Hampshire Services
HIAS SCHOOL IMPROVEMENT

MATHEMATICS ADVISORY CENTRE

Progression in Fractions, Decimals and Percentages, Ratio and Proportion

Hampshire County Council

www.hants.gov.uk

Version 1 - May 2017

How to use this document

This document is designed to give ideas about how to use concrete apparatus and images to support children's conceptual understanding of Fraction, decimals, percentages, ratio and proportion.

So often children are able to follow calculation processes and get the answer right in many cases without fully understanding how and why the method works. This is true of work in this area too.

This document is arranged into some of the 'big ideas' about teaching fractions. These are not exclusive to a particular year group but key misconceptions many children have. The example tasks and modelling will help to highlight whether these misconceptions are present and allow teachers to address them.

The Big ideas can be found here:

www.ncetm.org.uk/search?q=big+ideas+fractions

For more guidance use the Resource Tool https://www.ncetm.org.uk/resources/41211 on the NCETM website where exemplification material and activity ideas can be found using the following tabs of EXEMPLIFICATION and ACTIVITIES. The SUBJECT KNOWLEDGE Audit tool will also help to give an insight to the expectations under each NC objective. There is also a tab for VIDEOS under each domain where you can see teacher using concrete resources to develop childrens' conceptual understanding and reasoning.

The EYFS Framework

Mathematics involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measures

Early Learning Goals

Mathematics Numbers: children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Shape, space and measures: children use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems. They recognise, create and describe patterns. They explore characteristics of everyday objects and shapes and use mathematical language to describe them.

Additional Guidance can be found in Development Matters:

Key Stage 1

The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources [for example, concrete objects and measuring tools].

At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money.

By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.

National Curriculum 2014

Lower Key Stage 2

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure that pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure that they can use measuring instruments with accuracy and make connections between measure and number.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

National Curriculum 2014

Upper Key Stage 2

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems.

Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

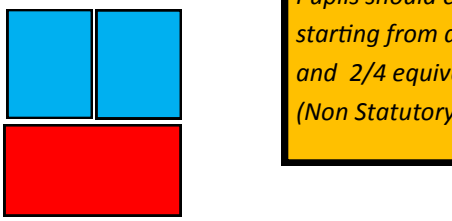
Pupils should read, spell and pronounce mathematical vocabulary correctly.

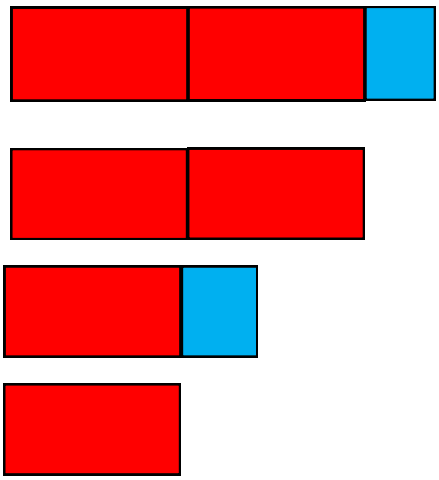
National Curriculum 2014

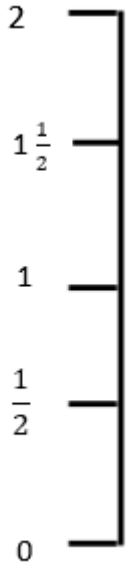
Counting in Fractional Steps

Year 2

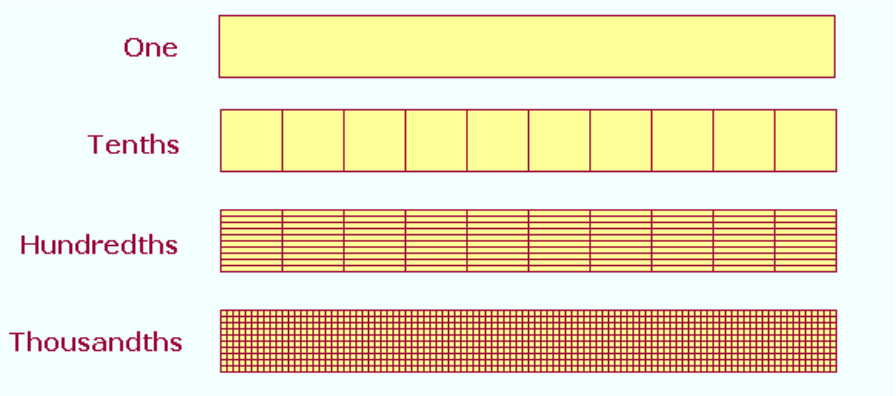
Pupils should count in fractions up to 10, starting from any number and using the 1/2 and 2/4 equivalence on the number line (Non Statutory Guidance)







Use fractions cards to help show relationships between the steps.



Link with Measures

Count using Number lines either horizontal or vertical and link to measures e.g. 1 litre, 11/2L , 2 litres etc.

If you count in steps of $\frac{1}{2}$ starting from 0, how many steps will it take to reach:
2, 4 or 6
What do you notice?

NCETM: Teaching for Mastery: Questions, tasks and activities to support assessment. Y2

Watch this video showing Counting in fractional Steps.

https://www.ncetm.org.uk/resources/43609 Counting in Fractional Steps

Counting in Fractional Steps

Year 4, Year 5,

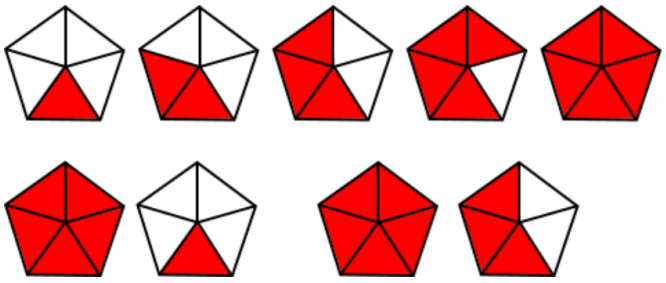
Pupils practice counting using simple fractions and decimal both forwards and backwards. (non Statutory)

Pupils in all year groups will benefit from regular counting in decimal and fractional steps.

Fractional Shapes

When counting in fractional steps, provide visual representations for each step in the count.

Play **this slide** to accompany counting in fifths.



