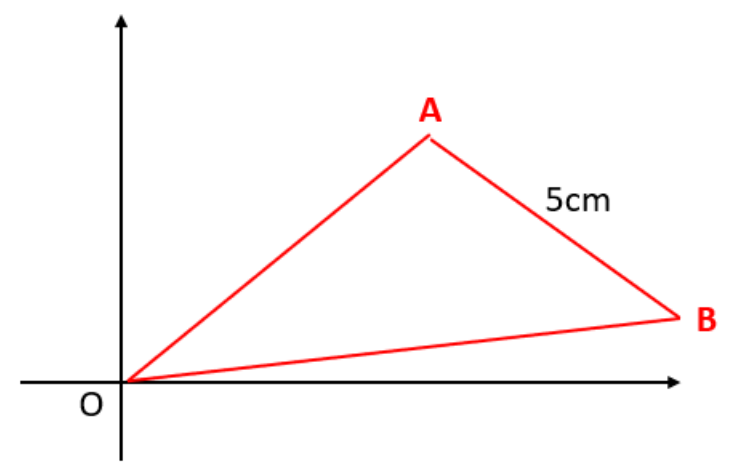
Find \overrightarrow{BE}

$$\overrightarrow{BE} = \overrightarrow{BC} - \overrightarrow{DC} - \overrightarrow{ED} = \underline{y} - \underline{z} - \underline{x}$$



$$\overrightarrow{AB} = \underline{b} - \underline{a}$$

A vector has magnitude and direction



$$AB = 5\text{cm}$$

A length has magnitude but no direction

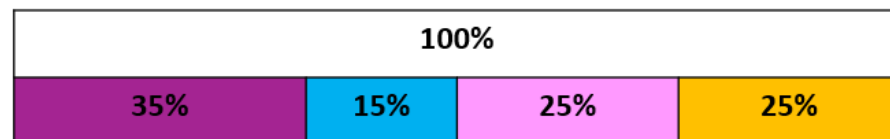
relative frequency

In a probability experiment, coloured counters were taken from a bag without looking and then replaced

Colour	Frequency	Relative Frequency
Purple	7	0.35
Blue	3	0.15
Pink	5	0.25
Orange	5	0.25
<i>Total</i>	<i>20</i>	<i>1.00</i>

The relative frequency of the event 'select blue' is 0.15

Picture, model, or diagram



The relative frequency shows the proportion of the total for each event occurring. It can be represented as a fraction, a decimal or a percentage.

Non-Example

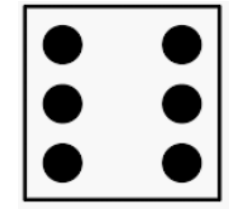
In a probability experiment, coloured counters were taken from a bag without looking and then replaced. This was repeated twenty times.

Colour	Frequency
Purple	7
Blue	3
Pink	5
Orange	5
<i>Total</i>	<i>20</i>

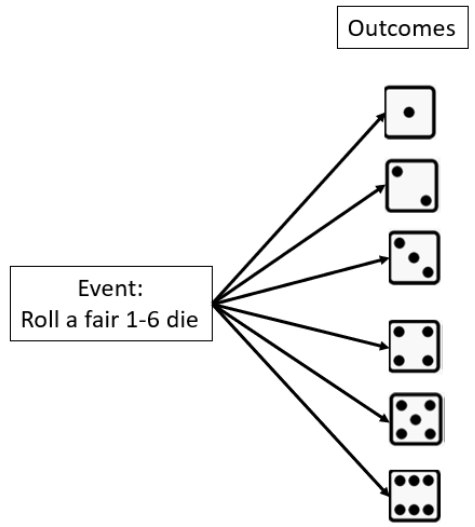
'Blue' was selected three times

outcome

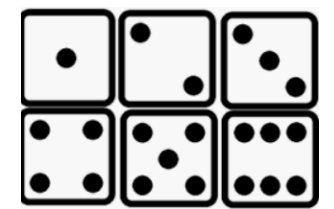
Roll a fair 1-6 die



'6' is one possible outcome



Roll a fair 1-6 die



'7' is impossible and so is **not** an outcome

random (variable)



A discrete **random variable**:

The number of marbles in a jar

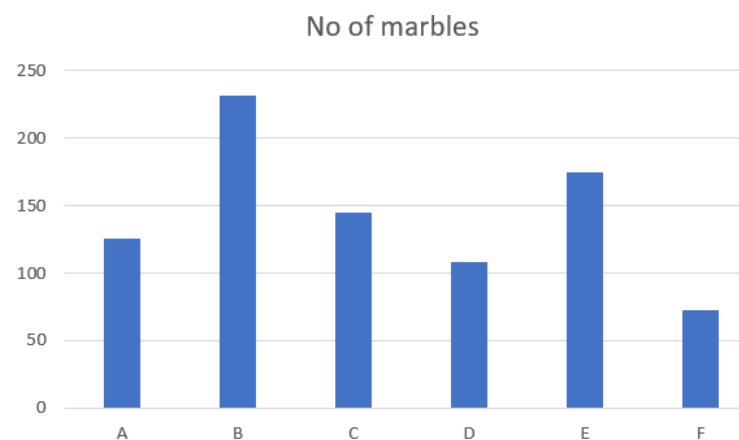
A continuous **random variable**:

The number of seconds taken to complete a race



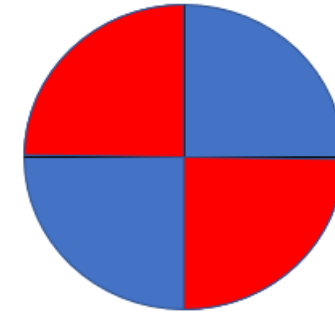
A variable that can take on different values

Marble Jar	Number of marbles
A	126
B	232
C	145
D	108
E	175
F	73



A sample of exactly 10 marbles from
each jar

fair

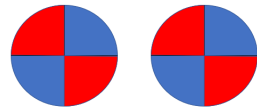


This spinner is fair.
There is a equal chance of 'blue' and 'red'.



