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# Magic squares

## An opportunity to explore patterns and relationships through problem solving

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# Overview

## In this document

Problem solving comes in all shapes and sizes, not just ‘word problems’. All types of mathematical problems serve a useful purpose in mathematics teaching, but different types of problem will achieve different learning outcomes.
This resource explores the opportunities for the use of “Magic Squares”.]

## Points to consider when using this resource

Magic squares present an interesting opportunity for pupils to explore patterns and relationships and to enable some to generalise, whilst others are developing addition and subtraction strategies

# Magic Squares

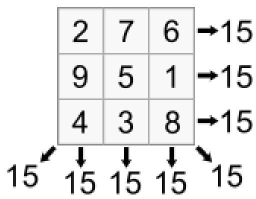

One of the three aims of the national curriculum for mathematics is to ensure that:

*‘All pupils can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.*

Problem solving comes in all shapes and sizes, not just ‘word problems’. All types of mathematical problems serve a useful purpose in mathematics teaching, but different types of problem will achieve different learning outcomes. Some problems require recall of facts and procedures, some stimulate different strategies, and some depend on logic and reasoning, some have multiple solutions and others demand decision making and creativity. In general, the more open-ended types of problems have greater potential for stimulating higher order mathematical thinking. This is partly because they usually involve a search for patterns and relationships between elements in the problem. Magic squares present an interesting opportunity for pupils to explore patterns and relationships and to enable some to generalise, whilst others are developing addition and subtraction strategies.

A magic square is an arrangement of distinct numbers (i.e. each number is used once), in a square grid, where the numbers in each row, and in each column, and the numbers in the diagonals, all add up to the same number. A magic square has the same number of rows as it has columns.

This example uses the numbers 1 – 9 and has 3 rows and 3 columns and is a magic square of *order 3*. Every row and column add up to 15. In addition to this, both diagonals also sum to 15. This is the *magic number*.




Any magic square can be rotated and reflected to produce 8 distinct squares. We would consider the eight squares to be equivalent since the same sets of three numbers are being added to make 15 each time.

Magic squares have a long history, dating back to 650 BC in China. At various times they have acquired magical or mythical significance, and have appeared as symbols in works of art. In modern times they have been generalized a number of ways, including using extra or different constraints, multiplying instead of adding cells, using alternate shapes or more than two dimensions, and replacing numbers with shapes and addition with geometric operations

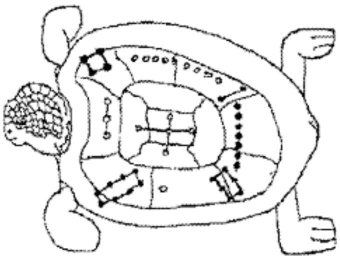
Magic squares were known to Chinese mathematicians as early as 650 BC, and to Islamic mathematicians possibly as early as the seventh century AD

## Lo Shu square (3×3 magic square)

The legend of Lo Shu dates from around 650 BC. The legends say that there was at one time in ancient China a huge flood. While the great king Yu was trying to channel water out to sea, a turtle emerged from it with a strange pattern on its shell. The pattern was a 3x3 grid in which circular dots of numbers were arranged, such that the sum of the numbers in each row, column and diagonal was the same: 15, which is also the number of days in each of the 24 cycles of the Chinese solar year.

According to the legend, thereafter people were able to use this pattern in a certain way to control the river and protect themselves from floods.

The Lo Shu square, as the magic square on the turtle shell is called, is the unique normal magic square of order three in which 1 is at the bottom and 2 is in the upper right corner. Every normal magic square of order three is obtained from the Lo Shu by rotation or reflection.



Not all magic squares use the numbers 1-9. Here is an example that uses the numbers 5-13, with a magic number of 27

| | | |
|---|---|---|
| 12 | 7 | 8 |
| 5 | 9 | 13 |
| 10 | 11 | 6 |

This one is a 4 x 4 magic square. The magic number in this case is 34. All rows, columns and main diagonals add up to 34. Also, each block of four squares sum to 34.

| | | | |
|---|---|---|---|
| 9 | 6 | 3 | 16 |
| 4 | 15 | 10 | 5 |
| 14 | 1 | 8 | 11 |
| 7 | 12 | 13 | 2 |

Magic squares are a great way to encourage problem solving with access for all pupils

Have a look at these magic squares of order 3, using different numbers. Check them.

Are they magic?
Have you noticed that the magic number is 3 x the middle number yet?

| | | |
|---|---|---|
| 10 | 2 | 9 |
| 6 | 7 | 8 |
| 5 | 12 | 4 |

| | | |
|---|---|---|
| 11 | 3 | 10 |
| 7 | 8 | 9 |
| 6 | 13 | 5 |

| | | |
|---|---|---|
| 12 | 4 | 11 |
| 8 | 9 | 10 |
| 7 | 14 | 6 |

When we want pupils to generalise, we may need to model and scaffold some of the main ideas first.
To begin with, ask pupils to check complete squares and find the magic number as above. Then ask them to fill in some gaps for incomplete squares such as the square below.

| | | |
|---|---|---|
| 13 | | 12 |
| | 10 | |
| 8 | | 7 |

For the pupils who are ready to look much more deeply into the problem, asking the right questions is the key to unlocking their understanding of the general square so that they can construct their own.

What is the same and what is different about these complete squares? What if I change the centre number to 7?
Can you show me an example of a magic square with a 7 in the middle …and another…and another…?
If I know the middle number is 7, then what else do I know? What if I change the middle number to another number?
Can I use any number in the middle?
Could it be negative, fractional, greater than 100?

| | | |
|---|---|---|
| 7+3 | | 7+2 |
| | 7 | |
| 7-2 | | 7-3 |

Magic squares come in all sizes. For primary pupils, start with the 3 x 3 version and consider how to help pupils to develop an understanding of the structure of the problem. As with all mathematical tasks, we need to rehearse the mathematics before we teach it. Pupils (and teachers!) will only achieve the mastery of such a problem is they are able to construct and de-construct it. I hope the diagram below will help you in making your own magic squares for the classroom.

| | | |
|---|---|---|
| $x+p$ | | $x+n$ |
| | $x$ | |
| $x-n$ | | $x-p$ |

Happy problem solving and generalising!

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