One fifth, two fifths…..5 fifths (1 whole) one and 1 fifth

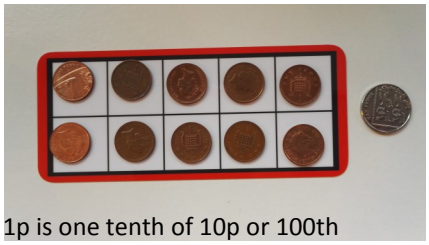
Year 4

count up and down in hundredths

(The decimal recording of money is introduced formally in Year 4 (Non Statutory)

0.1, 0.2...

https://www.ncetm.org.uk/resources/44557



1p is one tenth of 10p or 100th of £1

Year 3

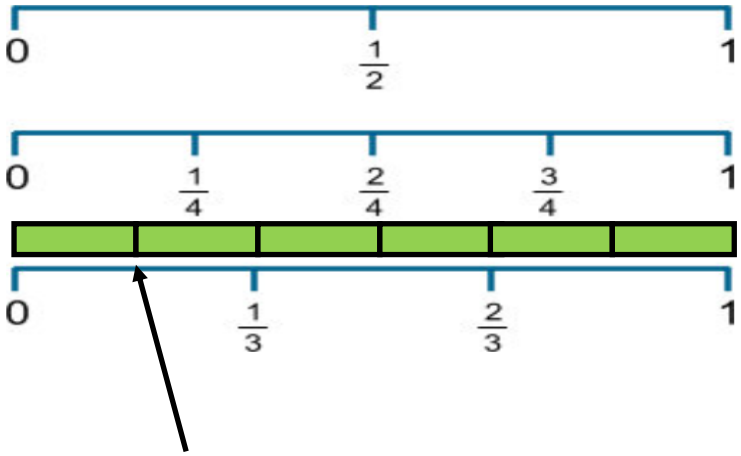
count up and down in tenths

1/10, 2/10, 3/10

0.1, 0.2 etc

Pupils connect tenths to place value and to division by ten. (non Statutory)

Common misconception: The number half always lies halfway between 2 labelled integers on a number line



What would go here? How do you know?

What is half of one third?

Use strips of paper to find out the missing fraction.

6 pieces of card fit in 1 whole. The missing fraction is 1 sixth.

Y4 Pupils should connect hundredths and tenths to place value and decimal measure. (Non Statutory guidance)

Link with money counting in 1 pence (hundredth of £)

10p (tenth of £)

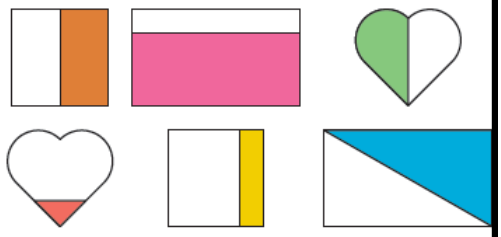
Cm is tenth of m

Recognising fractions, comparing and ordering fractions

Which of these show half of each whole shape?

Explain your reasoning.

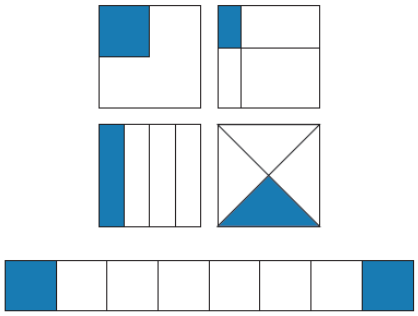
Children should talk about the two parts needing to be equal parts of the whole.



NCETM: Teaching for Mastery: Questions, tasks and activities to support assessment. Y1

NCETM: Teaching for Mastery: Questions, tasks and activities to support assessment. Y2

Which of these diagrams have $\frac{1}{4}$ of the whole shaded?



Explain your reasoning.

Odd one out

Which is the odd one out from the group (not half)?

Same and different

Tell me what is the same/different about these sets and images.

Sorting activity

Give set of images- how can you sort them? Explain how and why you have sorted them like that.

True or false?

All the images (like those shown above) represent a half. Explain.

All shapes split into 2 are then split into halves. Explain.

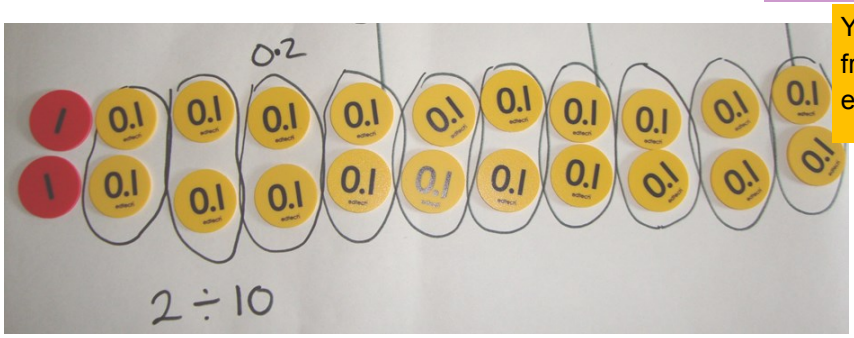
Give an example of…

Another shape or set of items- split into two half. Show 1 half.

Video link:

https://www.ncetm.org.uk/resources/43609

Sorting and Reasoning about fractions



Y3 recognise that tenths arise from dividing an object into 10 equal parts and in dividing

NC

Y2 recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity

Y3 recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators

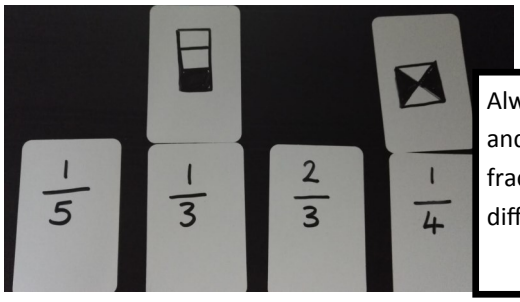
Y4 recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten

Y5 recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents (appears also in Equivalence)

Recognising fractions, comparing and ordering fractions

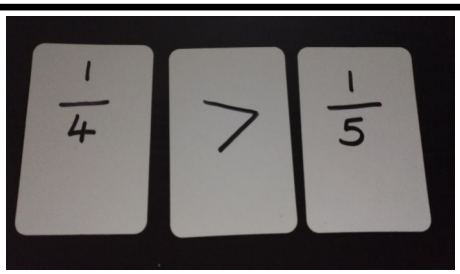
Put these Fractions in order

(mix images and written fractions)

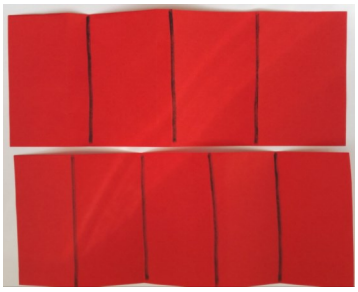


Always make links between images and written (symbolic form). Show fractional images in a variety of different ways.