Picture, model, or diagram

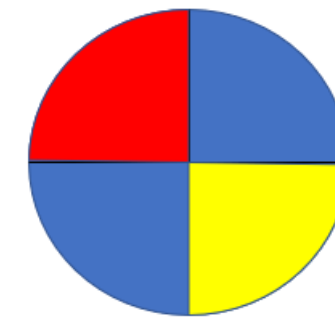
Non-Example



		Second spin	
		Red	Blue
First spin	Red	RR	RB
	Blue	BR	BB
		Outcomes	

		Second spin	
		Red	Blue
First spin	Red	0.25	0.25
	Blue	0.25	0.25
		Probability	

When using two fair spinners,
the probability of each outcome is the same (0.25)



This spinner is unfair.
There is a greater chance of 'blue' than 'red' or 'yellow'

probability scale (0-1)

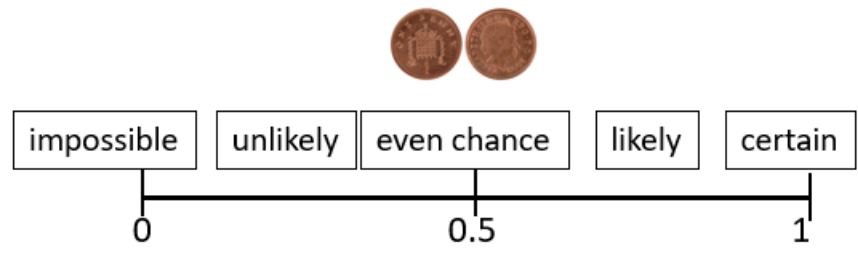


Event: Toss a coin
 The probability of getting a 'head' is 0.5
 The probability of getting a 'tail' is 0.5

Picture, model, or diagram

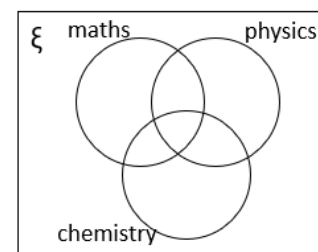


Non-Example



Event: Toss a coin
 A 'head' was the result 53 times

Venn diagram



In a year group of 142 students:
 23 study only maths
 18 study only physics
 60 study physics
 45 study only chemistry
 32 study physics and chemistry only
 8 study none of these subjects
 7 study all three subjects

How many study mathematics and chemistry only?

Step one: Fill all the information into the diagram

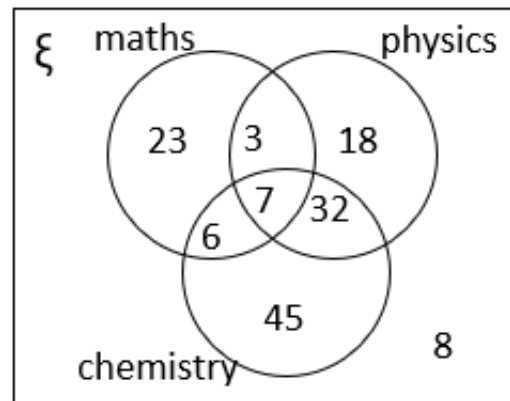
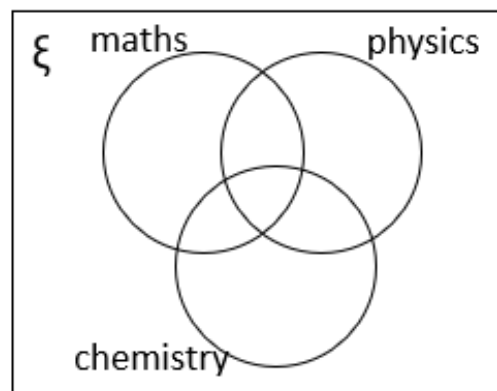
Step two: Subtract the number of students accounted for from 142

Solution: 6 students studied maths and chemistry only



Picture, model, or diagram

Non-Example



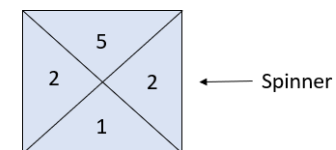
Subject	No of students
Maths	39
Physics	60
Chemistry	90

This table shows the number of students studying each subject

sample space

The spinner is spun **twice** and the score is **added** and recorded in the sample space table below

+	1	2	2	5
1				
2				
2				
5				



Use the sample space to record all possible outcomes and hence work out the probability of scoring **more than 4**

Picture, model, or diagram



+	1	2	2	5
1	2	3	3	6
2	3	4	4	7
2	3	4	4	7
5	6	8	7	10

$$P(\text{more than 4}) = \frac{7}{16}$$

Non-Example

Colour	Frequency
Purple	7
Blue	3
Pink	5
Orange	5
Total	20

exhaustive set



If a coin is tossed, there are two possible outcomes
Heads or Tails

The probability of getting a head or a tail is 100%

'Heads and Tails' are an **exhaustive set**

An **exhaustive set** contains **all** possible outcomes



Event: Roll a fair 1-6 die

100%					
1	2	3	4	5	6

Event: Toss a coin

100%	
H	T

If a fair six-sided (1-6) die is rolled ten times and the outcomes are:
6,6,4,4,3,4,3,5,6,2

The set of actual outcomes is not exhaustive, since 1 has not appeared

empirical

The theoretical probability of rolling a 6 on a fair 1-6-sided die is $\frac{1}{6}$

We can carry out a number of trials to gather **empirical** data to test this.