Fractions with larger denominators are smaller parts of the whole. How do you know? Prove it.



Child's work



Would you rather have …?

$\frac{1}{4}$ of 40p or $\frac{1}{2}$ of 30p?

Prove your choice.

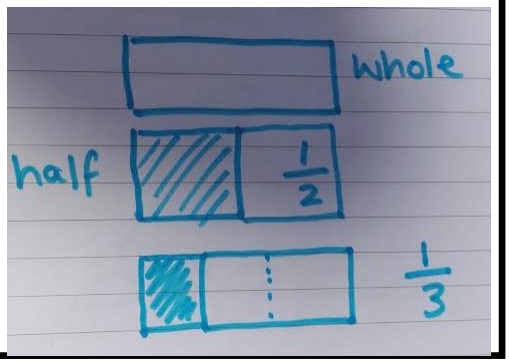
For children who are more confident try using more difficult continuous measures e.g.

$\frac{1}{4}$ of £1.20 or $\frac{1}{2}$ of £3.30?

Give an example of a fraction that is more than half but less than a whole.

How do you know?

Child's work



Odd one out

Which is the odd one out and why?

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{4}$

0.5 Add some other examples to the box which fit.

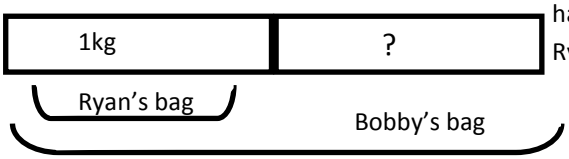
Recognising and comparing fractions

Simple Contextual problems (context of measures)

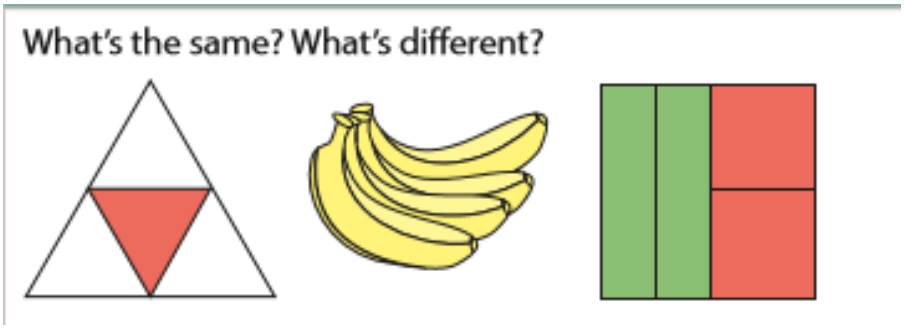
Sam grew a sunflower that is 12 multilink tall. His friend's grew half the height. How tall was his friend's plant?

Peter's flower grew 8cm tall and Lucy's only grew half the height. How tall was Lucy's? (use string to practice measures, and then folding for half.)

Ryan had a bag of sweets half as heavy as Bobby. Ryan's was 1 kg. How heavy was Bobby's?



Common misconception: Identical fractions of different 'wholes' are not the same fractional piece.



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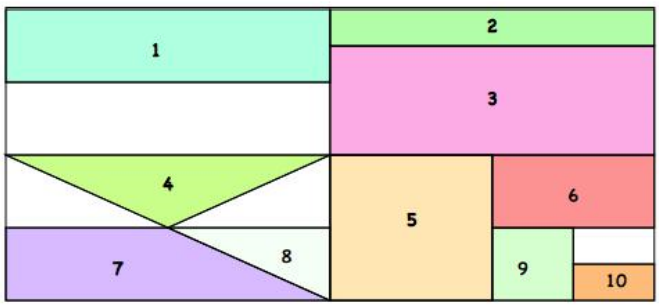
Images such as this help children to see that Fractional parts are not always congruent (same shape). see equivalence

Rectangle Tangle Nrich:

http://nrich.maths.org/1048 Rectangle Tangle



What equivalents can you see?

Describe fraction relationships

Shape 1 is equivalent to two shape 2 therefore shape 2 is half of shape 1.

Shape 2 and 6 are equivalent. I can prove this by…

I know this because...

The large rectangle above is divided into a series of smaller quadrilaterals and triangles. Each of the shapes is a fractional part of the large rectangle.

Can you untangle what fractional part is represented by each of the ten numbered shapes?

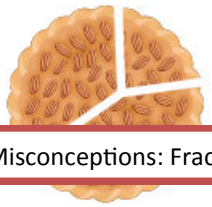
This idea can be made simpler with more basic rectilinear shapes and fewer shapes. Children should be encouraged to trace, cut out and overlay to compare fractional pieces.

Recognising and comparing fractions

Pupils write the shaded part as $\frac{3}{5}$. i.e. they see three parts green and five parts white and record these numbers as a fraction.

The denominator refers to the whole and the numerator is the part of this.

$\frac{1}{2}$



Pupils identify these parts a thirds.

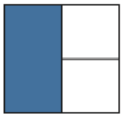
(Denominator is Down the bottom of the fraction)

Misconceptions: Fractions are read as pieces rather than ***equal*** part/whole relationships.

Jayne says that the shaded part of the whole square below does not show a half because there are three pieces, not two.

Do you agree?

Explain your reasoning.

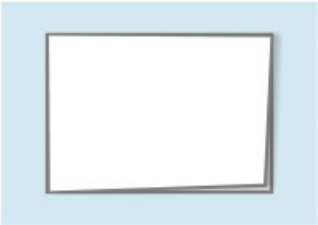


NCETM: Teaching for Mastery: Questions, tasks and activities to support assessment. Y2

Common misconception: Identical fractions of different 'wholes' are not the same fractional piece.

You need two pieces of A4 paper – two different colours works best.

Fold one piece of paper in half to make an A5 sized piece and overlay it on the unfolded sheet as central as you can to make a frame around the folded sheet. This is what you should see:



Prove it!

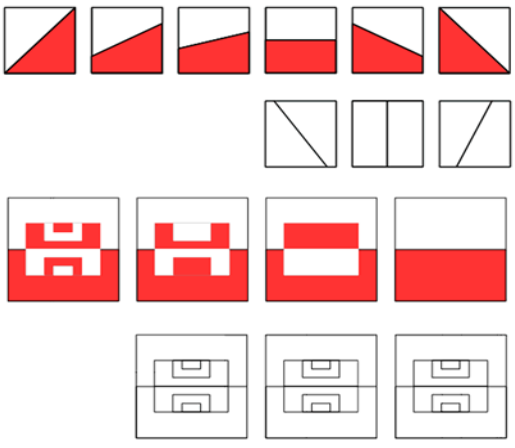
How can you check?

What fraction of the A4 sheet is the frame?

How can you reason that both the frame and the central piece are the same fraction of the

Halving

These images show squares split in half:



https://nrich.maths.org/1788

How might you check that each was correct?
Can you think of more ways to split a square into two halves?
You might like to use squared paper to draw your ideas on.

Recognising and comparing fractions

Only a fraction of each whole rod is shown. Using the given information, identify which whole rod is longer.

Explain your reasoning.

NCETM: Teaching for Mastery: Questions, tasks and activities to support assessment. Y6

Children may think that these fractions are actually the same as they visually look the same. They need to understand that the whole will therefore look different even though fractional pieces look the same they are a part of a different whole.

Common misconception: Identical fractions of different 'wholes' are not the same fractional piece.

If this is $\frac{1}{2}$ what would the whole look like?

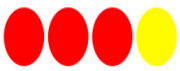
If this is $\frac{1}{3}$ what would the whole look like?

If this is $\frac{1}{4}$ what would the whole look like? What would $\frac{1}{2}$ look like?

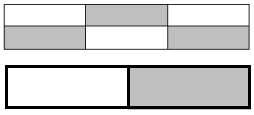
Expose children to fractions in a variety of ways otherwise they may have the common misconception:

Fractional pieces have to be congruent

Discrete quantities



What fraction of the counters are Yellow?



Continuous quantities

Show me 1/4 of …

This jug,

This set of cubes/counters

This measuring tape

Images from NCETM

See Video of Y4 lesson from NCETM

https://www.ncetm.org.uk/resources/49420 Concrete Representation leading to multiplicative reasoning

This video shows how fractions, decimals and measures can be linked together. The idea could also be used to support children to convert metric measures.

Comparing decimals and Rounding including decimals

Circle each decimal when rounded to one decimal place is 6.2

6.32 6.23 6.27 6.17

Can you write others?

Year 4,

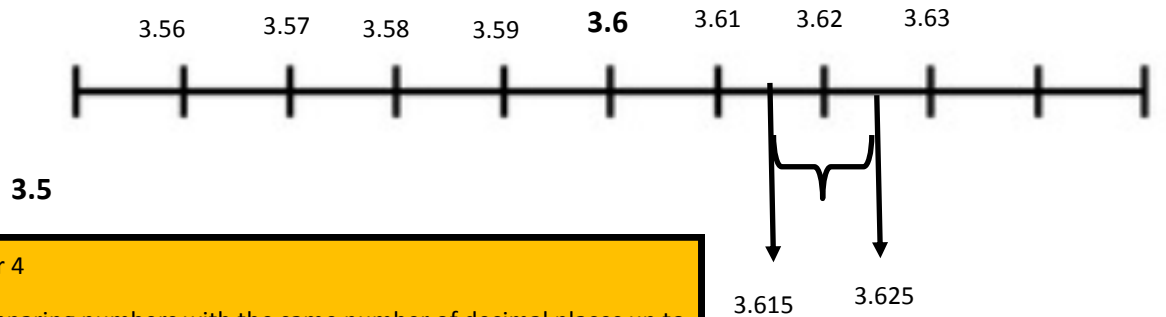
Round decimals with one decimal

Year 5

Round decimals with two decimal places to the nearest whole number and to one decimal place.

I am thinking of a decimal number. When I round it to the nearest tenth it is 0.5 what could it be? Prove it.

I am thinking of a number. When I round it to the nearest tenth it is 3.6. When I round it to the nearest hundredth it is 3.62. Which numbers could it be?



Year 4

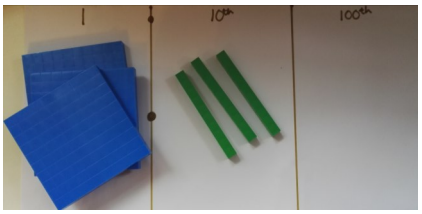
Comparing numbers with the same number of decimal places up to 2 decimal places.

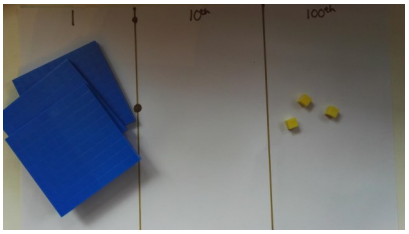
Year 5

Read, write, order and compare numbers with up to 3 decimal places.

Which is greater, £3.03 or £3.30?

Use < or >





Place these decimals on a line from 0 to 2:

0.3, 0.1, 0.9, 0.5, 1.2, 1.9

Which is lighter: 3.5kg or 5.5kg? 3.72kg or 3.27kg? Which is less: £4.50 or £4.05?

Put in order, largest/smallest first:

6.2, 5.7, 4.5, 7.6, 5.2, 99, 1.99, 1.2, 2.1

Convert pounds to pence and vice versa. For example: Write 578p in £.

How many pence is £5.98, £5.60, £7.06, £4.00? Write the total of ten £1 coins and seven 1p coins (£10.07)

Write centimetres in metres. For example, write: 125 cm in metres (1.25 metres)

From NCETM, Planning Resource Tool Year 4. https://www.ncetm.org.uk/resources/42649

Equivalence (Including fractions, decimals and %)

See the Hampshire Fractions Cards templates (10ths to halves)

LINK HERE

See NRich Rectangle tangle activity on page 6

Give an example of…

An equivalent fraction for $\frac{2}{5}$ … and another…

How can you prove it? Show me (use images or concrete resources)

Year 2

Recognise and show, using diagrams, equivalent fractions with small denominators.