Picture, model, or diagram

Event: Roll a fair 1-6 die

Result	Frequency
1	45
2	42
3	39
4	47
5	38
6	44

A probability experiment collects **empirical data**



Non-Example

The theoretical probability of rolling a 6 on a fair 1-6-sided die is $\frac{1}{6}$

We can use this **theoretical** probability to work out the probability of rolling a 4



theoretical (probability)

Event: Roll a fair 1-6 die

What is the **theoretical probability** of rolling a 4?



$$P(4) = \frac{1}{6}$$

Result	<i>number of each outcome</i> <i>number of possible outcomes</i>
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
4	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$

For a fair die, each number has an equal chance in **theory**



Event: Roll a fair 1-6 die

1	2	3	4	5	6
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Event: Roll a fair 1-6 die

Result	Frequency
1	45
2	42
3	39
4	47
5	38
6	44

A probability experiment collects **empirical data**

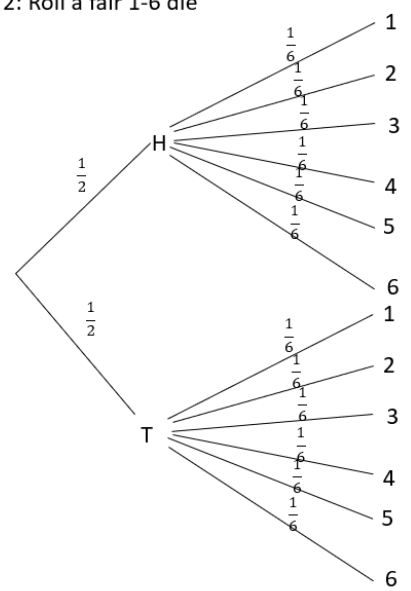
independent

Landing on heads after tossing a coin
AND
rolling a 5 on a single 6-sided die
are examples of **independent** events.



Picture, model, or diagram

Event 1: Toss a coin
Event 2: Roll a fair 1-6 die



The probability of Event 2 is **independent** of Event 1

Non-Example

A card is chosen at random from a standard deck of 52 playing cards.
Without replacing it, a second card is chosen.

What is the probability that the first card chosen is a queen
and the second card chosen is a jack?

$$P(\text{Queen}) = 4 / 52 ; P(\text{Jack}) = 4 / 51$$

$$P(\text{Queen and a Jack}) = 4 / 52 \times 4 / 51 = 16 / 2652$$

$$P(\text{Queen and a Jack}) = 4 / 663$$

The probability of the Jack is dependent on the probability of the Queen.

dependent

A card is chosen at random from a standard deck of 52 playing cards.
Without replacing it, a second card is chosen.

What is the probability that the first card chosen is a queen and the second card chosen is a jack?

$$P(\text{Queen}) = 4 / 52 ; P(\text{Jack}) = 4 / 51$$

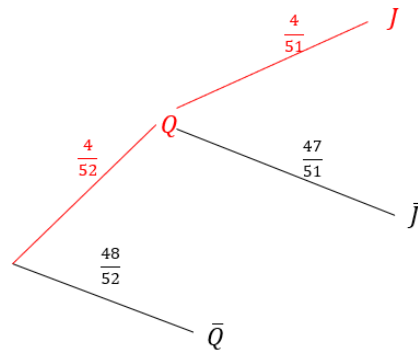
$$P(\text{Queen and a Jack}) = 4 / 52 \times 4 / 51 = 16 / 2652$$

$$P(\text{Queen and a Jack}) = 4 / 663$$

The probability of the Jack is **dependent** on the probability of the Queen.

Picture, model, or diagram

Event 1: Draw a card, do **not** replace
 Event 2: Draw a card



The probability of drawing a Jack second is **dependent** on whether or not a Queen was drawn first

Non-Example

Landing on heads after tossing a coin
 AND
 rolling a 5 on a single 6-sided die
 are examples of **independent** events.

conditional

	Have pets	Do not have pets	Total
Male	0.41	0.08	0.49
Female	0.45	0.06	0.51
Total	0.86	0.14	1

What is the **(conditional)** probability that a randomly selected person is male, given that they have a pet?

$$P(\text{male and have a pet}) = 0.41$$

$$P(\text{have a pet}) = 0.86$$

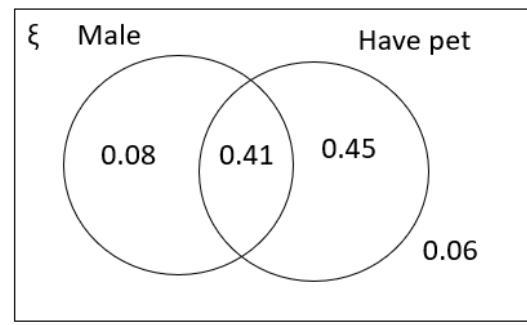
$$P(\text{male} \mid \text{have a pet}) = 0.41 / 0.86 = 0.477$$



Picture, model, or diagram

Non-Example

$$P(\text{male} \mid \text{have a pet}) = \frac{P(\text{male} \cap \text{have a pet})}{P(\text{have a pet})}$$

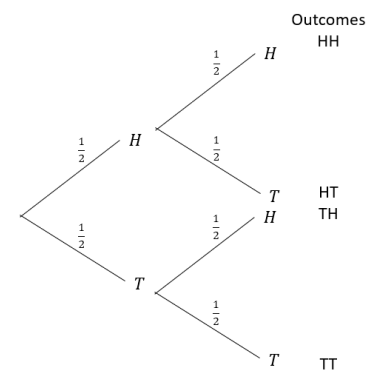


Event	H	T
H	HH	HT
T	TH	TT

There are four possible outcomes when a fair coin is tossed twice

tree diagram

Event 1: Toss a coin
 Event 2: Toss a coin
 Use a **tree diagram** to work out the probability of HH