Year 3

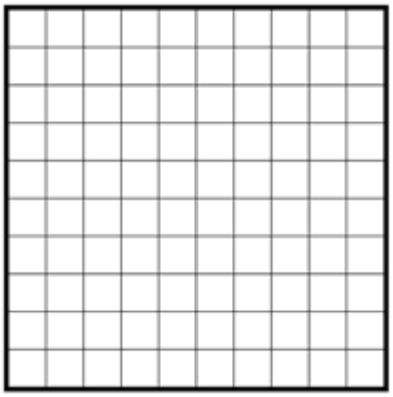
Recognise and show, using diagrams, families of common equivalent fractions

Year 5

Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.

NCETM planning resource tool Y2 https://www.ncetm.org.uk/resources/42635

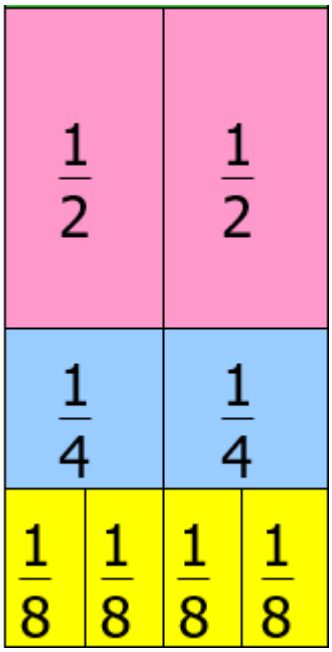
Would a chocolate lover rather have $\frac{1}{2}$ or $\frac{3}{5}$ of this chocolate bar?



Show 3 tenths. Show 30 hundredths. What do you notice?

How could you write these as a decimal, percentage and fraction?

Equivalence (Including fractions, decimals and %)



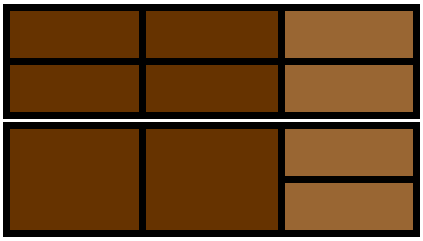
Use Fraction walls to show equivalence.

See Hampshire fraction cards– cut up and use to model equivalence.

Link from Moodle

How many eights are equal to one half?
How do you know?

Would you rather have 2/3 of the box of chocolates or 4/6 of the box? Explain why.



"I would have either as they are equivalent."

I have two different boxes of chocolates. One has 9 in. Would you rather have 2/3 of this one or 4/6 of the other which has 18 in?

Making generalisations

$\frac{1}{2}$ $\frac{12}{24}$ (1 x 12) (2 x 12) $\frac{84}{168}$ (1 x 84) (2 x 84)

"The numerator is always half the denominator"

So $\frac{n}{2n}$

Show how $\frac{n}{3n}$ for $\frac{1}{3}$ or $\frac{8}{24}$

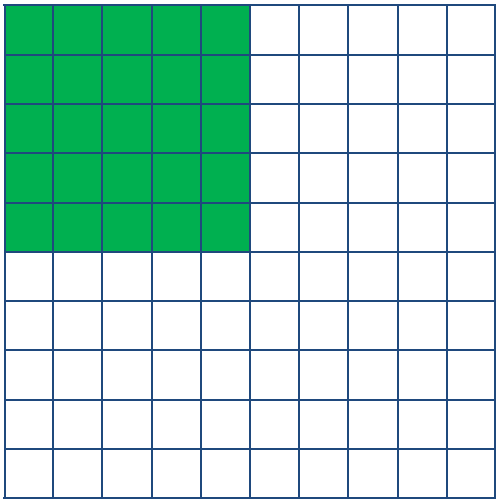
And $\frac{2n}{3n}$ for $\frac{2}{3}$ $\frac{6}{9}$ (2 x 3) (3 x 3)

What if n was 6? What if it was 4?

Equivalence (Including fractions, decimals and %)

$\frac{1}{4}$ is equivalent to $\frac{25}{100}$

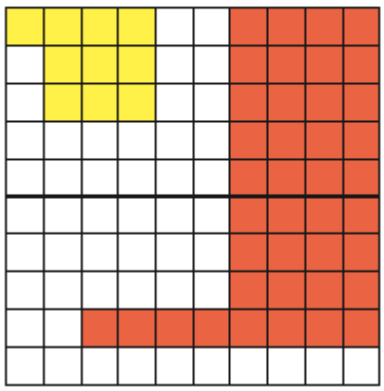
So therefore it is 0.25 as a decimal



Y6 Fractions

Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

Express the yellow section of the grid in hundredths, tenths, as a decimal and as a percentage of the whole grid.
Do the same for the red section.



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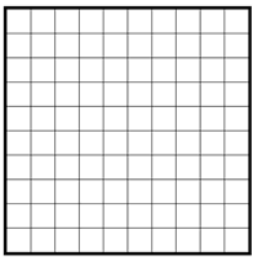
Show 3 tenths. Show 30 hundredths. What do you notice?

How could you write these as a decimal, percentage and fraction?



$\frac{3}{10} = \frac{30}{100}$

Ones	**Decimal point**	Tenths	hundredths
0	.	3	
0	.	3	0

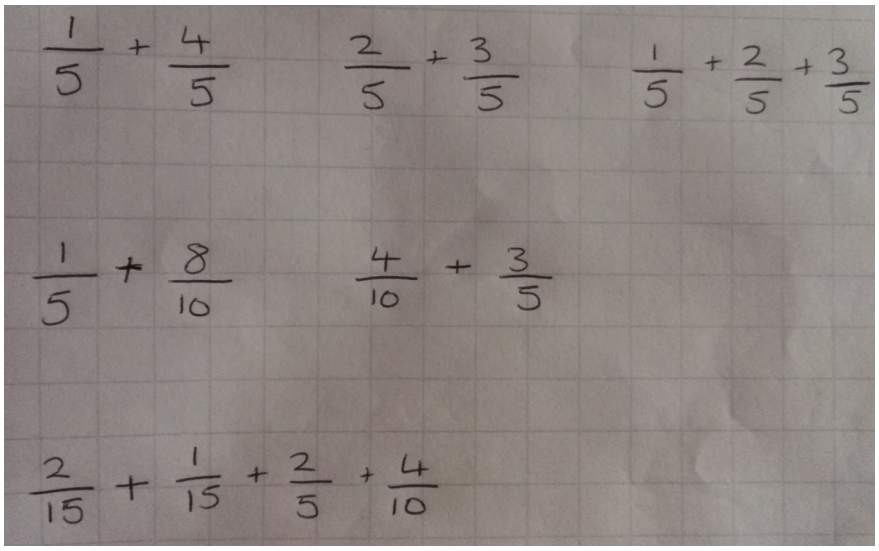
Addition and Subtraction of Fractions (including equivalence)

How many ways can you make 1 using addition of fractions?

Try with the same denominator.

Try different denominators that have denominators that are multiples of each other.

A range of different denominators.



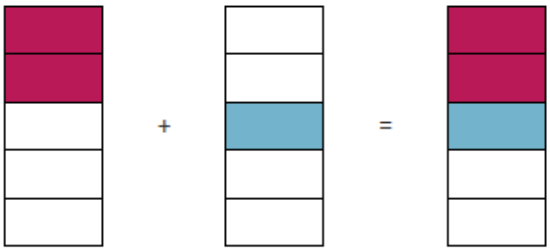
Now prove it– draw images

https://www.ncetm.org.uk/resources/43609

Addition and Subtraction of Fractions

Reasoning about Addition and Subtraction of Fractions

As the diagram shows, it is true that $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$



Fractions can be added or subtracted if they have the same denominator. See Equivalence also. This key idea needs to be secure first.

Calculating fractions, bar model and problems

See Hampshire Fraction card templates.

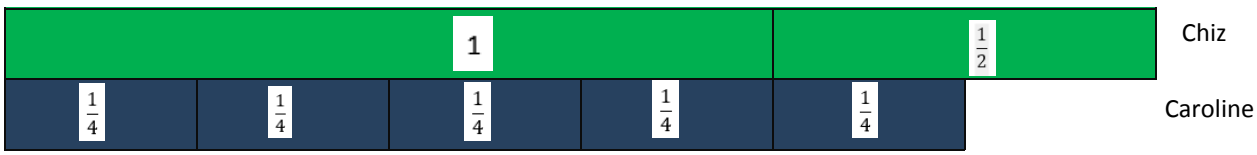
Chiz and Caroline each had two sandwiches of the same size.

Chiz ate $1\frac{1}{2}$ of his sandwiches.

Caroline ate $\frac{5}{4}$ of her sandwiches.

Draw diagrams to show how much Chiz and Caroline each ate.

Who ate more? How much more?



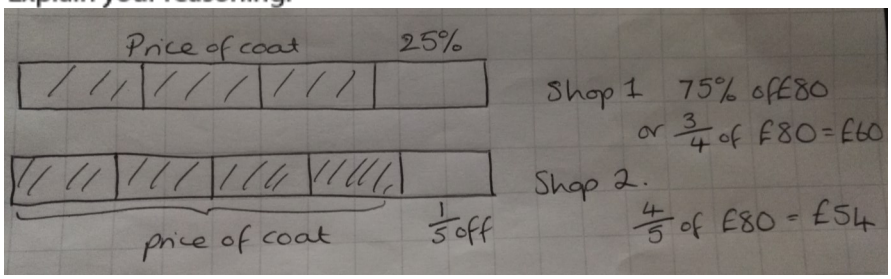
4/4 = 1 whole so 5/4 is more than 1. There are 4/4 = 1 whole and 2/4 = 1/2

Problem solving and calculating with Fractions, decimals and %

Krysia wanted to buy a coat that cost £80. She saw the coat on sale in one shop at $\frac{1}{5}$ off. She saw the same coat on sale in another shop at 25% off.

Which shop has the coat at a cheaper price?

Explain your reasoning.



Going further...
Show me other ways of working it out.

Andy's marbles– Nrich

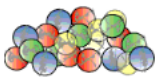
https://nrich.maths.org/2421

Unfortunately the bottom of the bag split and all the marbles spilled out. Poor Andy!

One third ($\frac{1}{3}$) of the marbles rolled down the slope too quickly for Andy to pick them up. One sixth ($\frac{1}{6}$) of all the marbles disappeared into the rain-water drain.

Andy and Sam picked up all they could but half ($\frac{1}{2}$) of the marbles that remained nearby were picked up by other children who ran off with them.

Andy counted all the marbles he and Sam had rescued.



He gave one third ($\frac{1}{3}$) of these to Sam for helping him pick them up. Andy put his remaining marbles into his pocket. There were 14 of them.

How many marbles were there in Andy's bag before the bottom split?

What fraction of the total number that had been in the bag had he lost or given away?

What do you know about factors? How does this help you find equivalents to help you calculate?

Y6 Ratio and Proportion

Solve problems involving the calculation of percentages.

Show me several different ways you can calculate:

1. 75% of 500cm

50% of 500cm = 250cm

25% of 500cm = 125cm

250+125cm= 325cm

So 75% = 375cm

10% of 500cm = 50cm

5 % of 500cm is 50cm ÷ 2= 25cm

10% x 7 = 70%

50cm x 7 = 350cm

350cm + 25cm = 375cm

Can you show me other ways?

Draw a model

Sam added two fractions together and got $\frac{7}{8}$ as the answer.
Write down two fractions that Sam could have added.

Tom wrote down two fractions. He subtracted the smaller fraction from the larger and got $\frac{1}{5}$ as the answer.
Write down two fractions that Tom could have subtracted.

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Multiplication and Division of fractions

https://www.ncetm.org.uk/self-evaluation/assess/893

Planning resource Tool NCETM Y5

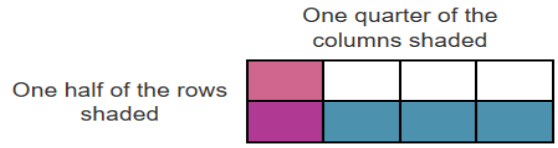
$3 \times \frac{1}{4}$

is three lots of one quarter. Therefore the answer must be three quarters.

Y5

Multiply proper fractions by whole numbers, supported by diagrams and materials.

Multiplying fractions can be thought of as finding the intersection of the representations of the fractions on a grid. For example, to work out $\frac{1}{2} \times \frac{1}{4}$, create a grid so that one half can be shaded in one direction, and one quarter in the other direction:



X = of

The intersection of the shaded sections demonstrates that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Y6

Multiply simple pairs of proper fractions, writing the answer in it's simplest form.

Divide proper fractions by whole numbers.