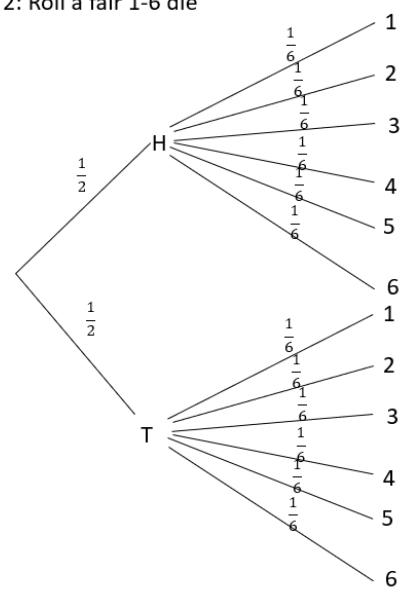


$$P(HH) = \frac{1}{4}$$



Picture, model, or diagram

Event 1: Toss a coin
 Event 2: Roll a fair 1-6 die



Non-Example

Event	H	T
H	HH	HT
T	TH	TT

grouped (data)

Observed data arising from counts and grouped into non-overlapping intervals is called **grouped data**.

The length of feet of 25 planks of wood were measured.

The lengths were grouped into classes of width 10 feet

Picture, model, or diagram



Non-Example

Length (feet)	Frequency (f)
$0 \leq \text{ft} < 10$	2
$10 \leq \text{ft} < 20$	6
$20 \leq \text{ft} < 30$	9
$30 \leq \text{ft} < 40$	5
$40 \leq \text{ft} < 50$	3

The number of different colour smarties in a pack of 25

It is not possible to group the data as each colour is separate

colour	Frequency (f)
Green	2
Orange	6
Blue	9
Yellow	5
Purple	3

mean (\bar{x})

The **mean** average of 6, 11, 16 is 11

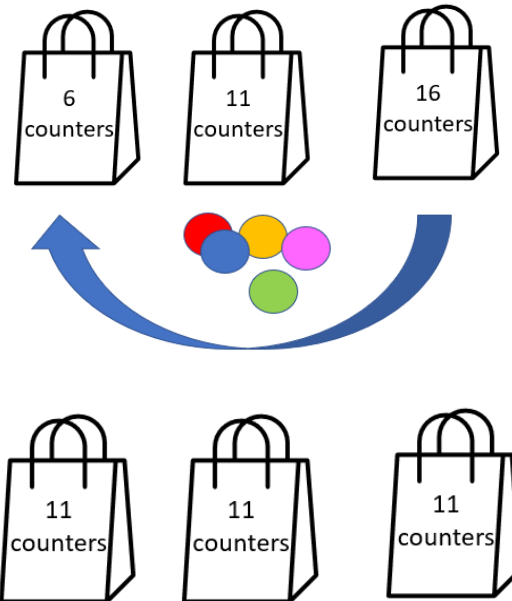
$$(6 + 11 + 16) \div 3 = 11$$

$$\bar{x} = 11$$

Picture, model, or diagram



Non-Example



The mean (\bar{x}) can be thought of as 'equal' shares

The **median** of 12, 6, 3, 5, 8 is 6

3, 5, 6, 8, 12

median

The **median** of 12, 6, 3, 5, 8 is 6

3, 5, 6, 8, 12



Middle value of this data set is 6

The mode of 8, 5, 6, 8, 9, 8 is 8

5, 6, 8, 8, 8, 9

mode

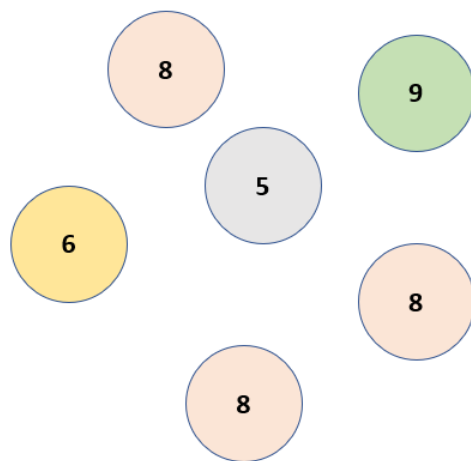
The **mode** of 8, 5, 6, 8, 9, 8 is 8

5, 6, 8, 8, 8, 9

Picture, model, or diagram



Non-Example



'8' occurs most frequently

The **median** of 12, 6, 3, 5, 8 is 6

3, 5, 6, 8, 12

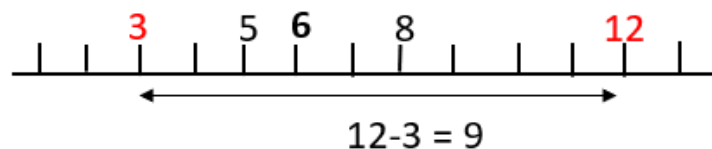
range

The **range** of 12, 6, 3, 5, 8 is 9

It is the difference between the maximum (12) and the minimum (3) value in the set.

3, 5, 6, 8, 12

$$12 - 3 = \underline{9}$$



The **range** of this data set is 9

The **median** of 12, 6, 3, 5, 8 is 6

3, 5, 6, 8, 12

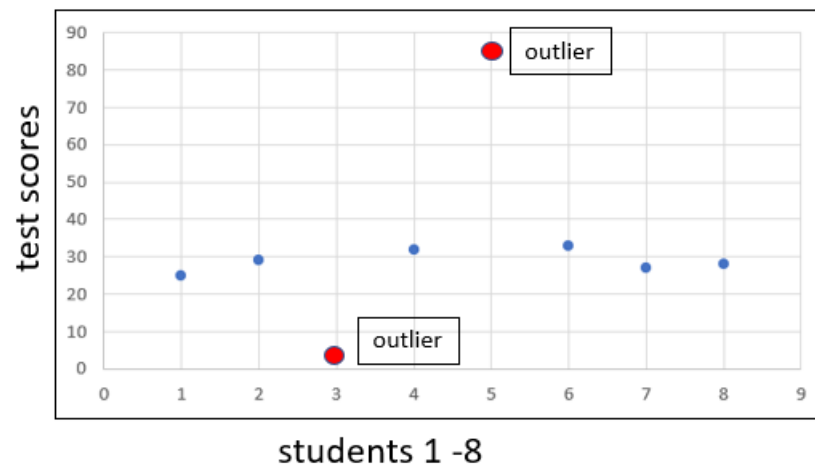
outlier

In a maths test, the following marks were scored:
25, 29, **3**, 32, **85**, 33, 27, 28
Both 3 and 85 are **outliers**.
They lie outside the main cluster of scores.

Picture, model, or diagram



Non-Example



In a maths test, the following marks were scored:
25, 29, 3, 32, 85, 33, 27, 28
The range of scores is 82

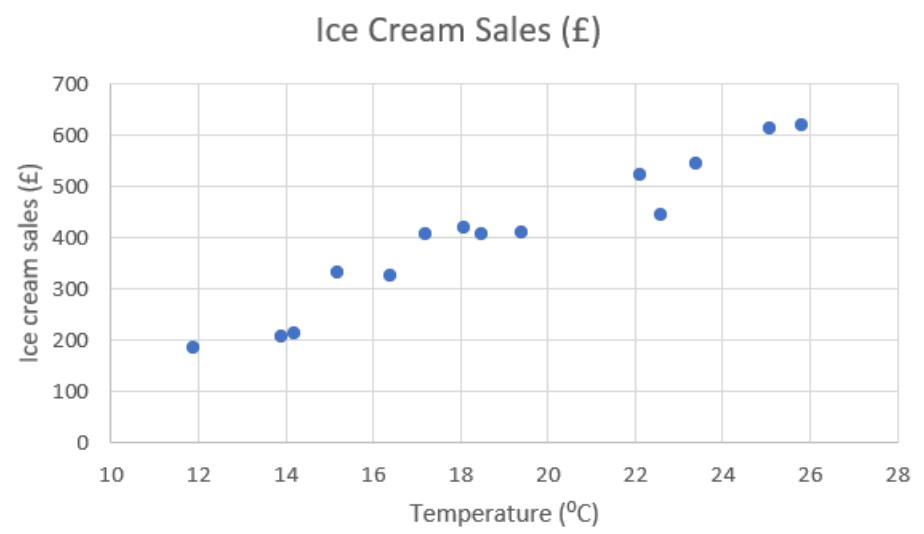
$$85 - 3 = 82$$

bivariate (data)

Ice cream sales versus the temperature on that day. The two variables are 'Ice cream sales' and 'Temperature'.

Temperature (°C)	Ice Cream Sales (£)
14.2	215
16.4	325
11.9	185
15.2	332
18.5	406
22.1	522
19.4	412
25.1	614
23.4	544
18.1	421
22.6	445
17.2	408
13.9	207
25.8	620

The warmer the temperature, the more ice creams are sold. 'Temperature' and 'Ice cream sales' are **bivariate**



Univariate means one variable (one type of data)

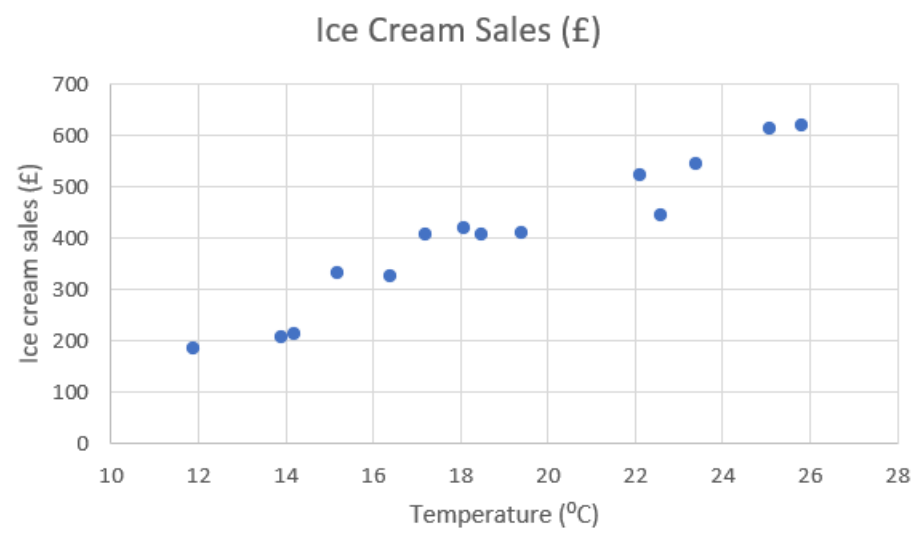
Example: Travel Time (minutes): **15, 29, 8, 42, 35, 21, 18, 42, 26**

The variable is **Travel Time**

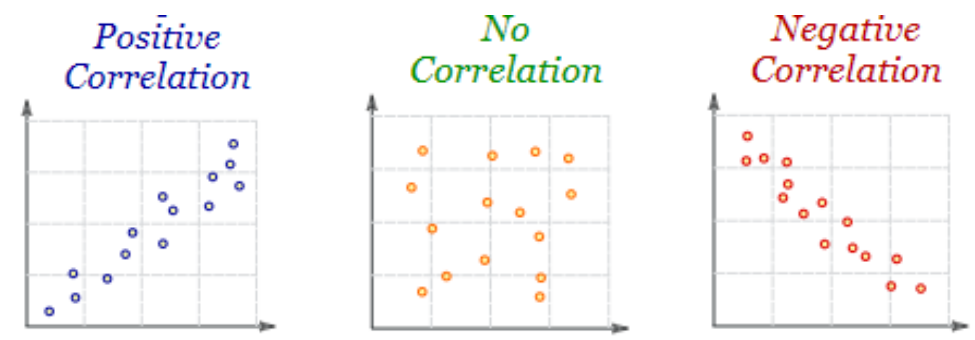
With **bivariate data** we have **two** sets of related data we want to **compare**.