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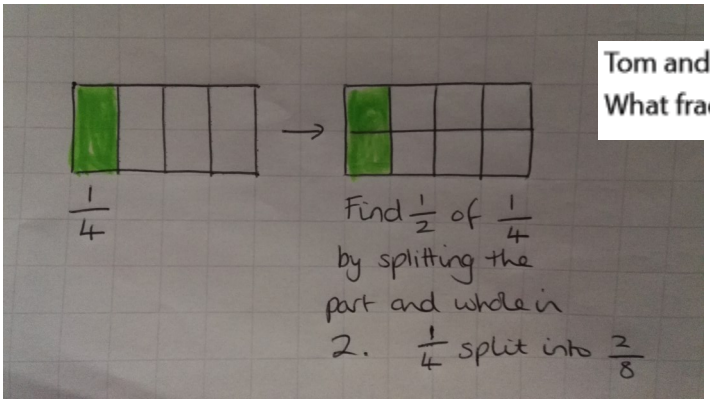
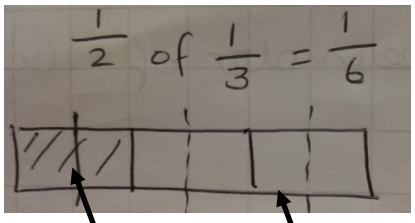


Tom and Sam shared equally one third of a chocolate bar.
What fraction of the chocolate bar did each child get?



1 third of chocolate bar to be split in

This is what the WHOLE would

Slick Jim

Slick Jim won the lottery.

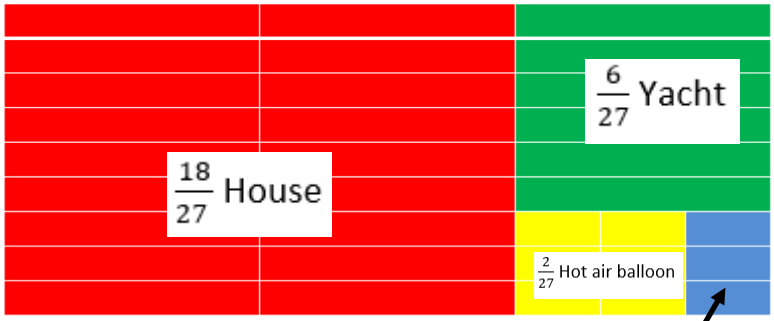
He spent two thirds of his winnings on a very posh house.

He spent two thirds of what he had left on a luxury yacht.

Then he spent two thirds of what he had left on a hot air balloon.

He spent his last £20000 on a flashy car.

How much did Slick Jim win on the lottery?



Can you show other ways to solve this?

From Mathematical challenges for more able pupils. NNS

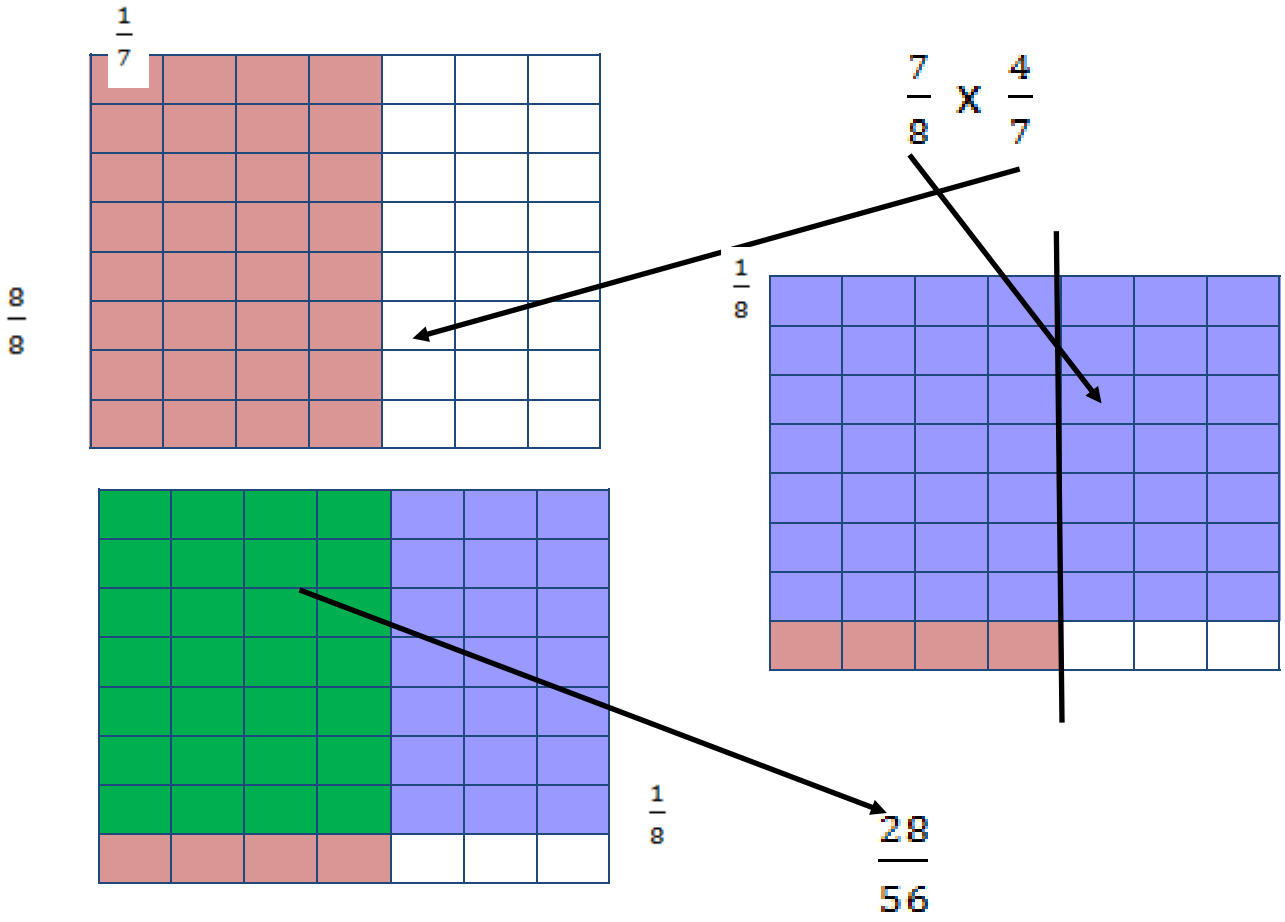
http://webarchive.nationalarchives.gov.uk/20110202093118/http:/nationalstrategies.standards.dcsf.gov.uk/node/85260

Multiplication and Division of fractions

$\frac{7}{7}$

Use the denominators to form your array.