scatter graphs

Bivariate data

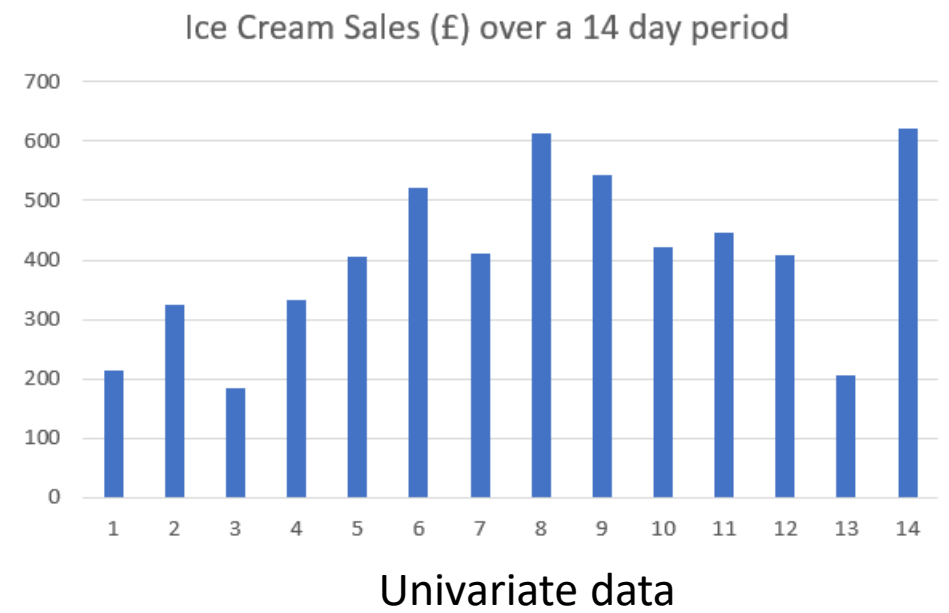


Picture, model, or diagram



Scatter graphs are used to compare two sets of data. Positive correlation means that as one variable increases, so does the other. No correlation means that one variable does not have an effect on the other. Negative correlations means that as one variable increase, the other decreases.

Non-Example



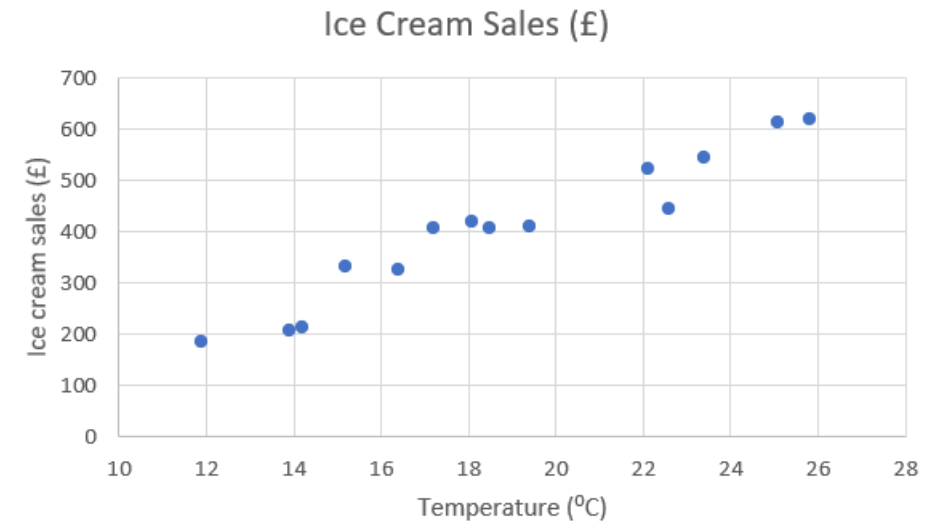
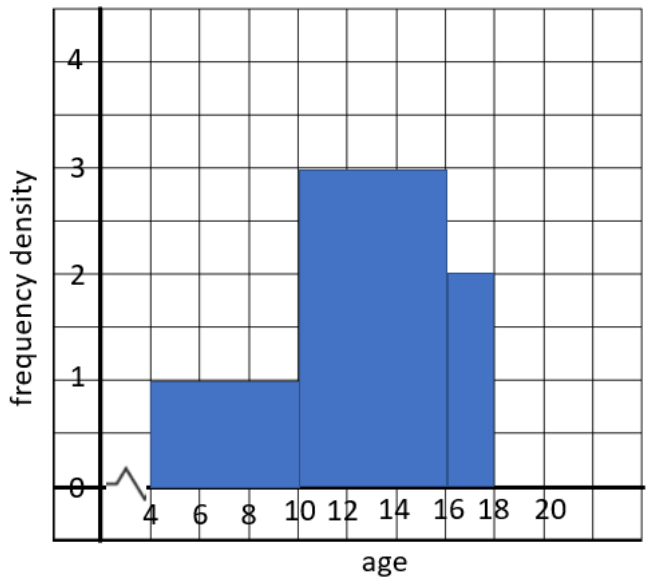
Univariate data

histogram

Histograms look like bar charts but the area of the bar represents the frequency, not the height. The class widths can be unequal in a **histogram**

The ages of 28 children on a school trip

Age	Frequency	Class width	Frequency density
4-9	6	6	$6 \div 6 = 1$
10-15	18	6	$18 \div 6 = 3$
16-17	4	2	$4 \div 2 = 2$



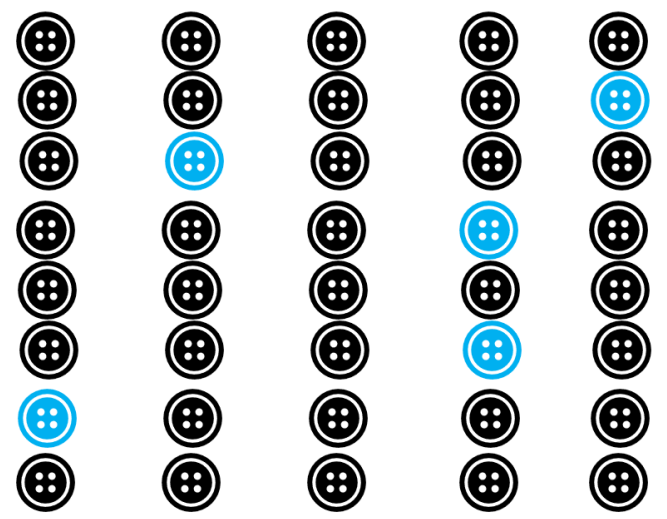
sample

Sample: A collection of data from **part** of the population

Example: A random selection of five buttons from a box of forty buttons

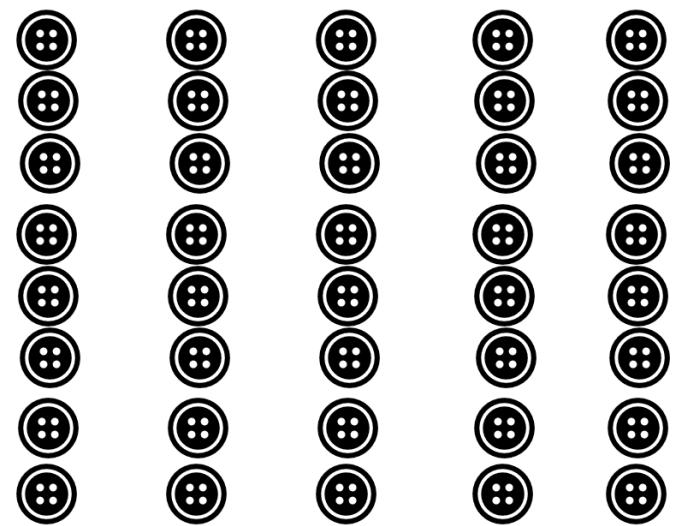


Picture, model, or diagram



The blue buttons are a random **sample** from the population

Non-Example



This is the population ~ all the 40 buttons in the box

population

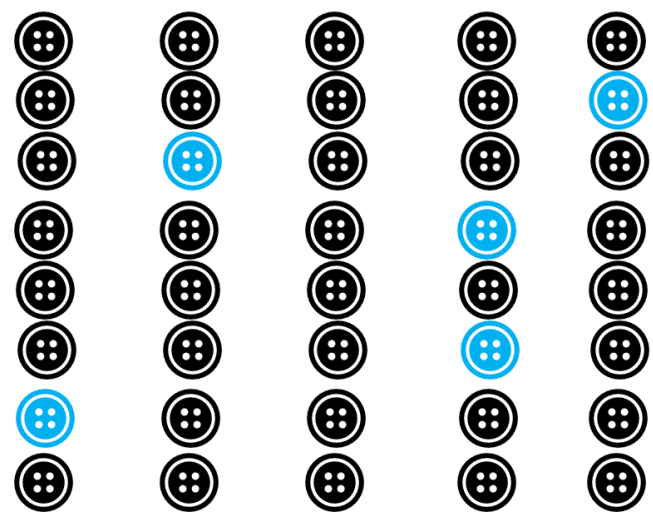
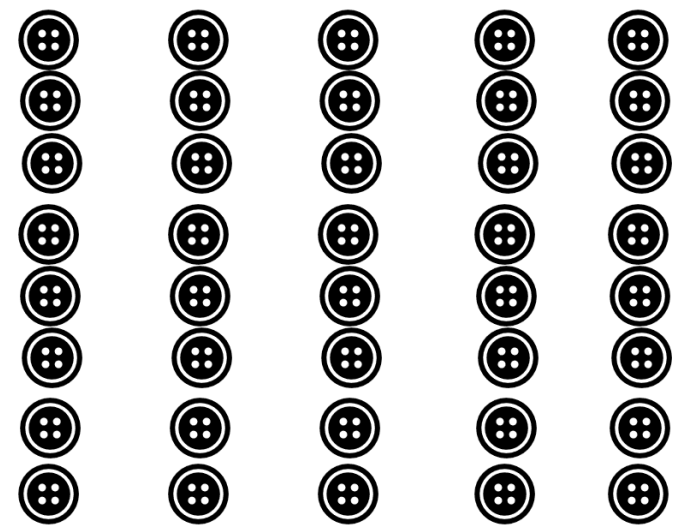
Population: The whole group we are interested in

Example: all the buttons in a box of 40 buttons



Picture, model, or diagram

Non-Example



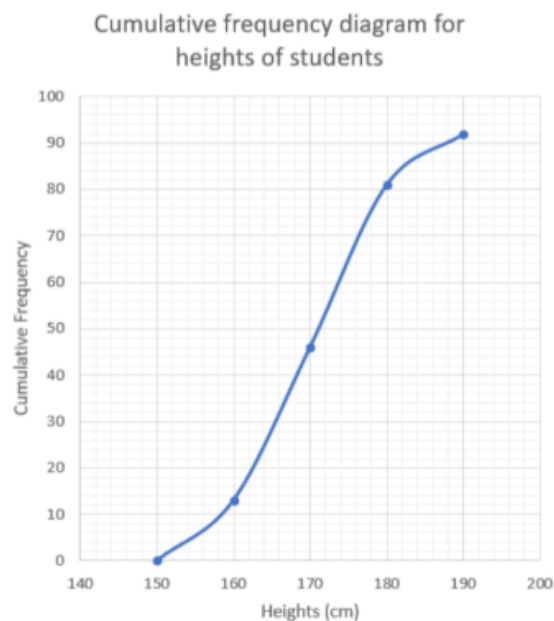
This is the **population** ~ all the 40 buttons in the box