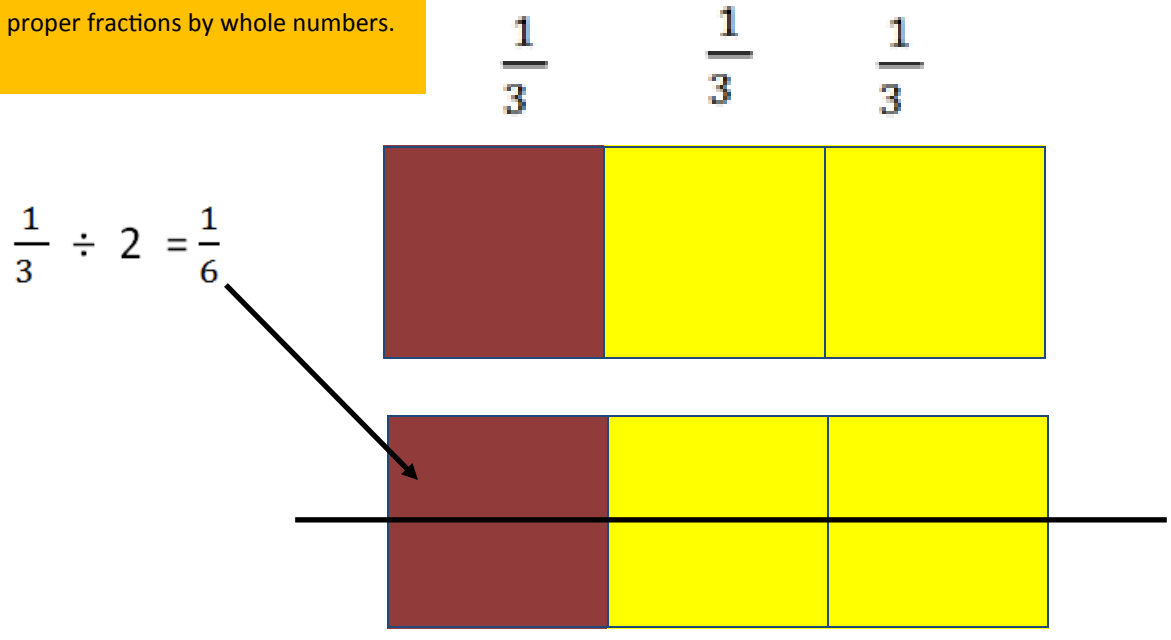
8 x 7 grid (56 squares)



Y6

Multiply simple pairs of proper fractions, writing the answer in it's simplest form.

Divide proper fractions by whole numbers.



Ratio and proportion

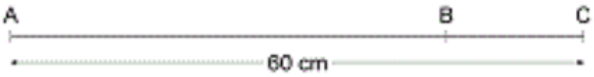
Solve simple problems involving direct proportion by scaling quantities up or down, for example:

Two rulers cost 80 pence. How much do three rulers cost?

Use the vocabulary of ratio and proportion to describe the relationships between two quantities solving problems such as:

Two letters have a total weight of 120 grams. One letter weighs twice as much as the other. Write the weight of the heavier letter.

The distance from A to B is three times as far as from B to C. The distance from A to C is 60 centimetres. Calculate the distance from A to B.

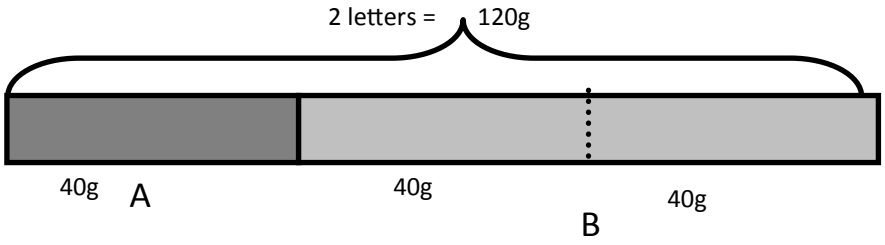


NCETM Planning Resource tool

https://www.ncetm.org.uk/

Y6

Solve problems involving similar shapes where the scale factor is known or can be found.



120g ÷ 3 = 40g

Letter A weighs 40g

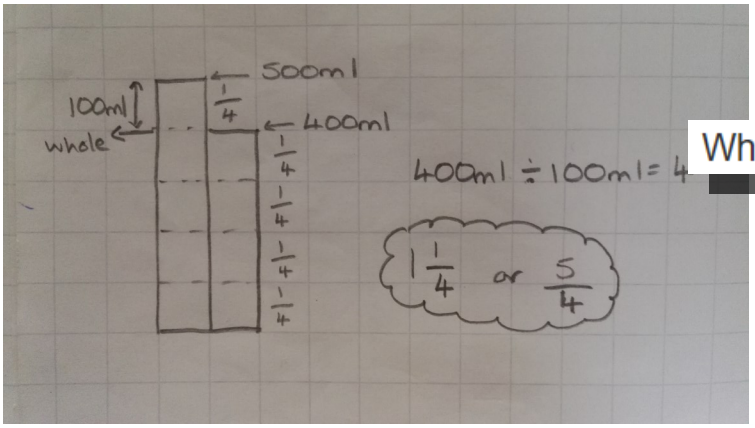
Letter B weighs 2 x 40g = 80g

Two letters have a total weight of 120 grams. One letter weighs twice as much as the other. Write the weight of the heavier letter.

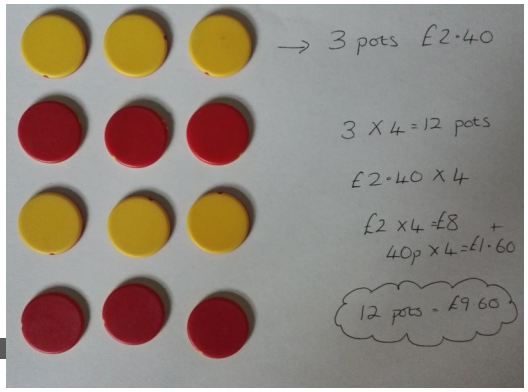
Y6 Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

https://www.ncetm.org.uk/resources/42671

NCETM Planning Resource Tool Y6



What fraction is 500ml of 400ml?



You can buy 3 pots of banana yoghurt for £2·40.

How much will it cost to buy 12 pots of banana yoghurt?

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