The blue buttons are a random sample from the population

cumulative frequency

Heights, h (cm)	Frequency	Cumulative frequency
$150 < h \leq 160$	13	13
$150 < h \leq 160$	33	46
$150 < h \leq 160$	35	81
$150 < h \leq 160$	11	92

Picture, model, or diagram



Non-Example

Heights, h (cm)	Frequency
$150 < h \leq 160$	13
$150 < h \leq 160$	33
$150 < h \leq 160$	35
$150 < h \leq 160$	11

box plot (box and whisker diagram)

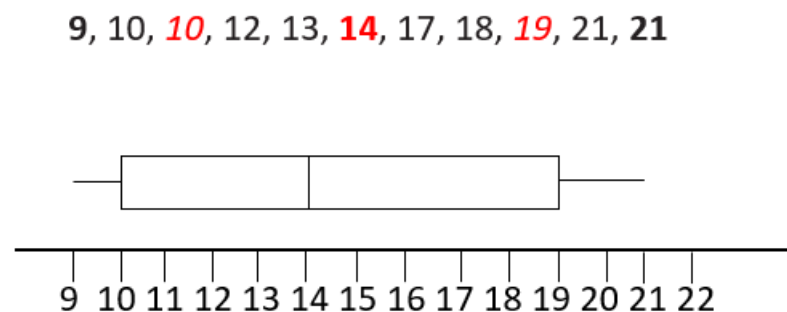
This data shows the ages of 11 people on a boat trip

9, 10, **10**, 12, 13, **14**, 17, 18, **19**, 21, 21

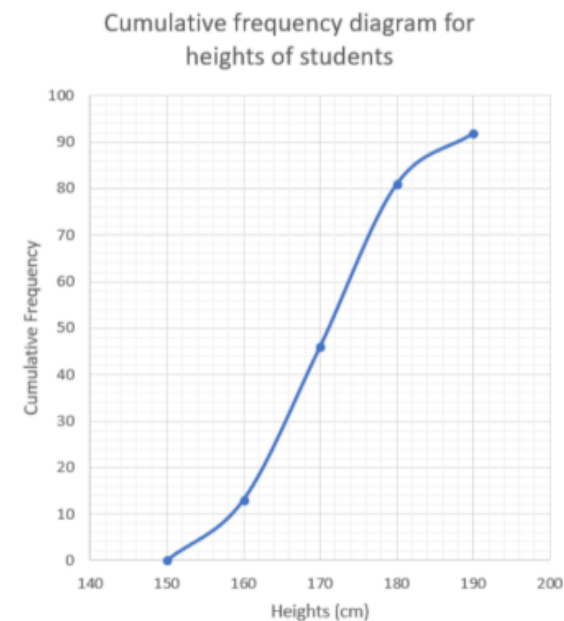
The data is ordered and the median, quartiles, maximum and minimum values identified.

This is then plotted on a horizontal axis as a box for the middle 50% of the data, with 'whiskers' to show the first and last 25%

Picture, model, or diagram



Non-Example



lower quartile

Quartiles are the values that divide a list of numbers into quarters

Example: 5,7,4,4,6,2,8

Order: 2,4,4,5,6,7,8

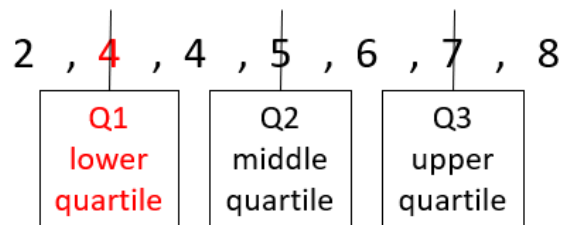
Quarter the list:

Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

Picture, model, or diagram



Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

Non-Example

The mode of 8, 5, 6, 8, 9, 8 is 8

5, 6, 8, 8, 8, 9

upper quartile

Quartiles are the values that divide a list of numbers into quarters

Example: 5,7,4,4,6,2,8

Order: 2,4,4,5,6,7,8

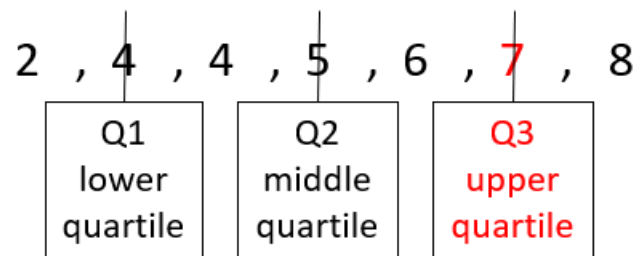
Quarter the list:

Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

Picture, model, or diagram



Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

Non-Example

The mode of 8, 5, 6, 8, 9, 8 is 8

5, 6, 8, 8, 8, 9

inter quartile range

Quartiles are the values that divide a list of numbers into quarters

Example: 5,7,4,4,6,2,8

Order: 2,4,4,5,6,7,8

Quarter the list and find the difference between the upper and lower quartiles

Lower quartile (Q1) = 4

Median (Q2) = 5

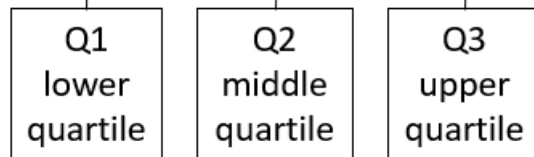
Upper quartile (Q3) = 7

Inter quartile range (IQR) = 3

Picture, model, or diagram



2 , 4 , 4 , 5 , 6 , 7 , 8



Lower quartile (Q1) = 4

Median (Q2) = 5

Upper quartile (Q3) = 7

Inter quartile range (IQR) = 7 - 4 = 3

Non-Example

2 , 4 , 4 , 5 , 6 , 7 , 8

The range for this data set is $8 - 2 = 6$

Word

Example

Picture, model, or diagram

Non-Example



Word

Example

Picture, model, or diagram

Non-Example

